2009.03.12.

TMK-ra:

$$ {}^{0}\bar{v}_1 = \begin{bmatrix} 0 \\ l_{c1} \\ 0 \end{bmatrix} \dot{q}_1 = \bar{\bar{\Omega}}_{c1} \cdot \dot{\bar{q}} \quad \text{és} \quad {}^{0}\bar{v}_2 = \begin{bmatrix} l_1 S_2 & 0 \\ l_1 C_2 + l_{c2} & l_{c2} \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \bar{\bar{\Omega}}_{c2} \cdot \dot{\bar{q}} $$

$$ \bar{\bar{\Omega}}_{c1} = \bar{\bar{\Omega}}_1 - [\bar{s}_{c1} \times] \bar{\bar{\Gamma}}_1 ; \quad \bar{s}_{c1} = \begin{bmatrix} -(l_1 - l_{c1}) \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \bar{\bar{\Omega}}_{c1} = \begin{bmatrix} 0 \\ l_{c1} \\ 0 \end{bmatrix} $$

$$ \bar{\bar{\Omega}}_{c2} = \bar{\bar{\Omega}}_2 - [\bar{s}_{c2} \times] \bar{\bar{\Gamma}}_2 ; \quad \bar{s}_{c2} = \begin{bmatrix} -(l_2 - l_{c2}) \\ 0 \\ 0 \end{bmatrix} \leftrightarrow \bar{\bar{\Omega}}_{c2} = \begin{bmatrix} l_1 S_2 & 0 \\ l_1 C_2 + l_{c2} & l_{c2} \\ 0 & 0 \end{bmatrix} $$

Jakobinus:

$$ {}^{2}\bar{\bar{J}}_2 = \begin{bmatrix} {}^{2}\bar{d}_0 & {}^{2}\bar{d}_1 \\ {}^{2}\bar{t}_0 & {}^{2}\bar{t}_1 \end{bmatrix} = \begin{bmatrix} \bar{\bar{\Omega}}_2 \\ \bar{\bar{\Gamma}}_2 \end{bmatrix} \qquad {}^{2}\bar{\bar{J}}_0 = \begin{bmatrix} \bar{\bar{A}}_{02} & \bar{\bar{0}} \\ \bar{\bar{0}} & \bar{\bar{A}}_{02} \end{bmatrix} \cdot {}^{2}\bar{\bar{J}}_2 = \text{HF} $$

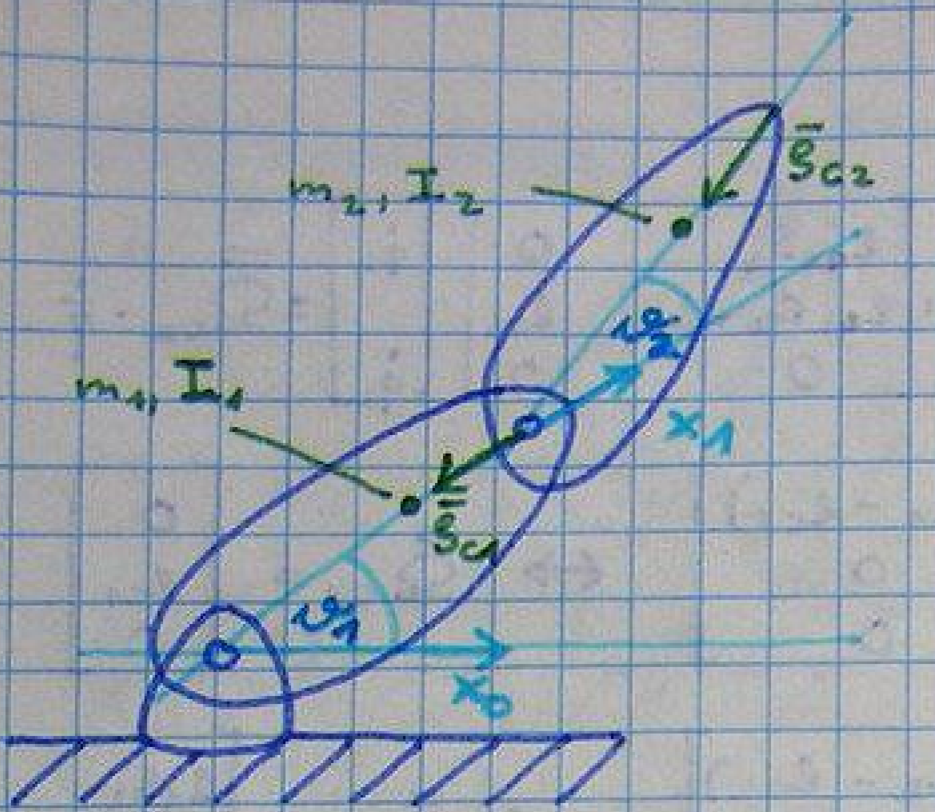
$$ {}^{2}\bar{\bar{J}}_2 = \begin{bmatrix} l_1 S_2 & 0 \\ l_1 C_2 + l_2 & l_2 \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad {}^{0}\bar{\bar{J}}_2 = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 S_{12} \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} \\ 0 & 0 \\ \hline 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} $$

