Jelek és Rendszerek 2 - Gyakorlat

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Periodikus áramú hálózatok:

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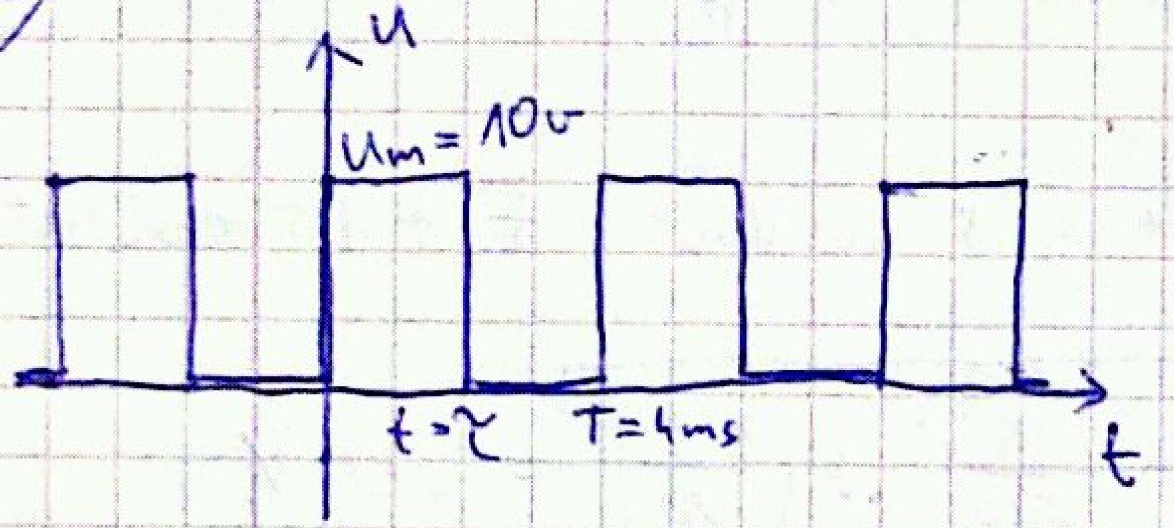

a, $\tau = ? \rightarrow U_{eff} = \frac{U_m}{2} = 5V$

b, Fourier - sor

c, hiba ?

a,

$$U_{eff} = \sqrt{\frac{1}{T}\int_0^{\tau} u^2(t)\,dt} = \sqrt{\frac{\tau}{T}} \cdot U_m = \frac{U_m}{2}$$

$$\tau = \frac{T}{4} = 1\,ms$$

b, $u(t) = U_0 + \sum_{k=1}^{\infty} \left(U_k^A \cdot \cos k\omega_0 t + U_k^B \cdot \sin k\omega_0 t \right)$

$$U_0 = \frac{1}{T}\int_0^T u(t)\,dt$$

$$U_k^A = \frac{2}{T}\int_0^T u(t) \cos k\omega_0 t\,dt$$

$$U_k^B = \frac{2}{T}\int_0^T u(t) \sin k\omega_0 t\,dt$$

$$\omega_0 = \frac{2\pi}{T}$$

$U_0 = \frac{U_m}{4} = 2{,}5\ V$

$U_k^A = \frac{2}{T} U_m \cdot \int_0^{T/4} \cos k\omega_0 t\, dt = \frac{2U_m}{T}\left[\frac{\sin k\omega_0 t}{k\omega_0}\right]_0^{T/4} =$

$= \frac{U_m}{k\pi} \cdot \sin\frac{k\pi}{2}$ $\quad k = 0 \dots \infty$

$U_k^B = \frac{U_m}{k\pi}\left(1 - \cos\frac{k\pi}{2}\right)$

$u(t) = 2{,}5 + \left(\frac{10}{\pi}\cos\omega_0 t + \frac{10}{\pi}\sin\omega_0 t\right) + \left(0 + \frac{10}{\pi}\sin 2\omega_0 t\right) + \left(-\frac{10}{3\pi}\cos 3\omega_0 t + \frac{10}{3\pi}\sin \dots\right)$ $\quad (k=1,\ k=2,\ k=3)$

$= 2{,}5 + 4{,}5\cos\left(\omega_0 t - \frac{\pi}{4}\right) + 3{,}18\cdot\cos\left(2\omega_0 t - \frac{\pi}{2}\right) + 1{,}5\cos\left(3\omega_0 t - \frac{3\pi}{4}\right)\dots$

c) $U_{\text{tama eff}} = \sqrt{2{,}5^2 + \left(\frac{4{,}5}{\sqrt2}\right)^2 + \left(\frac{3{,}18}{\sqrt2}\right)^2 + \left(\frac{1{,}5}{\sqrt2}\right)^2} = 4{,}75\ V$

hiba: $\frac{5V - 4{,}75V}{5V}\cdot 100\% = 5\%$

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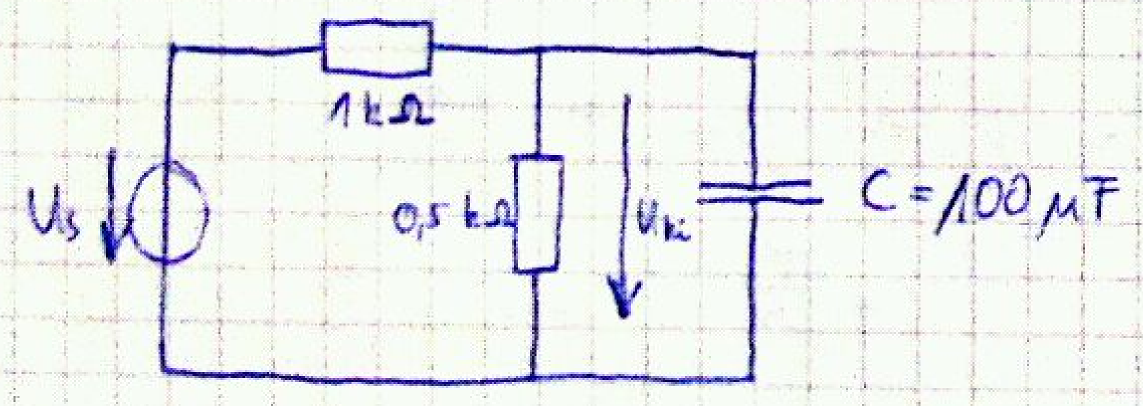
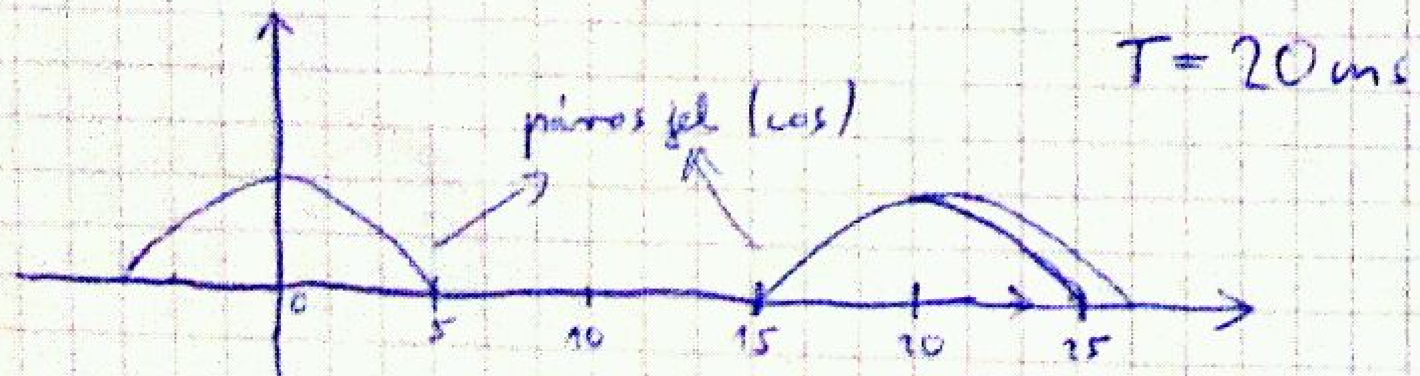


$U_s = 10\cdot\cos\omega t$ $\qquad U_{ki} = ?$

$T = 20\ ms$



Fourier sor: $U_0 = 2\cdot\frac{1}{T}\int_0^{T/4} 10\cos\omega t\, dt = \frac{10}{\pi}$

$U_k^B =$ mivel páros ezért nincs sin $= 0$

$U_k^A = 2\cdot\frac{2}{T}\int_0^{T/4} 10\cos\omega t\cdot\cos k\omega_0 t\ dt = \frac{20}{T}\int_0^{T/4}\cos(k+1)\omega t + \cos(k-1)\omega t =$

$= \frac{20}{T}\left[\frac{\sin(k+1)\omega t}{(k+1)\omega} + \frac{\sin(k-1)\omega t}{(k-1)\omega}\right]_0^{T/4} = \frac{10}{\pi}\left(\frac{\sin(k+1)\frac{\pi}{2}}{k+1} + \frac{\sin(k-1)\frac{\pi}{2}}{k-1}\right) =$

$\cos\alpha\pm\beta = \cos\alpha\cos\beta$

$k = 1 \rightarrow \quad U_1^A = \frac{10}{\pi}\cdot\left(0 + \lim_{k\to1}\frac{\sin(k-1)\frac{\pi}{2}}{(k-1)\frac{\pi}{2}\cdot\frac{2}{\pi}}\right) = 5$

$U_s(t) = \frac{10}{\pi} + 5\cos\omega t + \frac{20}{3\pi}\cdot\cos 2\omega t + \dots$

$H(j\omega) = ? = \frac{U_{ki}}{U_s} = \frac{R_2 \otimes \frac{1}{j\omega C}}{R_2\otimes\frac{1}{j\omega C} + R_1} = \frac{0{,}5}{1{,}5 + j\omega 50}$

átviteli tényezők:

