hiányzás: Bóra

2009. 03. 05.

m-DOF, $m < 6$ (hiányzó szabadsági fokok)

1.) Választunk m feltételt → ezt precízen betartjuk

$RR \rightarrow v_{x0}, v_{y0} \leftrightarrow \dot q_1, \dot q_2$

RRTR
(SCARA) → $v_{x0}, v_{y0}, v_{z0}, w_{z0} \leftrightarrow \dot q_1, \dot q_2, \dot q_3, \dot q_4$

2.) Ha nincs értelmes altér amelyben mozog a robot, akkor minimalizáljuk az egyenlethibák négyzetösszegét (L.S)
Ilyenkor nincs megoldás.

Séma: $\bar{\bar{A}}_{n\times m}$, $n > m$, $\bar x \in \mathbb{R}^m$, $\bar y \in \mathbb{R}^n \rightarrow \| \bar{\bar{A}}\bar x - \bar y \|^2 \rightarrow \min$

$$F = \langle \bar{\bar{A}}\bar x - \bar y, \bar{\bar{A}}\bar x - \bar y \rangle = \langle \bar{\bar{A}}\bar x, \bar{\bar{A}}\bar x \rangle - 2\langle \bar{\bar{A}}\bar x, \bar y \rangle + \langle \bar y, \bar y \rangle =$$

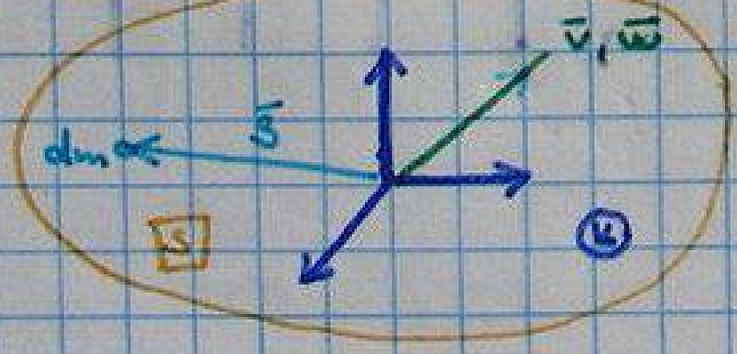
$$= \langle \bar{\bar{A}}^T \bar{\bar{A}}\bar x, \bar x \rangle - 2\langle \bar x, \bar{\bar{A}}^T \bar y \rangle + \langle \bar y, \bar y \rangle$$

$$F'_{\bar x} = \bar 0 = \cancel{2}\bar{\bar{A}}^T \bar{\bar{A}}\bar x - \cancel{2}\bar{\bar{A}}^T \bar y \Rightarrow \bar x = (\bar{\bar{A}}^T \bar{\bar{A}})^{-1} \bar{\bar{A}}^T \bar y$$

$$\begin{pmatrix} \bar v_m \\ \bar \omega_m \end{pmatrix} = \bar{\bar{J}}_m \dot{\bar q} \Rightarrow \dot{\bar q} = (\bar{\bar{J}}_m^T \bar{\bar{J}}_m)^{-1} \bar{\bar{J}}_m^T \begin{pmatrix} \bar v_m \\ \bar \omega_m \end{pmatrix}$$

$$\boxed{\bar{\bar{J}}_m^T} \; \boxed{\bar{\bar{J}}_m} = \boxed{\bar{\bar{J}}_m^T \bar{\bar{J}}_m}$$

$K = \frac{1}{2} m v^2$



$\bar v_s = \bar v + \bar\omega \times \bar s$

$K = \int_{S} \frac{1}{2} \bar v_s^2 \, dm$

$$\bar v_s^2 = \langle \bar v + \bar\omega \times \bar s, \bar v + \bar\omega \times \bar s \rangle = \langle \bar v, \bar v \rangle + \underbrace{2\langle \bar v, \bar\omega \times \bar s \rangle}_{2\langle \bar s, \bar v \times \bar\omega \rangle} + \langle \bar\omega \times \bar s, \underbrace{\bar\omega \times \bar s}_{-\bar s \times \bar\omega} \rangle$$

$\langle \bar s \times \bar\omega, \bar s \times \bar\omega \rangle$; $[\bar s \times]^T$, $[\bar s \times]$

$\Downarrow$

$\langle [\bar s \times]^T [\bar s \times] \bar\omega, \bar\omega \rangle$

$$K = \frac{1}{2}\langle \bar v, \bar v \rangle \int dm + \langle \int \bar s \, dm, \bar v \times \bar\omega \rangle +$$