$$ \begin{bmatrix} v_{2,x_0} \\ v_{2,y_0} \end{bmatrix} = \begin{bmatrix} -l_1 S_1 - l_2 S_{12} & -l_2 C_{12} \\ l_1 C_1 + l_2 C_{12} & l_2 C_{12} \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} $$

mj.: $$ \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\underbrace{ad - bc}_{\text{det.}}} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} $$

$$ \begin{vmatrix} v_{2,x_0} \\ v_{2,y_0} \end{vmatrix} = (-l_1 S_1 - l_2 S_{12}) l_2 C_{12} - (-l_2 C_{12})(l_1 C_1 + l_2 C_{12}) = l_1 l_2 S_2 $$

$$ \det = 0 \begin{cases} \vartheta_2 = 0^\circ \\ \vartheta_2 = 180^\circ \end{cases} $$



$$\overline{\overline{H}} = \begin{bmatrix} D_{11} & D_{12} \\ * & D_{22} \end{bmatrix} = \overline{\overline{\Omega}}_{c1}^{T} \cdot \overline{\overline{\Omega}}_{c1} m_1 + \overline{\overline{\Pi}}_1^{T} \overline{\overline{K}}_{c1} \overline{\overline{\Pi}}_1 + $$

$$+ \overline{\overline{\Omega}}_{c2}^{T} \cdot \overline{\overline{\Omega}}_{c2} m_2 + \overline{\overline{\Pi}}_2^{T} \overline{\overline{K}}_{c2} \overline{\overline{\Pi}}_2 =$$

$$= \begin{bmatrix} 0 & l_{c1} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ l_{c1} & 0 \\ 0 & 0 \end{bmatrix} m_1 + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & I_1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} l_1 S_2 & l_1 C_2 + l_{c2} & 0 \\ 0 & l_{c2} & 0 \end{bmatrix} \begin{bmatrix} l_1 S_2 & 0 \\ l_1 C_2 + l_{c2} & l_{c2} \\ 0 & 0 \end{bmatrix} m_2 + \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & I_2 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} l_{c1}^2 & 0 \\ 0 & 0 \end{bmatrix} m_1 + \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix} +$$

$$+ \begin{bmatrix} l_1 S_2^2 + l_1^2 C_2^2 + 2 l_1 C_2 l_{c2} + l_{c2}^2 & l_{c2}(l_1 C_2 + l_{c2}) \\ l_{c2}(l_1 C_2 + l_{c2}) & l_{c2}^2 \end{bmatrix} m_2 + \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

$$\Rightarrow \overline{\overline{H}} = \begin{bmatrix} l_{c1}^2 & 0 \\ 0 & 0 \end{bmatrix} m_1 + \begin{bmatrix} I_1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} C_2 & (l_1 C_2 + l_{c2}) l_{c2} \\ * & l_{c2}^2 \end{bmatrix} m_2 + \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

$$\boxed{\begin{aligned} D_{11} &= m_1 l_{c1}^2 + m_2 (l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} C_2) + I + I_2 \\ D_{12} &= m_2 (l_1 C_2 + l_{c2}) l_{c2} + I_2 = D_{21} \\ D_{22} &= m_2 l_{c2}^2 + I_2 \\ P &= m_1 g\, l_{c1} S_1 + m_2 g (l_1 S_1 + l_{c2} S_{12}) \\ D_{ijk} &= \frac{1}{2} \left(\frac{\partial D_{ij}}{\partial q_k} + \frac{\partial D_{ik}}{\partial q_j} + \frac{\partial D_{jk}}{\partial q_i} \right) \end{aligned}}$$