$H(j\cdot 0) = \frac{1}{3}$

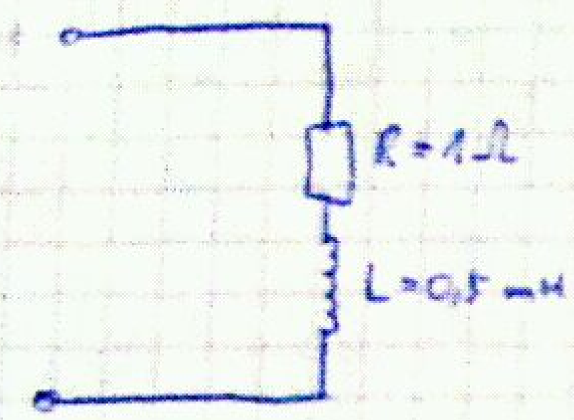
$H\left(j\frac{2\pi}{T} = 0{,}1\pi\right) = \frac{0{,}5}{1{,}5 + j5\pi} = 0{,}032\cdot e^{-j84{,}5^\circ}$

$H(j\,0{,}2\pi) = \frac{0{,}5}{1{,}5 + j10\pi} = 0{,}015\cdot e^{-j87{,}5^\circ}$

nincs mértékegység

$U_{ki}(t) = \frac{10}{\pi}\cdot\frac{1}{3} + 5\cdot 0{,}032\cdot\cos(\omega_0 t - 84{,}5^\circ) + \frac{20}{3\pi}\cdot 0{,}015\cdot\cos(2\omega_0 t - 87{,}5^\circ)$

2. gyak



$u(t) = 10\cos(\omega_0 t) + 3\cos(3\omega_0 t - 50^\circ)\ V$

$\omega_0 = 2\ \frac{krad}{s}$

$U_{eff} = \sqrt{\left(\frac{10}{\sqrt{2}}\right)^2 + \left(\frac{3}{\sqrt{2}}\right)^2} = 7{,}38\ V$

Mindkét frekvencián van munkavégzés → Σ-va meghagyjuk a hatásos teljesítményt

$U_{eff}^2 \cdot Re\{Y_1\} + U_{eff}^2 \cdot Re\{Y_2\}$

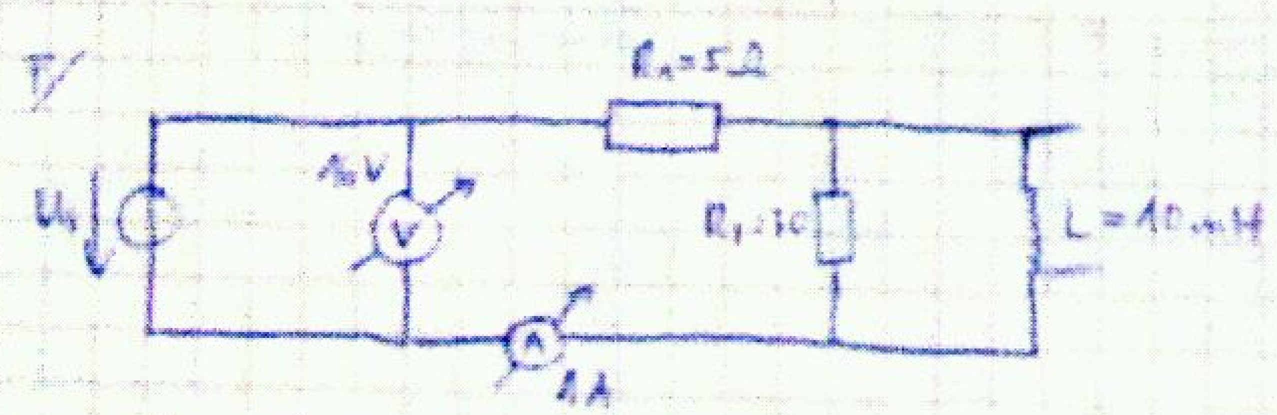
admittancia: $Y = \frac{1}{Z} = \frac{1}{R + j\omega L} = \underbrace{\frac{R}{R^2 + \omega^2 L^2}}_{Re} - j\underbrace{\frac{\omega L}{R^2 + \omega^2 L^2}}_{Im}$

frekvenciafüggő

$Re(Y_1) = \frac{1}{2}$

$Re(Y_2) = \frac{1}{10}$ $\Rightarrow$ $P =$

T/



$u_s(t) = \hat{U}_1 \cos(\omega_0 t) + \hat{U}_3 \cos(3\omega_0 t + 10^\circ)\ (V)$

$\omega_0 = 1\ \frac{krad}{s}$

Felvett hatásos teljesítmény? Műszer → effektív értéket mutat

1, $16^2 = U_{1\,eff}^2 + U_{3\,eff}^2$

$\bar{I} = \frac{\bar{U}}{Z_e}$ → komplex csúcs

$Z_e = R_2 \times (j\omega L) + R_1 = \frac{R_1 R_2 + (R_1 + R_2) \cdot j\omega L}{R_2 + j\omega L} =$

$= \quad Z_1 = 12\ e^{j\,48,5^\circ}$

$Z_2 = 25\ e^{j\,36,5^\circ}$

$|\bar{I}| = \frac{|\bar{U}|}{|Z_e|} \rightarrow 1^2 = \frac{U_{1\,eff}^2}{12^2} + \frac{U_{3\,eff}^2}{25^2}$

$U_{1\,eff} = 12{,}06\ V$

$U_{3\,eff} = 10{,}51\ V$

$u_s(t) = 12{,}06 \cdot \sqrt{2} \cos(\omega_0 t) + 10{,}51\sqrt{2}\cos(3\omega_0 t + 10^\circ)$

Forrás áramának időfüggvénye

$i(t) = \sqrt{2} \cdot \frac{12{,}06}{12} \cos(\omega_0 t - 48{,}5^\circ) + \sqrt{2} \cdot \frac{10{,}51}{25} \cdot \cos(3\omega_0 t + 10^\circ -$

$- 36{,}5^\circ)\ (A)$

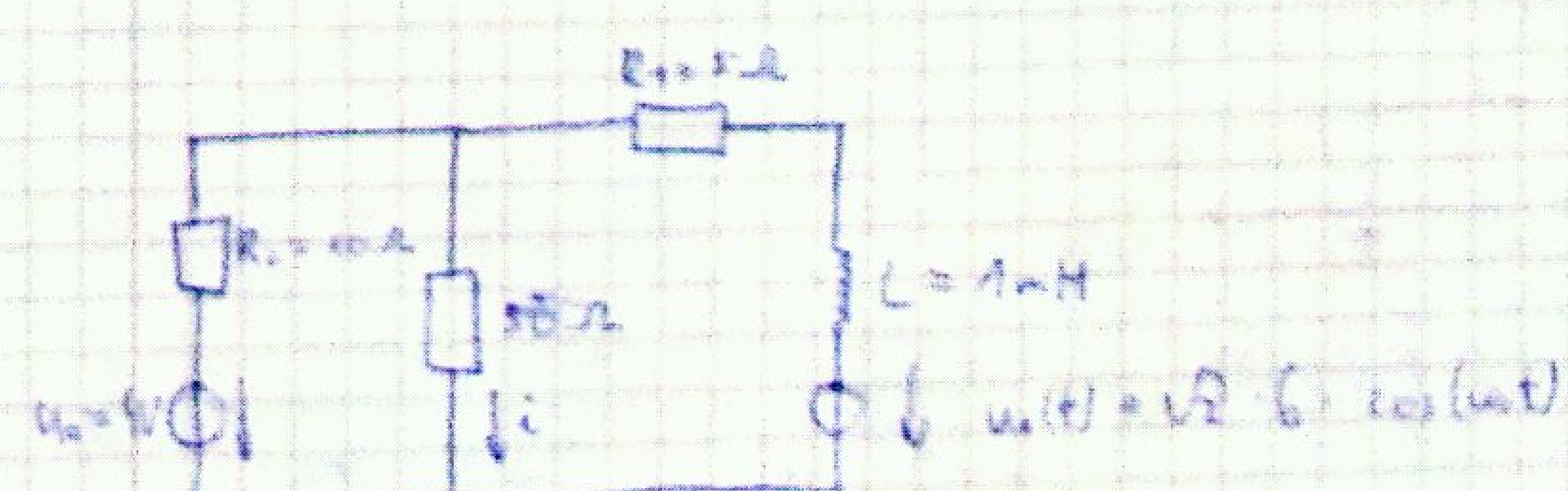
$P_{fel} = -P(u_s) = U_{eff}\ I_{eff} \cos\varphi$

ω_0: $U_{eff} = 12{,}06\ V\ \sqrt{2}$

$I_{eff} = \sqrt{2}\ \frac{12{,}06}{12}$ $\quad P_{\omega_0} =$

$\varphi = 48{,}5^\circ$ $\quad + = P = 11{,}6\ W$

$3\omega_0$ $\quad P_{3\omega_0} =$



$\omega_0 = 10^4\ \frac{rad}{s}$

$U_0 \longrightarrow L$ rövidzár

$\bar{U}_1 = \sqrt{2}\cdot 6 \qquad I'$

$j\omega L = j\cdot 10 \qquad \bar{I}''$

$I' = 4\cdot\frac{5+50}{5+50+10}\cdot\frac{1}{50} = 0{,}025\,A$

$\bar{I}'' = \sqrt{2}\cdot 6\cdot\frac{10\times 50}{10\times 50+5+j10}\cdot\frac{1}{50} = \sqrt{2}\cdot 0{,}06\cdot e^{-j36{,}5^\circ}$

$i(t) = 0{,}025 + \sqrt{2}\cdot 0{,}06\cdot\cos(\omega_0 t) - 36{,}5^\circ)$

Fourier-transzformáció:

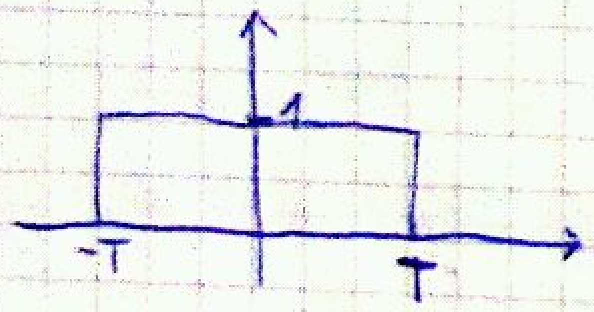
$x(t)$ jel $\xrightarrow{\mathcal{F}} X(j\omega) = \int_{-\infty}^{\infty} x(t)\cdot e^{-j\omega t}\,dt = \mathcal{F}(x(t)) =$

