1. $\boxed{8}$ $\sum\limits_{n=1}^{\infty} \underbrace{\frac{(-9)^n}{\sqrt[5]{n}}}_{a_n} \cdot x^n$; $\sqrt[n]{|a_n|} = \frac{9}{\sqrt[5]{\sqrt[n]{n}}} \longrightarrow 9 \Rightarrow \underline{R = \frac{1}{9}}$ (3)

Végpontok: $x_1 = +\frac{1}{9}$ esetén

$\sum\limits_{n=1}^{\infty} \frac{(-9)^n}{\sqrt[5]{n}} \cdot 9^{-n} = \sum\limits_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[5]{n}}$ konvergens, mert Leibniz, (1) de nem abszolút konvergens ($\frac{1}{5} \not> 1$)

$x_2 = -\frac{1}{9}$: $\sum\limits_{n=1}^{\infty} \frac{(-9)^n}{\sqrt[5]{n}} \cdot (-9)^{-n} = \sum\limits_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}} = \infty$ ($\frac{1}{5} \not> 1$) (1)

Tehát K.T. $= \underline{\underline{\left(-\frac{1}{9}, +\frac{1}{9}\right]}}$ (1), és azon abszolút konv. tart.: $\underline{\underline{\left(-\frac{1}{9}, +\frac{1}{9}\right)}}$ (2)

---

2. a) $\boxed{4}$ $f(x) = x\, e^{-x^2} = \underline{\underline{\sum\limits_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n+1}}}$ (3); $\underline{\underline{K.T. = \mathbb{R}.}}$ (1)

b) $\boxed{4}$ $I \approx \int_0^{0.2} T_4(x)\,dx = \int_0^{0.2} (x - x^3)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^{0.2} = \underline{\underline{2 \cdot 10^{-2} - 4 \cdot 10^{-4}}}$

c) $\boxed{2}$ $I = \int_0^{0.2} \left(\sum\limits_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{2n+1}\right) dx = \sum\limits_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \int_0^{0.2} x^{2n+1}\,dx =$

$= \sum\limits_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[\frac{x^{2n+2}}{2n+2}\right]_0^{0.2} = \sum\limits_{n=0}^{\infty} \underbrace{\frac{(-1)^n}{n!} \cdot \frac{(0.2)^{2n+2}}{2n+2}}_{b_n}$ Leibniz-sor, tehát

$\left|I - \int_0^{0.2} T_4(x)\,dx\right| \le |b_2| = \underline{\underline{\frac{0.2^6}{2 \cdot 6} = \frac{16}{3} \cdot 10^{-6}}}$

3, a, $\boxed{4}$ $f(x) = \dfrac{1}{\sqrt[3]{27-x^2}} = (27-x^2)^{-1/3} = 27^{-1/3}\left(1-\dfrac{x^2}{27}\right)^{-1/3} = \dfrac{1}{3}\left(1-\dfrac{x^2}{27}\right)^{-1/3} =$

$$= \frac{1}{3}\sum_{n=0}^{\infty}\binom{-1/3}{n}\left(\frac{-1}{27}\right)^{n} x^{2n}, \text{ ha } \left|-\frac{x^2}{27}\right| < 1$$

b, $\boxed{2}$ $\left|-\dfrac{x^2}{27}\right| < 1 \iff |x| < \sqrt{27} = R$

c, $\boxed{3}$ $f^{(6)}(0) = 6!\cdot a_6 = 6!\cdot\dfrac{1}{3}\cdot\left(\dfrac{-1}{27}\right)^3\cdot\binom{-1/3}{3} = \dfrac{6\cdot5\cdot4\cdot3\cdot2}{3}\cdot\dfrac{-1}{27^3}\cdot\dfrac{(-1/3)(-4/3)(-7/3)}{3\cdot2\cdot1}$

$2n=6;\ n=3$

4, a, $\boxed{4}$ $\lim\limits_{y\to0} f(0,y) = \lim\limits_{y\to0} 0 = 0;\ \lim\limits_{x\to0} f(x,0) = \lim\limits_{x\to0}\dfrac{10x}{x^2} = \nexists$

Tehát $\nexists \lim\limits_{(x,y)\to(0,0)} f(x,y)$.

b, $\boxed{4}$ $\lim\limits_{(x,y)\to(0,0)} g(x,y) = \lim\limits_{\rho\to0}\dfrac{3\rho^3\cos\varphi\sin^2\varphi}{\rho^2(\cos^2\varphi+2\sin^2\varphi)} = \lim\limits_{\rho\to0}\underset{\downarrow\atop 0}{3\rho}\ \underbrace{\dfrac{\cos\varphi\sin^2\varphi}{1+\sin^2\varphi}}_{\text{korlátos}} = 0$

5, a, $\boxed{10}$ ha $(x,y)\neq(0,0)$

$f'_x(x,y) = \dfrac{y^2(x^2+y^2)-(x-1)y^2\cdot2x}{(x^2+y^2)^2}$ 3 ; $f'_y(x,y) = \dfrac{2(x-1)y(x^2+y^2)-2(x-1)y^3}{(x^2+y^2)^2}$ 3

ha $(x,y)=(0,0)$

$f'_x(0,0) = \lim\limits_{x\to0}\dfrac{1}{x}\big(\underbrace{f(x,0)}_{0}-\underbrace{f(0,0)}_{0}\big) = 0$ 2 ; $f'_y(0,0) = \lim\limits_{y\to0}\dfrac{1}{y}\big(\underbrace{f(0,y)}_{-\frac{y^2}{y^2}=-1}-\underbrace{f(0,0)}_{0}\big) = \nexists$ 2

$\boxed{2}$ b, (f tot. diff.-ható $\mathbb{R}^2\setminus\{(0,0)\}$-on, mert itt a parc. deriváltak léteznek és folytonosak.) $\nexists$ grad $f(0,0)$, mert $\nexists\, f'_y(0,0)$ 2

c, $\boxed{3}$ $\operatorname{grad} f(2,1) = \begin{bmatrix}\dfrac{5-4}{25}\\[2mm]\dfrac{10-2}{25}\end{bmatrix} = \begin{bmatrix}1/25\\8/25\end{bmatrix}$

$$\frac{df}{d\underline{e}} = \langle \operatorname{grad} f(2,1),\ \underline{e}\rangle = \frac{1}{25\cdot\sqrt5}\big(1\cdot2+8\cdot(-1)\big) = \frac{-6}{25\cdot\sqrt5}$$