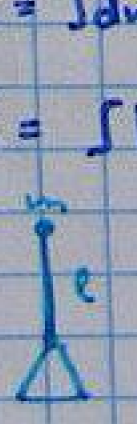
$$+ \frac{1}{2}\langle \int [\bar s \times]^T [s \times] \, dm \, \bar\omega, \bar\omega \rangle$$

$m = \int dm$, $\quad m\bar s_c = \int \bar s \, dm$

$\bar{\bar{K}} = \int [s\times]^T [s\times] \, dm$

$$\begin{bmatrix} 0 & s_z & -s_y \\ -s_z & 0 & s_x \\ s_y & -s_x & 0 \end{bmatrix} \begin{bmatrix} 0 & -s_z & s_y \\ s_z & 0 & s_x \\ -s_y & s_x & 0 \end{bmatrix} = \begin{bmatrix} s_y^2 + s_z^2 & -s_x s_y & -s_x s_z \\ -s_x s_y & s_x^2 + s_z^2 & -s_y s_z \\ -s_x s_z & -s_y s_z & s_x^2 + s_y^2 \end{bmatrix}$$

identitás mátrix Σ



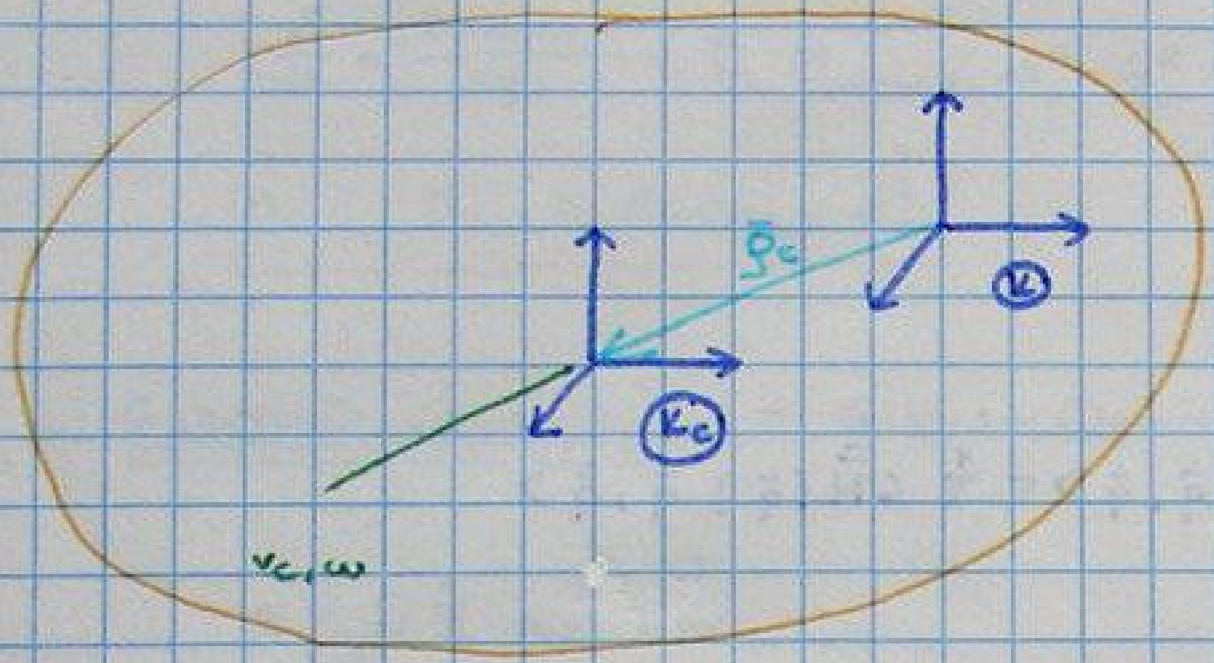
$\Theta = ml^2$

$$\bar{\bar{K}} = \int [\bar{s}\times]^T [\bar{s}\times]\, dm = \begin{bmatrix} K_x & K_{xy} & K_{xz} \\ * & K_y & K_{yz} \\ * & * & K_z \end{bmatrix} \qquad K_z = \int (s_x^2 + s_y^2)\, dm \qquad K_{xz} = -\int s_x s_z\, dm$$