- az integrálás során az idő eltűnik de helyén változónak az ω

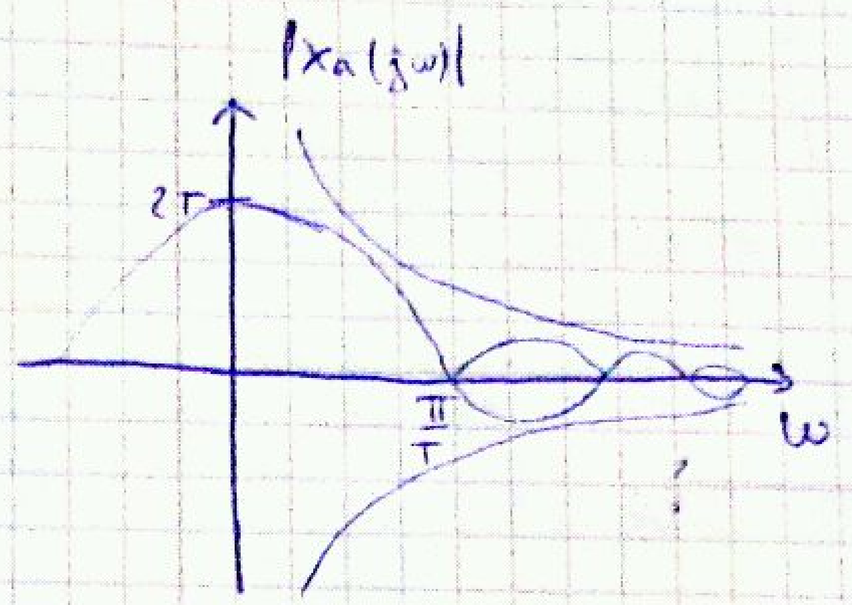
$= |X(j\omega)|\cdot e^{j\cdot \operatorname{arc}(X(j\omega))}$

amplitúdó spektrum — fázis spektrum

$X_a(t) = [\varepsilon(t+T) - \varepsilon(t-T)]\cdot 1$

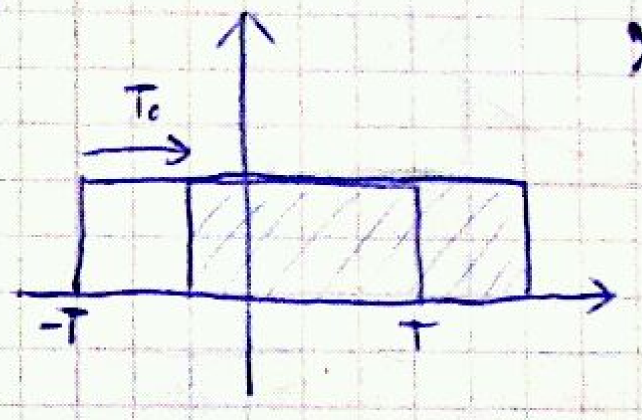


$X_a(j\omega) = \int_{-T}^{T} 1\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-T}^{T} = \frac{e^{-j\omega T} - e^{j\omega T}}{-j\omega} = \frac{2}{\omega}\cdot\sin(\omega T)$



$\frac{2}{\omega} \rightarrow$ hiperbolikus burkológörbe

b,



$X_b(t) = \varepsilon(\underbrace{t+T-T_0}_{t-(T+T_0)}) - \varepsilon(\underbrace{t-T-T_0}_{t-(T+T_0)})$

Eltolási tétel:

$X_b(j\omega) = X_a(j\omega)\cdot e^{-j\omega T_0}\cdot 1$

$|X_b(j\omega)| = |X_a(j\omega)|\cdot 1$

$e^{-j\omega T_0} = e^{j\operatorname{arc}(X(j\omega))}$

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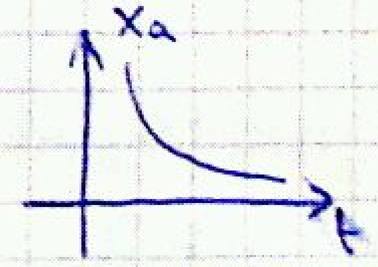
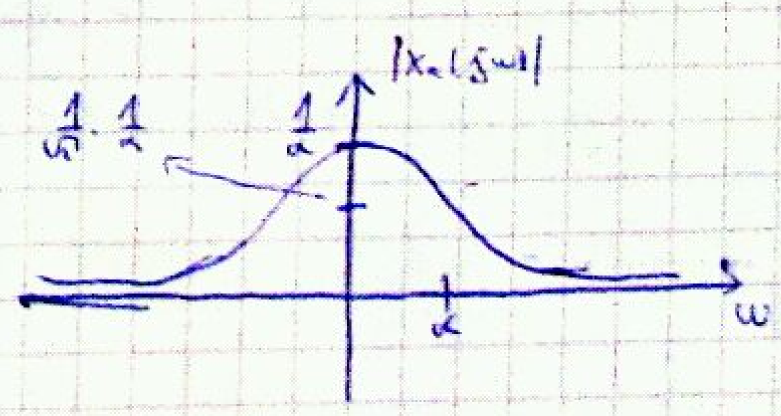
$X_a(t) = \varepsilon(t)\cdot e^{-\alpha t}$

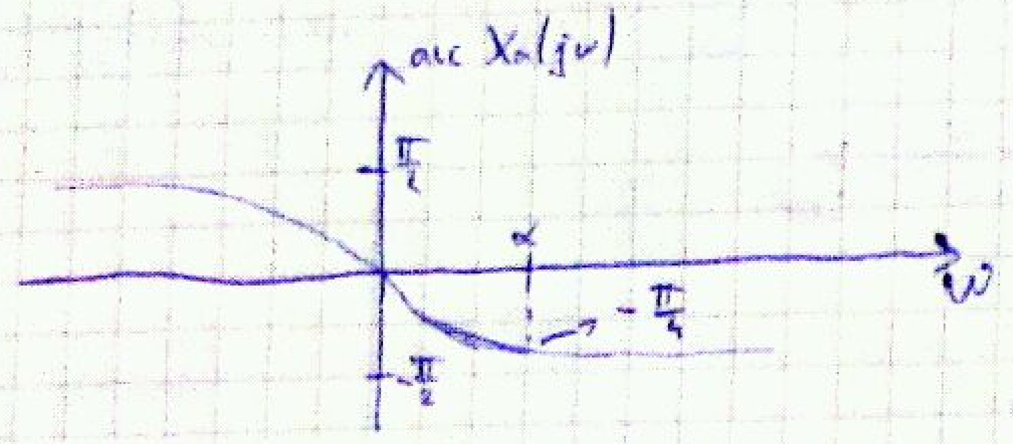


$X_a(j\omega) = \int_0^\infty e^{-\alpha t}\cdot e^{-j\omega t}\,dt = \left[\frac{e^{-(\alpha+j\omega)t}}{-(\alpha+j\omega)}\right]_0^\infty = \frac{1}{\alpha+j\omega} \to$ komplex spektrum

$|X_a(j\omega)| = \frac{1}{\sqrt{\alpha^2+\omega^2}}$

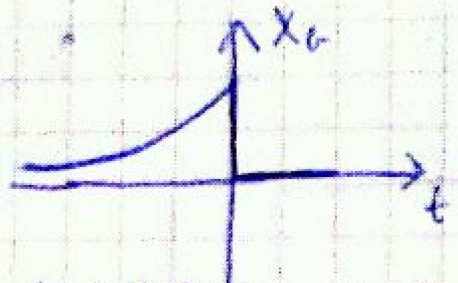


$$\text{arc}\, X_a(j\omega) = -\text{arc tg}\left(\frac{\omega}{\alpha}\right) \begin{matrix} \to \text{Im}\{\} \\ \to \text{Re}\{\} \end{matrix}$$



4.2.2/ ~~...~~

b, $x_b(t) = (1-\varepsilon(t))\, e^{\alpha t}$



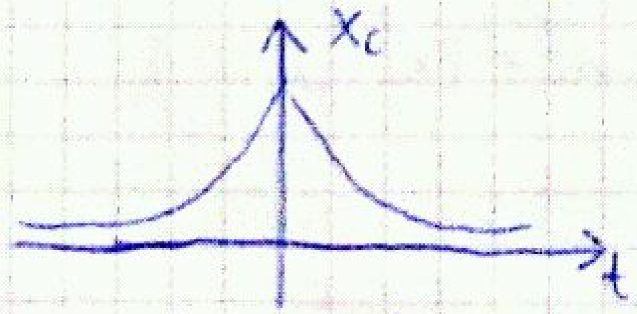
$$X_b(j\omega) = \int_{-\infty}^{0} e^{(\alpha - j\omega)t}\, dt = \left[\frac{e^{(\alpha-j\omega)t}}{\alpha - j\omega}\right]_{-\infty}^{0} = \frac{1}{\alpha - j\omega}$$

$|X_b(j\omega)| = |X_a(j\omega)|$

$\text{arc}\, X_b(j\omega) = -\,\text{arc}\, X_a(j\omega)$

c, $x_c(t) = e^{-\alpha|t|}$

$x_c(t) = x_a(t) + x_b(t)$



$$X_c(j\omega) = X_a(j\omega) + X_b(j\omega) = \frac{1}{\alpha + j\omega} + \frac{1}{\alpha - j\omega} = \frac{2\alpha}{\alpha^2 + \omega^2} = |X_c(j\omega)|$$

$\text{arc}\, X_c(j\omega) \equiv 0$

Ha az időfüggvény páros ⇒ komplex spektrum valós

Ha az időfüggvény páratlan ⇒ komplex spektrum képzetes

4.2.7/ Modulációs tétel

$$\mathcal{F}\{x(t)\cdot \cos\omega_0 t\} = \frac{1}{2}\left(X(j(\omega+\omega_0)) + X(j(\omega-\omega_0))\right)$$

a, $F_a(j\omega) = \frac{1}{2}\left(\frac{2}{\omega+\omega_0}\sin(\omega+\omega_0)T + \frac{2}{\omega-\omega_0}\sin(\omega-\omega_0)T\right)$

