
1 Дзугашвили-spline

Az a együtthatókat keressük.

$$f_i(t) = a_4(t - t_i)^4 + a_3(t - t_i)^3 + a_2(t - t_i)^2 + a_1(t - t_i) + a_0$$

Tudjuk, hogy

$$\begin{aligned} f_i(t_i) &= r_i & f_i(t_{i+1}) &= r_{i+1} \\ f'_i(t_i) &= v_i & f'_i(t_{i+1}) &= v_{i+1} \\ f''_i(t_i) &= a_i \end{aligned}$$

Nézzük az első oszlopot:

$$r_i = f_i(t_i) = a_4(t_i - t_i)^4 + a_3(t_i - t_i)^3 + a_2(t_i - t_i)^2 + a_1(t_i - t_i) + a_0$$

$$r_i = a_4 \cdot 0 + a_3 \cdot 0 + a_2 \cdot 0 + a_1 \cdot 0 + a_0$$

$$\boxed{r_i = a_0}$$

$$v_i = f'_i(t_i) = 4a_4(t_i - t_i)^3 + 3a_3(t_i - t_i)^2 + 2a_2(t_i - t_i) + a_1$$

$$v_i = a_4 \cdot 0 + a_3 \cdot 0 + a_2 \cdot 0 + a_1$$

$$\boxed{v_i = a_1}$$

$$a_i = f''_i(t_i) = 12a_4(t_i - t_i)^2 + 6a_3(t_i - t_i) + 2a_2$$

$$a_i = a_4 \cdot 0 + a_3 \cdot 0 + 2a_2$$

$$a_i = 2a_2$$

$$\boxed{a_2 = \frac{a_i}{2}}$$

Második oszlop:

$$\begin{aligned} r_{i+1} &= f_i(t_{i+1}) \\ &= a_4(t_{i+1} - t_i)^4 + a_3(t_{i+1} - t_i)^3 + a_2(t_{i+1} - t_i)^2 + a_1(t_{i+1} - t_i) + a_0 \end{aligned} \quad (1)$$

$$\begin{aligned} v_{i+1} &= f'_i(t_{i+1}) \\ &= 4a_4(t_{i+1} - t_i)^3 + 3a_3(t_{i+1} - t_i)^2 + 2a_2(t_{i+1} - t_i) + a_1 \end{aligned} \quad (2)$$

Leosztjuk (1)-t $(t_{i+1} - t_i)$ -vel (ezt megtehetjük, hiszen nem 0):

$$\frac{r_{i+1}}{t_{i+1} - t_i} = a_4(t_{i+1} - t_i)^3 + a_3(t_{i+1} - t_i)^2 + a_2(t_{i+1} - t_i) + a_1 + \frac{a_0}{t_{i+1} - t_i} \quad (3)$$

(2)-ből kivonunk 3(3)-t, átrendezzük a_4 -re:

$$\begin{aligned}
v_{i+1} - 3\frac{r_{i+1}}{t_{i+1} - t_i} &= 4a_4(t_{i+1} - t_i)^3 - 3a_4(t_{i+1} - t_i)^3 + 3a_3(t_{i+1} - t_i)^2 - 3a_3(t_{i+1} - t_i)^2 + \\
&\quad 2a_2(t_{i+1} - t_i) - 3a_2(t_{i+1} - t_i) + a_1 - 3a_1 - 3\frac{a_0}{t_{i+1} - t_i} \\
&= a_4(t_{i+1} - t_i)^3 - a_2(t_{i+1} - t_i) - 2a_1 - 3\frac{a_0}{t_{i+1} - t_i} \\
a_4 &= \frac{v_{i+1} - 3\frac{r_{i+1}}{t_{i+1} - t_i} + a_2(t_{i+1} - t_i) + 2a_1 + 3\frac{a_0}{t_{i+1} - t_i}}{(t_{i+1} - t_i)^3} \\
a_4 &= \frac{v_{i+1} - 3\frac{r_{i+1}}{t_{i+1} - t_i} + \frac{a_i(t_{i+1} - t_i)}{2} + 2v_i + 3\frac{r_i}{t_{i+1} - t_i}}{(t_{i+1} - t_i)^3} \\
\boxed{a_4} &= 3\frac{r_i - r_{i+1}}{(t_{i+1} - t_i)^4} + \frac{v_{i+1} + 2v_i}{(t_{i+1} - t_i)^3} + \frac{a_i}{2(t_{i+1} - t_i)^2}
\end{aligned}$$

Majd a_4 -t visszahelyettesítve (1)-be megkapjuk a_3 -t.

$$\begin{aligned}
r_{i+1} &= a_4(t_{i+1} - t_i)^4 + a_3(t_{i+1} - t_i)^3 + a_2(t_{i+1} - t_i)^2 + a_1(t_{i+1} - t_i) + a_0 \\
&= 3(r_i - r_{i+1}) + (v_{i+1} + 2v_i)(t_{i+1} - t_i) + \frac{a_i}{2}(t_{i+1} - t_i)^2 + \\
&\quad a_3(t_{i+1} - t_i)^3 + \frac{a_i}{2}(t_{i+1} - t_i)^2 + v_i(t_{i+1} - t_i) + r_i \\
a_3 &= \frac{-4(r_i - r_{i+1}) - (v_{i+1} + 3v_i)(t_{i+1} - t_i) - a_i(t_{i+1} - t_i)^2}{(t_{i+1} - t_i)^3} \\
\boxed{a_3} &= -4\frac{r_i - r_{i+1}}{(t_{i+1} - t_i)^3} - \frac{v_{i+1} + 3v_i}{(t_{i+1} - t_i)^2} - \frac{a_i}{t_{i+1} - t_i}
\end{aligned}$$