*: értéke az átlós párja

kinetikus energia: $K = \frac{1}{2} \langle \bar{v}, \bar{v} \rangle m + \langle m\bar{s}_c, \bar{v} \times \bar{\omega} \rangle + \frac{1}{2} \langle \bar{\bar{K}} \bar{\omega}, \bar{\omega} \rangle$

tömegközéppont



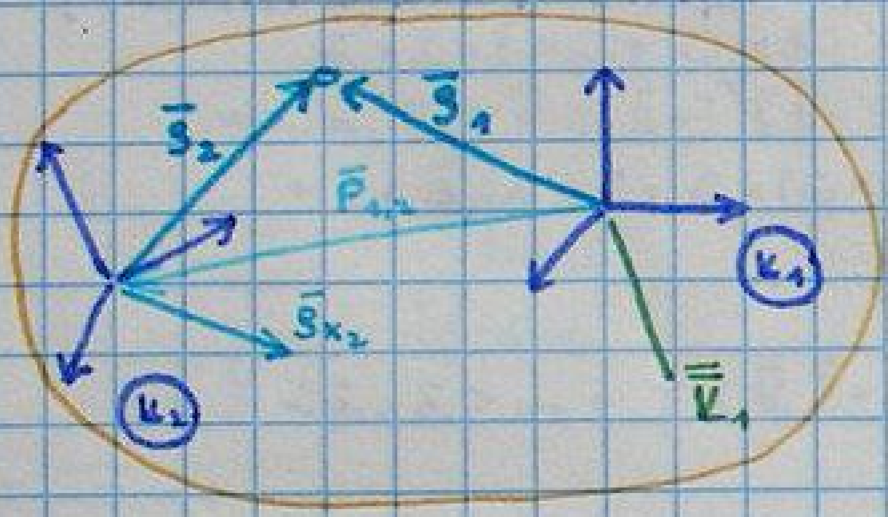
$$K = \frac{1}{2} \langle \bar{v}_c, \bar{v}_c \rangle m + \frac{1}{2} \langle \bar{\bar{K}}_c \bar{\omega}, \bar{\omega} \rangle$$

$$\bar{v} = \bar{\bar{\Omega}} \dot{\bar{q}} \qquad \bar{v}_c = \bar{v} + \bar{\omega} \times \bar{s}_c = \bar{v} - \bar{s}_c \times \bar{\omega} = \bar{\bar{\Omega}}_c \dot{\bar{q}}$$

$$\bar{\omega} = \bar{\bar{\Gamma}} \dot{\bar{q}} \qquad \bar{\bar{\Omega}}_c = \bar{\bar{\Omega}} - [\bar{s}_c \times] \bar{\bar{\Gamma}}$$

$$\bar{a}_c = \bar{a} + \bar{\varepsilon} \times \bar{s}_c + \bar{\omega} \times (\bar{\omega} \times \bar{s}_c)$$

$$\left.\begin{aligned} \bar{a} &= \bar{\bar{\Omega}} \ddot{\bar{q}} + \bar{\Theta} \\ \bar{\varepsilon} &= \bar{\bar{\Gamma}} \ddot{\bar{q}} + \bar{\Phi} \end{aligned}\right\} \bar{a}_c = \bar{\bar{\Omega}}_c \ddot{\bar{q}} + \bar{\Theta}_c, \quad \bar{\Theta}_c = \Theta + \bar{\Phi} \times \bar{s}_c + \bar{\omega} \times (\bar{\omega} \times \bar{s}_c)$$