$f_a(t) = (\varepsilon(t+T) - \varepsilon(t-T))\cos\omega_0 t$

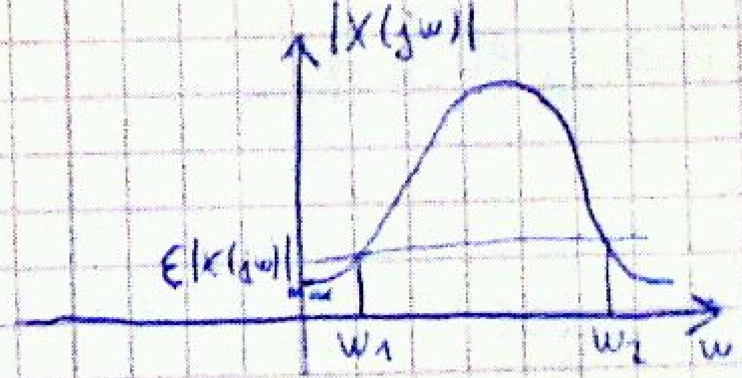
b, $f_b(t) = \cos^2\omega_0 t = \frac{1+\cos 2\omega_0 t}{2}$

$\mathcal{F}\{1\} = 2\pi\cdot\delta(\omega) \longrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\,\delta(\omega)\cdot e^{j\omega t}\, d\omega = 1$

csak 0-ban kell nézni

$$F_b(j\omega) = \frac{1}{2}2\pi\,\delta(\omega) + \frac{\pi}{2}\left(\delta(\omega+2\omega_0) + \delta(\omega-2\omega_0)\right)$$

Jel sávszélessége: az a tartomány melybe a jelet döntően alakító amplitúdók tartoznak

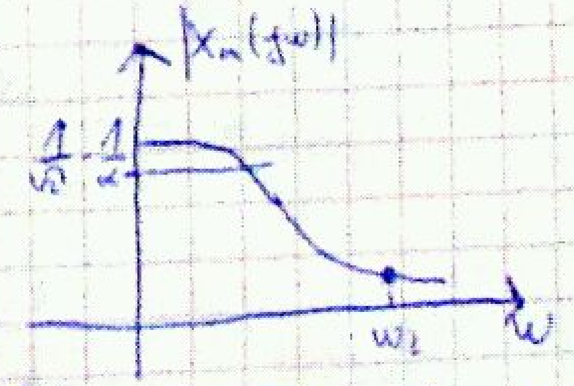


$\Delta\omega_{jel} = \omega_2 - \omega_2$, amelyen kívül $|X(j\omega)| \leq \varepsilon\, |X(j\omega)|_{max}$

4.2.13/ a, $\varepsilon = 0{,}1$ $\Delta\omega_{jel} = ?$

$x_a(t) = \varepsilon(t)\, e^{-\alpha t}$

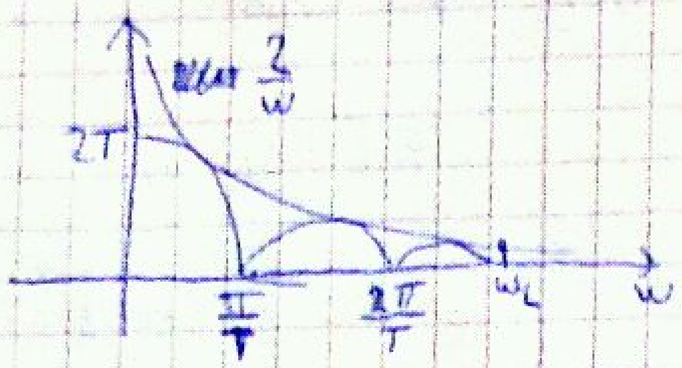
$|X_a(j\omega)| = \frac{1}{\sqrt{\alpha^2 + \omega^2}}$



$$\frac{1}{\sqrt{\alpha^2 + \omega_2^2}} = \varepsilon\cdot\frac{1}{\alpha} \longrightarrow \omega_2 = \Delta\omega_{jel} = \frac{1}{\varepsilon}\,\alpha\sqrt{1-\varepsilon^2} \approx \frac{\alpha}{\varepsilon} \approx 10\alpha$$

b, $X_e(t) = \varepsilon(t+T) - \varepsilon(t-T)$

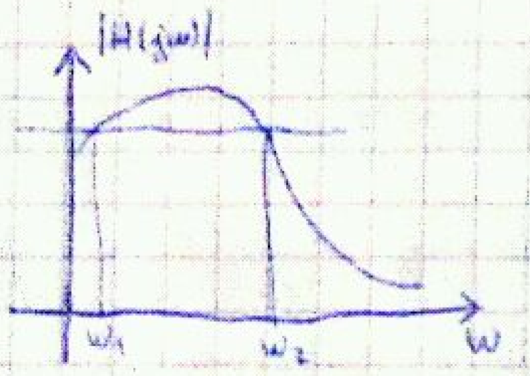
$X_e(j\omega) = 2T \cdot \frac{\sin \omega T}{\omega T}$



$\frac{2}{\omega_2} = \varepsilon \cdot 2T \longrightarrow \omega_2 = \Delta\omega_{jel} = \frac{1}{\varepsilon T} = \frac{10}{T}$

Alakhű (torzítatlan) jelátvitel:

rendszer sávszél



$\Delta\omega_r = \omega_2 - \omega_1$, ahol $|H(j\omega)| \geq \frac{1}{\sqrt{2}} |H(j\omega)|_{max}$

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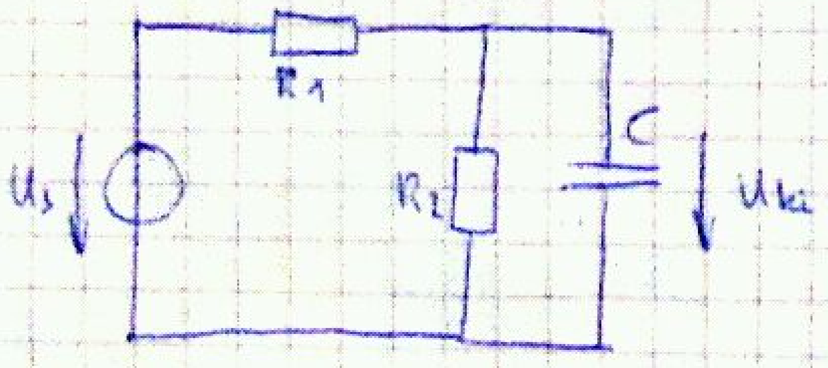

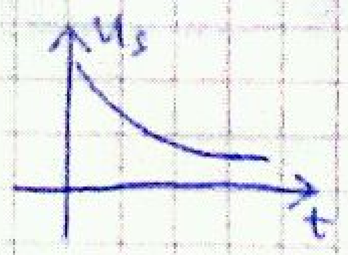

$$H(j\omega) = \frac{R_2 \times \frac{1}{j\omega C}}{R_1 + R_2 \times \frac{1}{j\omega C}} = \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2}$$

$$|H(j\omega)| = \frac{R_2}{\sqrt{\underbrace{(R_1+R_2)^2}_{Re^2} + \underbrace{\omega_2^2 C^2 R_1^2 R_2^2}_{Im^2}}} \equiv \frac{1}{\sqrt{2}} \cdot \frac{R_2}{R_1 + R_2}$$

$\omega_1 = 0$

$\omega_2 = \Delta\omega_r = \frac{R_1 + R_2}{C \cdot R_1 + R_2}$

alakhű, ha $\frac{R_1 + R_2}{C R_1 + R_2} \geq 10\alpha$

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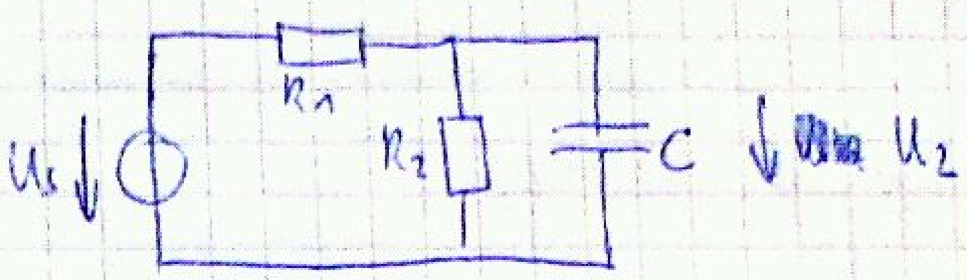


$\alpha = \frac{R_1 + R_2}{C R_1 R_2}$

$u_s(t) = U_0 (1 - \varepsilon(t)) e^{\alpha t}$

a, $U_2(j\omega) = U_1(j\omega) \cdot H(j\omega) = \frac{U_0}{\alpha - j\omega} \cdot \frac{R_2}{R_1 + R_2 + j\omega C R_1 R_2}$

b, $u_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U_{ki}(j\omega) e^{j\omega t} d\omega$

c, $U_{ki}(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{U_0 R_2}{C R_1 R_2} \cdot \frac{1}{(\alpha - j\omega)(\alpha + j\omega)} d\omega = \frac{U_0}{2\pi C R_1 \alpha^2} \int_{-\infty}^{\infty} \frac{1}{1 + \left(\frac{\omega}{\alpha}\right)^2} d\omega$

$= \frac{U_0}{2\pi C \cdot R_1 \alpha^2} \cdot \alpha \left[\operatorname{arc\,tg} \frac{\omega}{\alpha} \right]_{-\infty}^{\infty} = \frac{U_0}{2 C R_1 \alpha}$

$u_2(t) = \frac{U_0}{2 C R_1 \alpha} \cdot e^{-\frac{R_1 + R_2}{C R_1 R_2} t}$

Laplace transzformáció:

$X(t) \longrightarrow X(s) = \mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

$X(t)$	$X(s)$
$\delta(t)$	1
$\varepsilon(t)$	$\frac{1}{s}$
t	$\frac{1}{s^2}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$
$\cos \omega_0 t$	$\frac{s}{s^2 + \omega^2}$
$\sin \omega_0 t$	$\frac{\omega_0}{s^2 + \omega^2}$
$X(t-\tau) \cdot \varepsilon(t-\tau)$	$X(s) e^{-s\tau}$

4.3-1/ a, $x(t) = A \cdot e^{-\alpha t} \cdot \cos \omega t \cdot \varepsilon(t)$

$X(s) = A \cdot \frac{s+\alpha}{(s+\alpha)^2+\omega^2}$

b, $x(t) = A\, e^{-\alpha t} \cdot \cos(\omega t + \delta)\, \varepsilon(t)$

$x(t) = A \cdot e^{-\alpha t} \cdot (\cos \omega t \cdot \cos \delta - \sin \omega t \cdot \sin \delta)\, \varepsilon(t)$