Elmélet (nyíltláncú): 1.) $D_{ijk} = -D_{kji}$, ha $i,k \geq j$

2.) $D_{iji} = 0$, ha $i \geq j$

$$\overline{\overline{D}}^{1} = \begin{bmatrix} D_{111} & D_{112} \\ * & D_{122} \end{bmatrix} = \begin{bmatrix} 0 & D_{112} \\ * & D_{122} \end{bmatrix}, \qquad \overline{\overline{D}}^{2} = \begin{bmatrix} D_{211} & D_{212} \\ * & D_{222} \end{bmatrix} = \begin{bmatrix} -D_{112} & 0 \\ * & 0 \end{bmatrix}$$

$$\underset{ijk}{D_{112}} = \frac{1}{2} \left(\frac{\partial D_{11}}{\partial q_2} + \frac{\partial D_{12}}{\partial q_1} - \frac{\partial D_{12}}{\partial q_1} \right) = -m_2 l_1 l_{c2} S_2$$

$$\underset{ijk}{D_{122}} = \frac{1}{2} \left(\frac{\partial D_{12}}{\partial q_2} + \frac{\partial D_{12}}{\partial q_2} - \frac{\partial D_{22}}{\partial q_1} \right) = -m_2 l_1 l_{c2} S_2$$

$$\Rightarrow \overline{\overline{D}}^{1} = \begin{bmatrix} 0 & D_{112} \\ D_{112} & D_{122} \end{bmatrix}, \qquad \overline{\overline{D}}^{2} = \begin{bmatrix} -D_{112} & 0 \\ 0 & 0 \end{bmatrix}$$

$$D_1 = \frac{\partial P}{\partial q_1} = m_1 g\, l_{c1} C_1 + m_2 g\,(l_1 C_1 + l_{c2} C_{12})$$

$$D_2 = \frac{\partial P}{\partial q_2} = m_2 g\, l_{c2} C_{12}$$

$$\boxed{\begin{aligned} &D_{11}\ddot{q}_1 + D_{12}\ddot{q}_2 + 2D_{112}\dot{q}_1\dot{q}_2 + D_{122}\dot{q}_2^2 + D_1 = \tau_1 \\ &D_{12}\ddot{q}_1 + D_{22}\ddot{q}_2 - D_{112}\dot{q}_1^2 + D_2 = \tau_2 \end{aligned}}$$

$$\bar{\bar{H}}\ddot{\bar{q}} + \bar{h} = \tau, \quad \ddot{\bar{q}} = \bar{\bar{H}}^{-1}(-\bar{h} + \bar{\tau}) = -\bar{\bar{H}}^{-1}\bar{h} + \bar{\bar{H}}^{-1}\overbrace{\bar{\tau}}^{\bar{u}}$$

$$\bar{x} = \begin{bmatrix} \bar{q} \\ \dot{\bar{q}} \end{bmatrix} \begin{matrix} \}\ \bar{x}_1 \\ \}\ \bar{x}_2 \end{matrix}, \quad \dot{\bar{x}} = \bar{f}(\bar{x}) + \bar{g}(x)\bar{u}$$

$$\boxed{\dot{\bar{x}}_1 = \bar{x}_2, \quad \dot{\bar{x}}_2 = -\bar{\bar{H}}^{-1}(\bar{x}_1)\bar{h}(\bar{x}_1, \bar{x}_2) + \bar{\bar{H}}^{-1}(x_1)\bar{u}}$$

Lagrange: $\sum_i D_{ij}(\bar{q})\ddot{q}_j + \sum_j\sum_k D_{ijk}(\bar{q})\dot{q}_j\dot{q}_k + D_i(\bar{q}) = \tau_i$

Appell: $\bar{\bar{H}}(\bar{q})\ddot{\bar{q}} + \bar{h}(\bar{q}, \dot{\bar{q}}) = \bar{\tau}$

⇒ 3. alak: $\bar{\bar{C}}(\bar{q}, \dot{\bar{q}}) = [c_{ik}], \quad c_{ik} = \sum_j D_{ijk}(\bar{q})\dot{q}_j \Rightarrow \bar{\bar{C}}(\bar{q}, \dot{\bar{q}})\dot{\bar{q}}$

$\bar{D}(\bar{q})$

$\bar{\bar{H}}(\bar{q})\ddot{\bar{q}} + \bar{\bar{C}}(\bar{q}, \dot{\bar{q}})\dot{\bar{q}} + \bar{D}(\bar{q}) = \bar{\tau}$

$\dot{\bar{\bar{H}}} - 2\bar{\bar{C}}$ antiszimmetrikus ⇒ bármely változóban nulla a kvadratikus alak

4. alak: $\bar{\alpha}$ független paraméterek

$$\bar{\bar{Y}}(\underbrace{\bar{q}, \dot{\bar{q}}}_{\substack{\text{állapotváltozók} \\ \bar{x}}}, \underbrace{\dot{\bar{q}}}_{\dot{\bar{q}}_r}, \underbrace{\ddot{\bar{q}}}_{\ddot{\bar{q}}_r})\bar{\alpha} = \bar{\tau}, \quad \bar{\alpha} = ?$$