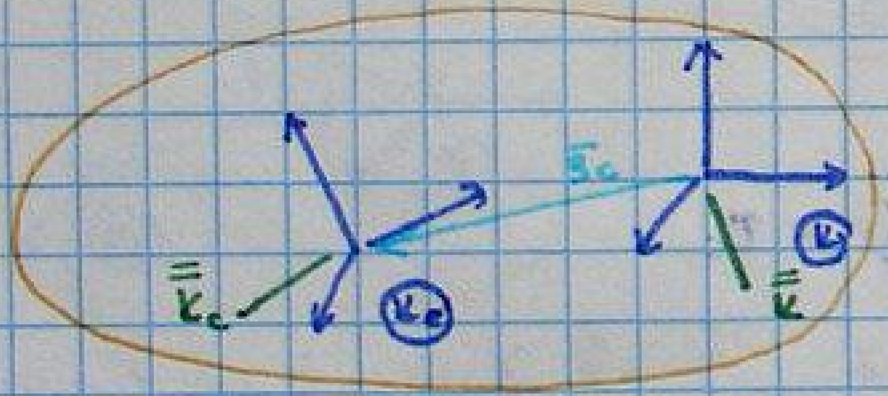


$$\bar{s}_1 = \bar{\rho}_{12} + \bar{s}_2$$

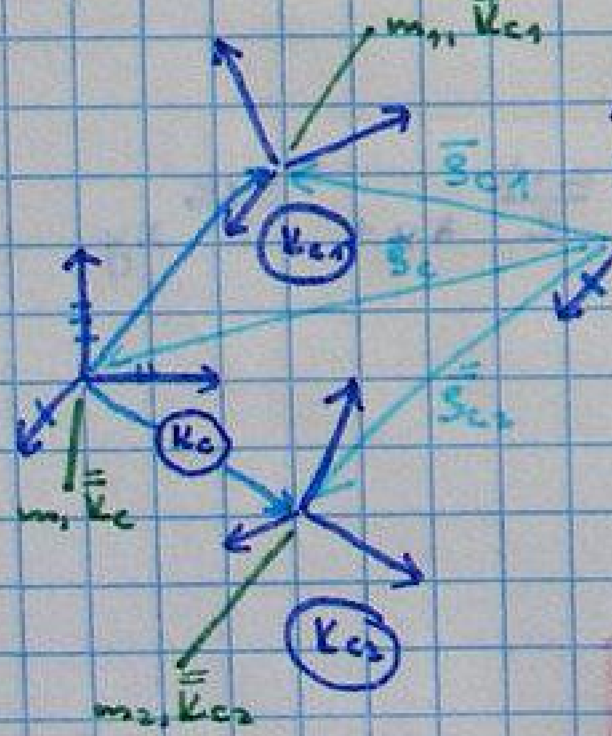
Huygens-elv:

$$\bar{\bar{K}}_1 = \bar{\bar{A}}_{12} \bar{\bar{K}}_2 \bar{\bar{A}}_{12}^T - [(\bar{\bar{A}}_{12} m \bar{\rho}_{2c})\times][\bar{\rho}_{12}\times] - [\bar{\rho}_{12}\times][(\bar{\bar{A}}_{12} m \bar{\rho}_{2c})\times] - m [\bar{\rho}_{12}\times][\bar{\rho}_{12}\times]$$

$$\bar{a} \times (\bar{b} \times \bar{c}) = \bar{b} \langle \bar{a}, \bar{c} \rangle - \bar{c} \langle \bar{a}, \bar{b} \rangle$$



$$\bar{\bar{K}}_c = \bar{\bar{A}}_{K,K_c} \bar{\bar{K}} \bar{\bar{A}}_{K,K_c}^T + m \|\bar{s}_c\|^2 \bar{\bar{I}} - m [\bar{s}_c \circ \bar{s}_c]$$

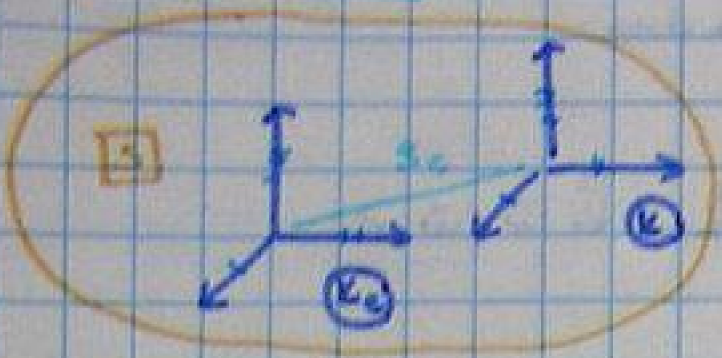


$$m = m_1 + m_2$$

$$m\bar{s}_c = m_1 \bar{s}_{c1} + m_2 \bar{s}_{c2} \Rightarrow \bar{s}_c = \frac{m_1 \bar{s}_{c1} + m_2 \bar{s}_{c2}}{m_1 + m_2}$$

$$\bar{\bar{K}}_c = \bar{\bar{A}}_{K,K_{c1}} \bar{\bar{K}}_{c1} \bar{\bar{A}}_{K,K_{c1}}^T + m_1 \|\bar{s}_{c1} - \bar{s}_{c2}\|^2 \bar{\bar{I}} - m_1 [(\bar{s}_{c1} - \bar{s}_c) \circ (\bar{s}_{c1} - \bar{s}_c)] + \bar{\bar{A}}_{K,K_{c2}} \bar{\bar{K}}_{c2} \bar{\bar{A}}_{K,K_{c2}}^T + m_2 \|\bar{s}_{c2} - \bar{s}_{c1}\|^2 \bar{\bar{I}} - m_2 [(\bar{s}_{c2} - \bar{s}_c) \circ (\bar{s}_{c2} - \bar{s}_c)]$$