$X(s) =$

c, $x(t) = A \cdot t \cdot e^{-\alpha t} \cdot \varepsilon(t)$

$X(s) = A \cdot \frac{1}{(s+\alpha)^2}$

d, $x(t) = A \cdot t \cdot \cos \omega t\, \varepsilon(t)$

$x(t) = A \cdot t \left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) \cdot \varepsilon(t)$

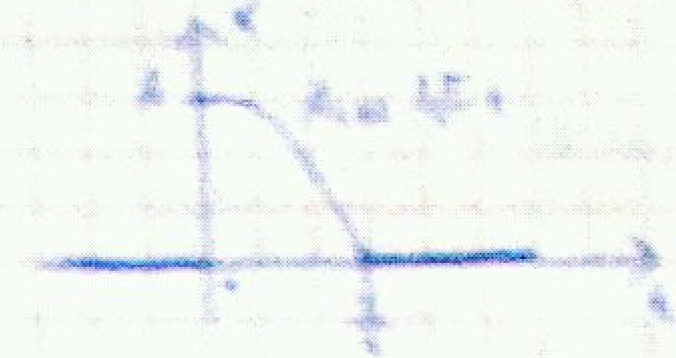
$X(s) = \frac{A}{2}\left(\frac{1}{(s+j\omega)^2} + \frac{1}{(s-j\omega)^2}\right) = \frac{A}{2} \cdot \frac{2s^2 - 2\omega^2}{(s^2+\omega^2)^2}$

4.3-3/ a,

$x(t) = \varepsilon(t) \cdot \frac{A}{T} \cdot t - \frac{A}{T}(t-T) \cdot \varepsilon(t-T)$

$X(s) = \frac{A}{T} \cdot \frac{1}{s^2}(1 - e^{-sT})$

b,



$\omega = \frac{\pi}{2T}$

$x(t) = A \cos \frac{\pi t}{2T} \cdot [\varepsilon(t) - \varepsilon(t-T)] = A \cos \frac{\pi}{2T} t\, \varepsilon(t) - A\, \varepsilon(t-T)$ [illegible]

$X(s) = A \cdot \frac{s}{s^2+\omega^2} + A \cdot \frac{[illegible]}{s^2+\omega^2} \cdot e^{-sT}$

c,

$x(t) = [\varepsilon(t) + \varepsilon(t-T) + \varepsilon(t-2T) + \dots$

$X(s) = \frac{1}{s}(1 + e^{-sT} + e^{-2sT} + \dots) = \frac{1}{s(1-e^{-sT})}$

4.3-5/ a, $Y(s) = \frac{[illegible]}{(s+1)(s+4)} = \frac{[illegible]}{s+1} + \frac{[illegible]}{s+4}$

$y(t) = (e^{-t} + 2e^{-4t})\, \varepsilon(t)$

b, $H(s) = 3 \frac{(s+2)^2}{(s+1)(s+4)}$ $h(t) = \mathcal{L}^{-1}\{H(s)\}$

$H(s) = 3 \cdot \frac{[illegible]}{[illegible]} = 3 \cdot 1 + \frac{-3s}{(s+1)(s+4)} =$

$= 3 + \frac{1}{s+1} - \frac{4}{s+4}$

$h(t) = 3\,\delta(t) + \varepsilon(t)\left(e^{-t} - 4e^{-4t}\right)$

4.3-5

c) $Y(s) = \frac{3s+2}{(s+1)(s+2)^2} = \frac{-1}{s+1} + \frac{4}{(s+2)^2} + \frac{1}{s+2}$

$y(t) = \varepsilon(t)\left(-e^{-t} + 4te^{-2t} + e^{-2t}\right)$

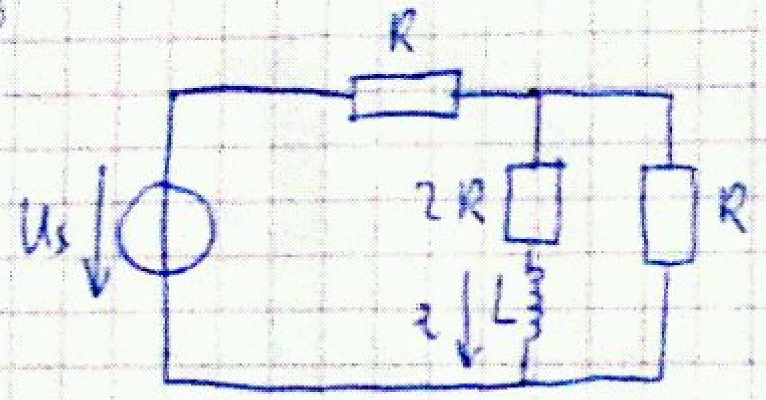
d) $H(s) = \frac{1}{s^2+8s+25} \longrightarrow h(t) = ?$

$H(s) = \frac{1}{(s+4+j3)(s+4-j3)} = \frac{\frac{1}{-6j} = \frac{1}{6}e^{j\frac{\pi}{2}}}{s+4+j3} + \frac{\frac{1}{6}e^{-j\frac{\pi}{2}}}{s+4-j3}$

$h(t) = \varepsilon(t)\cdot 2\,\mathrm{Re}\left\{\frac{1}{6}\cdot e^{j\frac{\pi}{2}}\cdot e^{-(4+j3)t}\right\} = \varepsilon(t)\cdot\frac{1}{3}e^{-4t}\cos\left(-3t+\frac{\pi}{2}\right)$

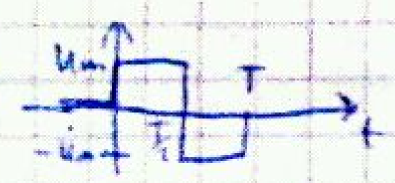
$= \varepsilon(t)\cdot\frac{1}{3}e^{-4t}\sin 3t$

4.3-15

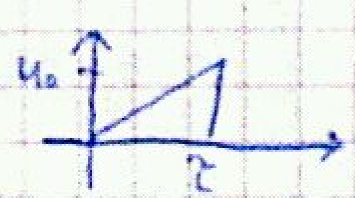


a) $u_s(t) = \varepsilon(t)$

b)



c)



a) $H(s) = \frac{R\times(2R+sL)}{R+R\times(2R+sL)}\cdot\frac{1}{2R+sL} = \frac{1}{2sL+5R} = \frac{1}{2L}\cdot\frac{1}{s+\frac{5R}{2L}}$

$U_s(s) = \frac{1}{s} \longrightarrow I(s) = U_s(s)\cdot H(s) = \frac{1}{2L}\cdot\frac{1}{s(s+\alpha)} =$

$= \frac{1}{2L}\left(\frac{\frac{1}{\alpha}}{s} + \frac{-\frac{1}{\alpha}}{s+\alpha}\right)$

$i(t) = \frac{1}{2L\alpha}\,\varepsilon(t)\left(1-e^{-\alpha t}\right) = g(t)$

b) $u_s(t) = U_m\left(\varepsilon(t) - 2\varepsilon(t-\frac{T}{2}) + \varepsilon(t-T)\right)$

$U_s(s) = U_m\cdot\frac{1}{s}\left(1 - 2e^{-s\frac{T}{2}} + e^{-sT}\right)$

$i(t) = U_m\left(g(t) - 2g(t-\frac{T}{2}) + g(t-T)\right)$

c) $u_s(t) = \frac{U_0}{\tau}t\left(\varepsilon(t) - \varepsilon(t-\tau)\right) = \frac{U_0}{\tau}t\,\varepsilon(t) - \frac{U_0}{\tau}\varepsilon(t-\tau)\cdot(t-\tau) -$

$- U_0\,\varepsilon(t-\tau)$

$U_s(s) = \frac{U_0}{\tau}\cdot\frac{1}{s^2}\left(1-e^{-s\tau}\right) - U_0\cdot\frac{1}{s}e^{-s\tau}$

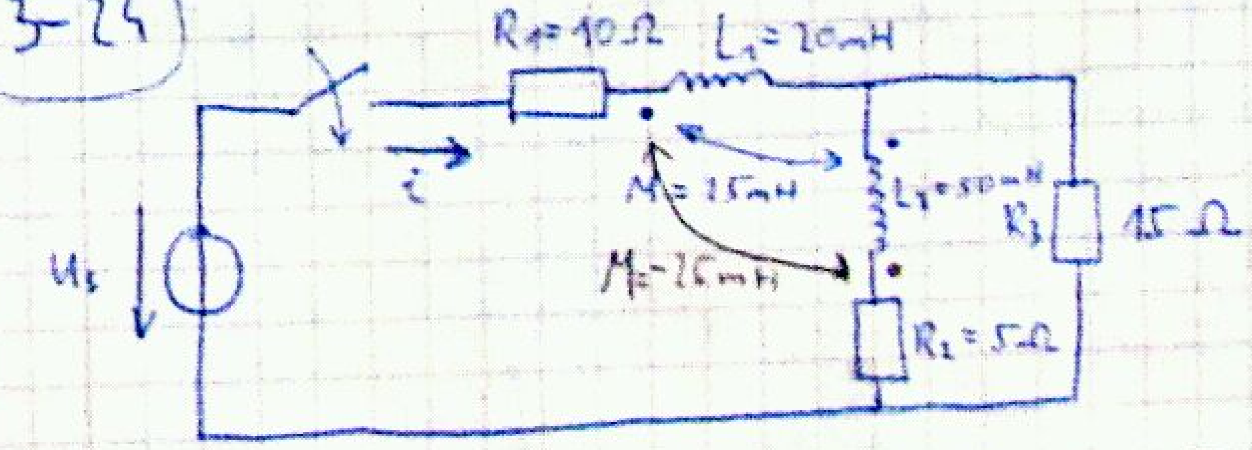
$I(s) = \frac{U_0}{2L\tau}\cdot\frac{1}{s^2(s+\alpha)}\left(1-e^{-s\tau}\right) - \frac{U_0}{2L}\cdot\frac{1}{s(s+\alpha)}e^{-s\tau} =$

$= \frac{U_0}{2L\tau}\left(\frac{\frac{1}{\alpha}}{s^2} + \frac{-\frac{1}{\alpha^2}}{s} + \frac{\frac{1}{\alpha^2}}{s+\alpha}\right)\left(1-e^{-s\tau}\right) - \frac{U_0}{2L}\left(\frac{\frac{1}{\alpha}}{s} + \frac{-\frac{1}{\alpha}}{s+\alpha}\right)e^{-s\tau}$

$i(t) = \frac{U_0}{2L\tau\alpha}\left[\varepsilon(t)\left(t-\frac{1}{\alpha}+\frac{1}{\alpha}e^{-\alpha t}\right) - \varepsilon(t-\tau)\left(t-\tau-\frac{1}{\alpha}+\frac{1}{\alpha}e^{-\alpha(t-\tau)}\right)\right] -$

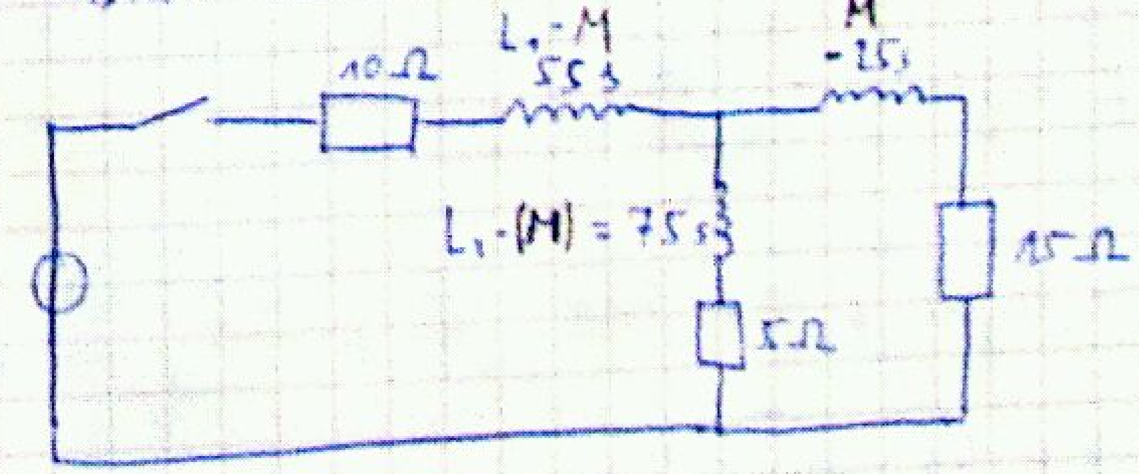
$- \frac{U_0}{2L\alpha}\,\varepsilon(t-\tau)\left(1-e^{-\alpha(t-\tau)}\right)$

4.3-24



$i(t) = ?$

$u_s(t) = 170\cdot\sin(\omega_0 t + 30°)$ $\omega_0 = 314\,\frac{rad}{s} = 0{,}314\,\frac{krad}{s}$



$$H(s) = \frac{1}{10 + 55s + (5+75s) \otimes (15-25s)} = 0{,}057 \cdot \frac{s+0{,}4}{s^2 + 2{,}97s + 0{,}314}$$

$$u_s(t) = \varepsilon(t) \cdot 170 \left(\sin \omega t \cdot \cos 30^\circ + \cos \omega t \cdot \sin 30^\circ \right)$$

$$U_s(s) = 85 \cdot \frac{0{,}314 \cdot \sqrt{3} + s}{s^2 + 0{,}314^2}$$

$$I(s) = U_s(s) \cdot H(s) = \frac{(2{,}64 + 4{,}86s)(s+0{,}4)}{(s - j0{,}314)(s + 0{,}314j)(s + 0{,}11)(s + 2{,}862)} =$$

$$= \frac{A}{s - 0{,}314j} + \frac{A^*}{s + 0{,}314j} + \frac{2}{s + 0{,}11} + \frac{-1{,}21}{s + 2{,}862}$$

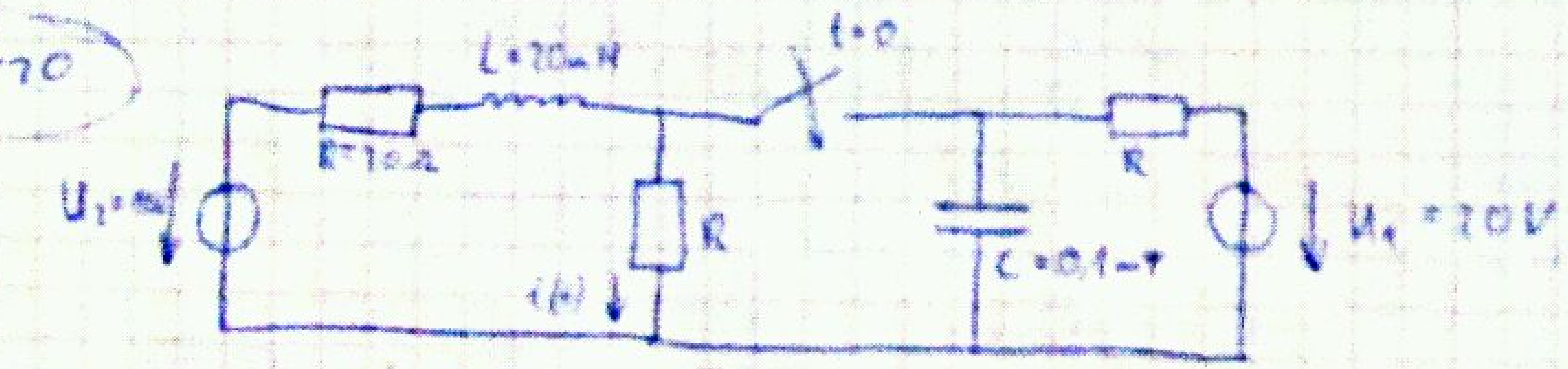
$\frac{1}{s+a}$

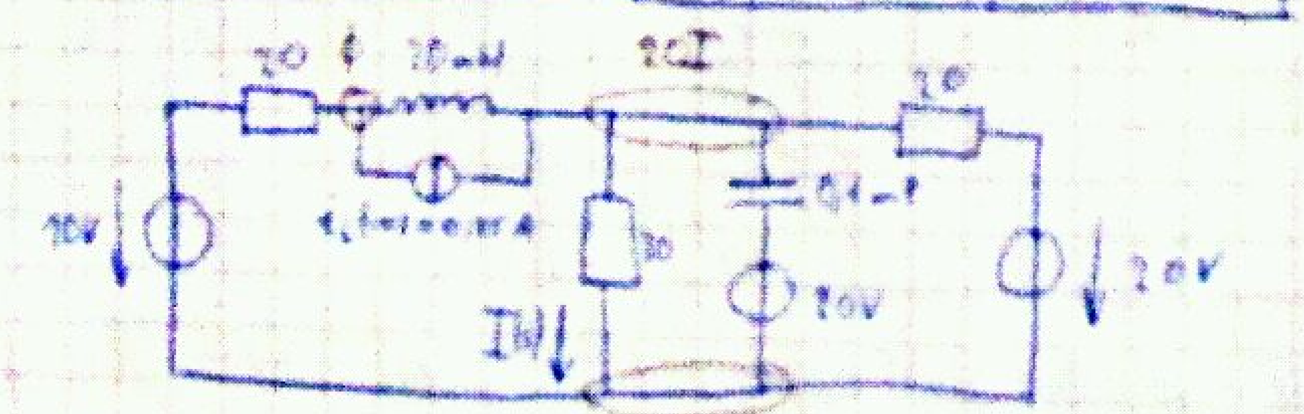
$$A = \frac{(2{,}64 + 4{,}86 \cdot 0{,}314j)(0{,}4 + 0{,}314j)}{2 \cdot 0{,}314j(0{,}11 + j0{,}314)(2{,}862 + 0{,}314j)} = 2{,}6\, e^{-j33^\circ}$$

$$i(t) = \varepsilon(t) \left(2\,\mathrm{Re}\left\{ 2{,}6\, e^{-j33^\circ} \cdot e^{j0{,}314t} \right\} + 2e^{-0{,}11t} - 1{,}21 e^{-2{,}862t} \right.$$

$$i(t) = \varepsilon(t) \left(5{,}2 \cdot \cos(0{,}314t - 33^\circ) + 2e^{-0{,}11t} - 1{,}21 e^{-2{,}862t} \right)$$

4.3-70





$$\Phi: \quad \frac{\Phi - 20I}{20s} + \frac{0{,}15}{s} + \frac{\Phi - \frac{10}{s}}{10} = 0$$

$$20I \quad \frac{20I - \Phi}{20s} - \frac{0{,}15}{s} + I + \frac{20I - \frac{20}{s}}{20} + \frac{20I - \frac{40}{s}}{\frac{1}{0{,}1s}} = 0$$

$$I(s) = \frac{2s^2 + 3{,}25s + 1{,}5}{s(2s^2 + 4s + 3)}$$

6. számítógépes gyak

Matlab 7.0.4

1, Fourier-sor

$$x_1(t) = 10\left(\varepsilon(t) - \varepsilon\left(t - \tfrac{t_n}{4}\right)\right) - 10\left(\varepsilon\left(t - \tfrac{t_n}{4}\right) - \varepsilon(t - t_n)\right)$$

```
>> xt1      n = 512
>> fs       tn = 5
            m = 3
>> ifs      np = 500
```

2, Fourier-transzformáció

$$x_2(t) = a\left(\varepsilon(t + t_m) - \varepsilon(t - t_m)\right)$$

```
>> xt2      t2 = 2  =10. idővált. [-2, 2]
            n = 256
            tm = 1
            a = 5
>> ftr           spektrum
```

3, Laplace-transzformáció

$$H(s) = \frac{3s + 6}{s^2 + 5s + 4} = \frac{2}{s+1} + \frac{1}{s+4}$$

```
>> sz = [0 3 6]
>> n = [1 5 4]
>> [r,p,c] = residue(sz,n)
   r = 2      -> konstans
       1

   p = -4     -> pólusok
       -1

   c =
       []
```

6. gyak

(*)

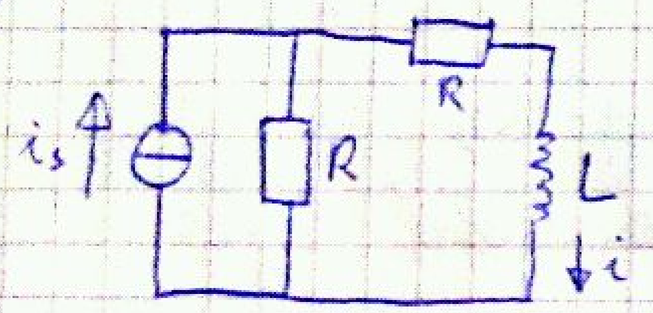
$$I(s) = \frac{2s^2+3{,}25s+1{,}5}{s(2s^2+4s+3)} = \frac{s^2+1{,}625s+0{,}75}{s(s+1-j0{,}707)(s+1+j0{,}707)} =$$

$$= \frac{0{,}5}{s} + \frac{0{,}265\,e^{-j19{,}5^\circ}}{s+1-j0{,}707} + \frac{0{,}265\,e^{j19{,}5^\circ}}{s+1+j0{,}707}$$

$$i(t) = \varepsilon(t)\left(0{,}5 + 2\,\mathrm{Re}\left\{0{,}2652\,e^{-j19{,}5^\circ}\cdot e^{-(1-j0{,}707)t}\right\}\right) =$$