1 merev test:



$$K = \frac{1}{2}\langle \bar{v}_c, \bar{v}_c\rangle m + \frac{1}{2}\langle \bar{\bar{K}}_c \bar{\omega}, \bar{\omega}\rangle = \frac{1}{2}\langle \bar{\bar{\Omega}}_c \dot{\bar{q}}, \bar{\bar{\Omega}}_c \dot{\bar{q}}\rangle m + \frac{1}{2}\langle \bar{\bar{K}}_c \bar{\bar{\Gamma}} \dot{\bar{q}}, \bar{\bar{\Gamma}} \dot{\bar{q}}\rangle$$

$$K = \frac{1}{2}\langle (\bar{\bar{\Omega}}_c^T \bar{\bar{\Omega}}_c m + \bar{\bar{\Gamma}}^T \bar{\bar{K}}_c \bar{\bar{\Gamma}}) \dot{\bar{q}}, \dot{\bar{q}}\rangle$$

$$\bar{v}_c = \bar{\bar{\Omega}}_c \dot{\bar{q}}$$
$$\bar{\omega} = \bar{\bar{\Gamma}} \dot{\bar{q}}$$
$$\bar{\bar{\Omega}}_c = \bar{\bar{\Omega}} - [s_c \times]\bar{\bar{\Gamma}}$$

m-DOF robot:

$$K = \frac{1}{2}\langle \underbrace{\sum_{s=1}^{m} \{\Omega_{cs}^T \Omega_{cs} m_s + \bar{\bar{\Gamma}}_s^T \bar{\bar{K}}_{cs} \bar{\bar{\Gamma}}_s\}}_{\bar{\bar{H}}(\bar{q})} \dot{\bar{q}}, \dot{\bar{q}}\rangle = \frac{1}{2}\langle \bar{\bar{H}}(\bar{q}) \dot{\bar{q}}, \dot{\bar{q}}\rangle$$

$\bar{\bar{H}}(\bar{q}) = [D_{jk}(\bar{q})]_{m \times m}$ $\quad \bar{\bar{H}}$ szimmetrikus és pozitív definit ($\Rightarrow$ invertálható)

$$K = \frac{1}{2}\sum_j \sum_k D_{jk}(\bar{q})\, \dot{q}_j \dot{q}_k$$

Lagrange-egyenletek

$L = K - P$

- P → potenciális energia
- K → kinetikus energia

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad \text{vagy} \quad \frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} + \frac{\partial P}{\partial q_i} = \tau_i$$

$$K = \frac{1}{2}\sum_j \sum_k D_{jk}\, \dot{q}_j \dot{q}_k$$

$$\frac{\partial K}{\partial \dot{q}_i} = \frac{1}{2}\sum_k D_{ik}\dot{q}_k + \frac{1}{2}\sum_j D_{ji}\dot{q}_j = \sum_j D_{ij}\dot{q}_j$$

$$\frac{d}{dt}\frac{\partial K}{\partial \dot{q}_i} = \sum_j D_{ij}\ddot{q}_j + \sum_j\sum_k \frac{\partial D_{ij}}{\partial q_k}\dot{q}_j\dot{q}_k = \qquad axy + bxy = \frac{a+b}{2}xy + \frac{a+b}{2}xy$$

$$= \sum_j D_{ij}\ddot{q}_j + \underbrace{\sum_j\sum_k \frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j}\right)\dot{q}_j\dot{q}_k}_{\text{szimmetrizált}}$$

$$\frac{\partial K}{\partial q_i} = \frac{1}{2}\sum_j\sum_k \frac{\partial D_{jk}}{\partial q_i}$$

$$\frac{\partial P}{\partial q_i} =: D_i \qquad \Rightarrow \qquad \boxed{\sum_j D_{ij}\ddot{q}_j + \sum_j\sum_k D_{ijk}\dot{q}_j\dot{q}_k + D_i = \tau}$$

$$\bar{\bar{H}}(\bar{q}) = [D_{jk}(\bar{q})], \qquad D_{ijk} = \frac{1}{2}\left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j} - \frac{\partial D_{jk}}{\partial q_i}\right)$$

Hatások!
- D_{ii} – effektív inercia
- D_{ij}, $i \neq j$ – csatoló inercia
- D_{ijj} – centripetális hatás
- D_{ijk}, $j \neq k$ – Coriolis hatás
- D_i – gravitációs hatás