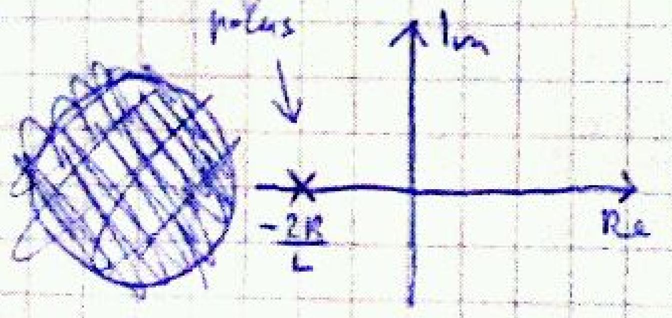
$$i(t) = \varepsilon(t)\left(0{,}5 + 0{,}53\cdot e^{-t}\cos(0{,}707t - 19{,}5^\circ)\right)$$

4.4-1/3



a, $H(s) = \frac{R}{R+R+sL} = \frac{\frac{R}{L}}{s+\frac{2R}{L}}$

pólus - zérus elrendezés:



összes pólus a bal félsíkon → GV stabil

b, $H(j\omega) = \frac{\frac{R}{L}}{j\omega+\frac{2R}{L}}$

c, $h(t) = \frac{R}{L}\,e^{-\frac{2R}{L}t}\,\varepsilon(t)$

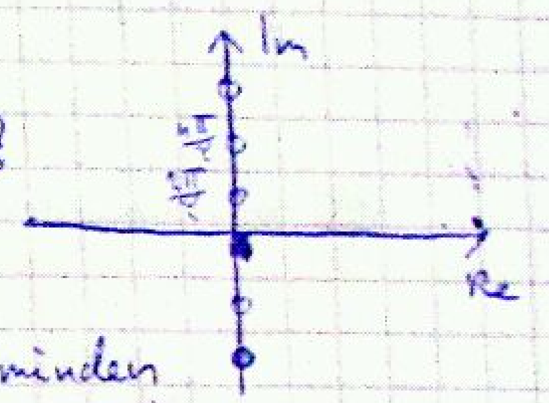
7. gyak

4.4-7/ $h(t) = \varepsilon(t) - \varepsilon(t-\tau)$

a, $h(t)$ belépő → kauzális r. → $H_s = \frac{1-e^{-s\tau}}{s}$

pólusok: —

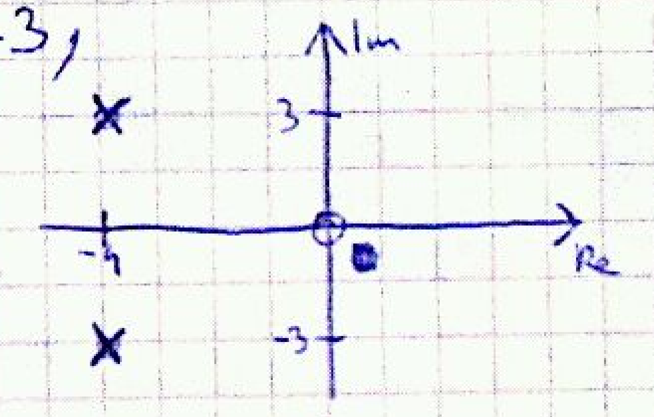
zérusok: $s = j\frac{2\pi k}{\tau}$, ahol $k \in \mathbb{Z}\setminus\{0\}$



GV-stabil, mert minden pólus negatív (mivel nincs)

b, $H(j\omega) = \frac{1-e^{-j\omega\tau}}{j\omega}$ → nem realizálható, mert nem polinom \ polinom

4.4-3/



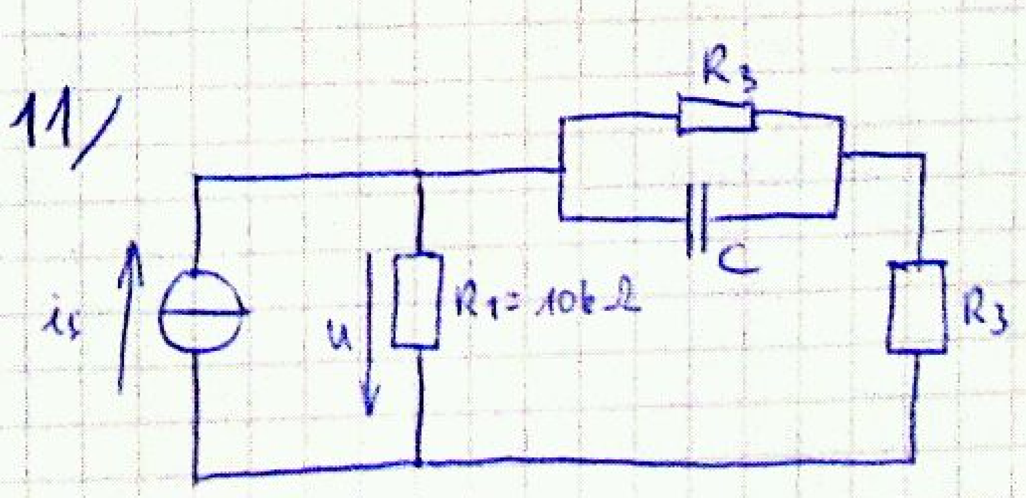
$H(j5) = 0{,}25$

$$H(s) = K\cdot\frac{s}{(s+4-j3)(s+4+j3)} = K\cdot\frac{s}{s^2+8s+25}$$

GV-stabil → $H(j\omega) = K\cdot\frac{j\omega}{(j\omega)^2+8j\omega+25}$ → $H(j5) = K\cdot\frac{1}{8} = 0{,}25$

$K = 2$

4.4.-11/



[V, mA, kΩ, ms, μF]

$g(t) = \varepsilon(t)(5 - 3e^{-t})$

a) $R_2 ; R_3 ; C = ?$

$$H(s) = \frac{G(s)}{\frac{1}{s}} = s \cdot \left(\frac{5}{s} - \frac{3}{s+1} \right) = \frac{2s+5}{s+1}$$

$$H(s) = Z_e = \underset{10}{R_1} \times \left(R_3 + R_2 \times \frac{1}{sC} \right) = \frac{s \cdot \overset{2}{\frac{R_2 R_3 C \cdot 10}{(10+R_3) R_2 C}} + \overset{5}{\frac{10(R_2+R_3)}{(10+R_3) R_2 C}}}{s + \underset{1}{\frac{10+R_2+R_3}{(10+R_3) R_2 C}}}$$

$u = H(s) \cdot i_s$

$u = R \cdot I$

más megoldás:

$g(+0) = 2 = 10 \times R_3 \rightarrow R_3 = \ldots$

$g(+\infty) = 5 = 10 \times (R_3 + R_2) \rightarrow R_2 = \ldots$

$\varepsilon(t)(5 - 3e^{-\frac{t}{\tau}}) \rightarrow \tau = 1 = R_2 \times (10 + R_3) \cdot C \rightarrow C = \ldots$

b) $I_s(s) = \frac{10}{s} - \frac{10}{s+2} = \frac{20}{s(s+2)}$ $\quad i(t) = 10(1 - e^{-2t})\varepsilon(t)$

$$U(s) = I(s) \cdot H(s) = \frac{20(2s+5)}{s(s+1)(s+2)} = \frac{50}{s} + \frac{-60}{s+1} + \frac{10}{s+2}$$

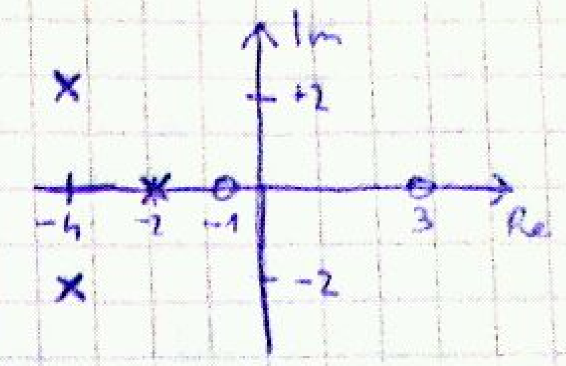
$u(t) = \varepsilon(t)(50 - 60e^{-t} + 10e^{-2t})$

c) $i_s(t) = 10 - 10(1 - e^{-2t})\varepsilon(t) \rightarrow$ nem belépő, nincs Laplace

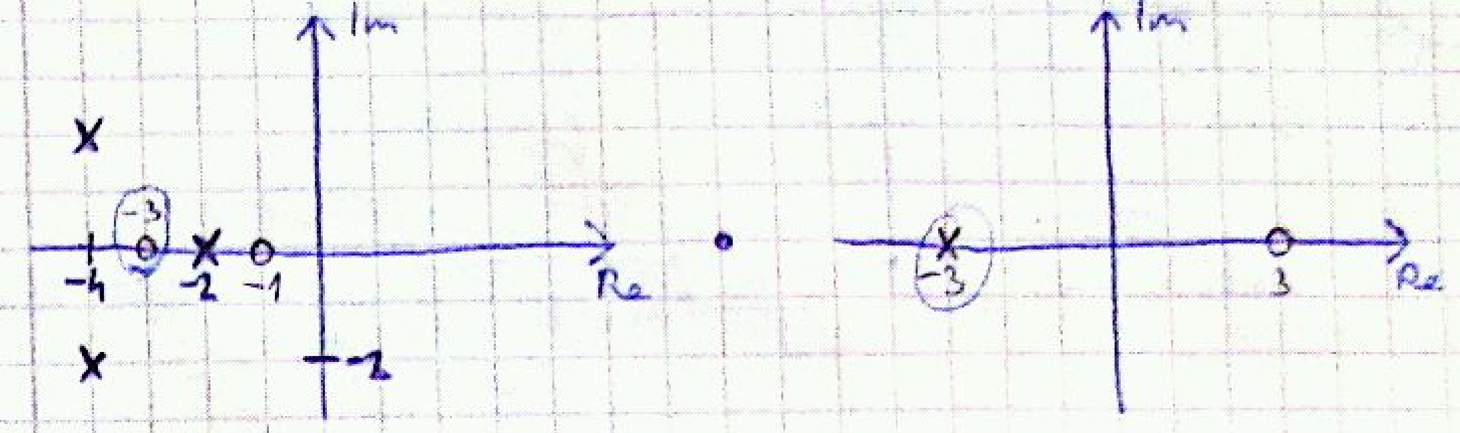
szuperpozícióval

$u(t) = 5 \cdot 10 - \varepsilon(t)(50 - 60e^{-t} + 10e^{-2t})$

F/



Bontsuk fel mindent áteresztőre és minimál fázisúra!

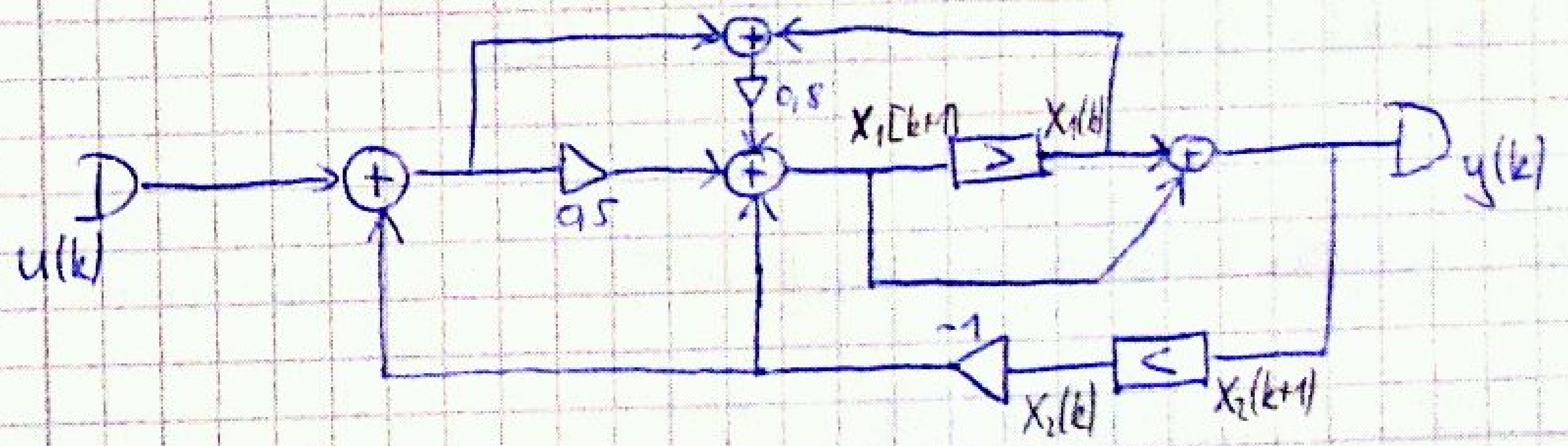


HF MA

csak negatív tartományban lehet

$$H_{HF}(s) = \frac{(s+1)(s+3)}{(s+2)(s+4-j2)(s+4+j2)} \qquad H_{MA}(s) = \frac{s-3}{s+3}$$

Diszkrét idejű hálózatok



$-1 \cdot 0{,}5 - 0{,}8$

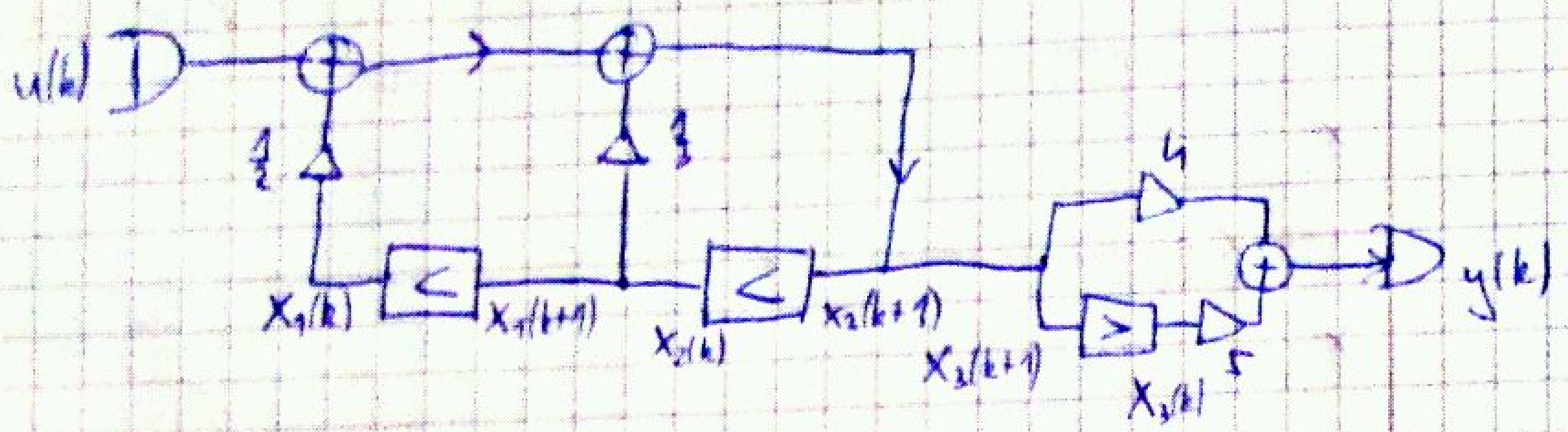
$x_1(k+1) = 0{,}8 x_1(k) - 2{,}3 x_2(k) + 1{,}3 u(k)$

$x_2(k+1) = x_1(k) + x_1(k+1) = 1{,}8 x_1(k) - 2{,}3 x_2(k) + 1{,}3 u(k)$

$y(k) = 1{,}8 x_1(k) - 2{,}3 x_2(k) + 1{,}3 u(k)$

$\boxed{\text{8. gyak}}$

3-4/



a,

állapotváltozós leírás normál alakja:

$$u(k) = \varepsilon(k) \longrightarrow y(k) = ?$$

$$x_1(k+1) = x_2(k)$$
$$x_2(k+1) = \frac{1}{2}x_1(k) + \frac{1}{3}x_2(k) + u(k)$$
$$x_3(k+1) = \frac{1}{2}x_1(k) + \frac{1}{3}x_2(k) + u(k)$$
$$y(k) = 2x_1(k) + \frac{4}{3}x_2(k) + 5x_3(k) + 4u(k)$$

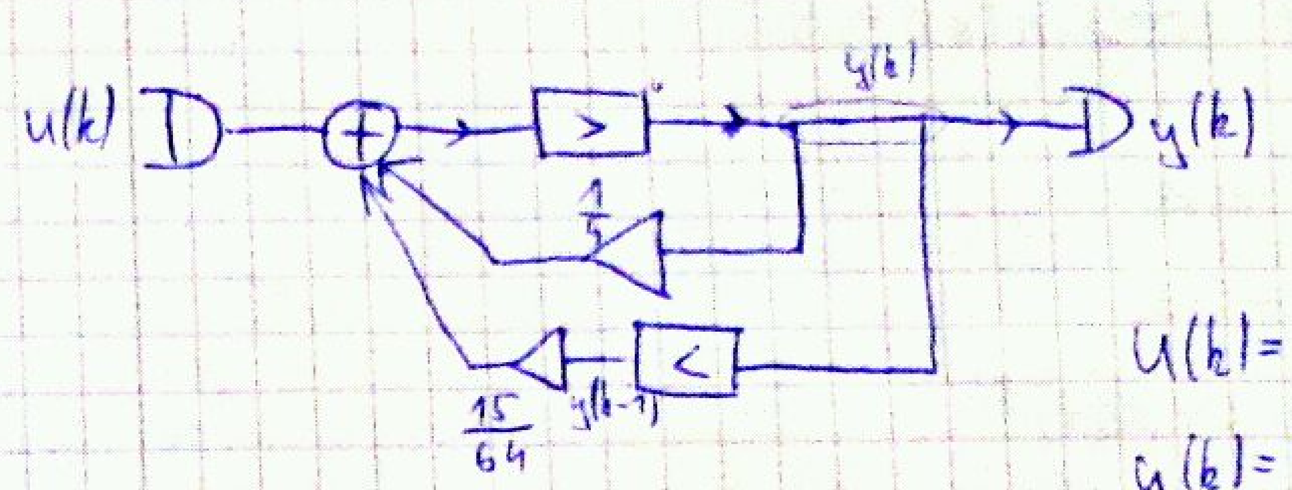
b, fokozatos behelyettesítés (step-by-step)

k	$u(k)$	$x_1(k)$	$x_2(k) = x_3(k)$	$y(k)$
0	1	0	0	4
1	1	0	1	$\frac{31}{3}$
2	1	1	$\frac{4}{3}$	$\frac{130}{9}$
3	1	$\frac{4}{3}$	$\frac{35}{18}$	$\frac{1025}{54}$

$x_1(k+1) = x_2(k)$

$k=0 \to$ hogy $x(1) = x_2(0)$

Rendszeregyenlet:



$u(k) = \left(\frac{1}{4}\right)^k \varepsilon(k)$

$y(k) = ?$

$$y(k) = \frac{1}{4}y(k-1) + \frac{45}{64}y(k-2) + u(k-1)$$

$$y(k) - \frac{1}{4}y(k-1) - \frac{45}{64}y(k-2) = u(k-1) \rightarrow \text{másodrendű differencia egyenlet}$$

homogén megoldás: $y_h(k) = c \cdot \lambda^k \quad (\approx c \cdot e^{x})$

$$c \cdot \lambda^k - \frac{1}{4}c \cdot \lambda^{k-1} - \frac{45}{64}c\lambda^{k-2} = 0 \qquad /:c \; /:\lambda^k \; /\cdot\lambda^2$$

$$\lambda^2 - \frac{1}{4}\lambda - \frac{45}{64} = 0$$

$$\lambda_1 = \frac{5}{8} \qquad \lambda_2 = -\frac{3}{8} \qquad |\lambda_i| < 1 \Rightarrow \text{GV-stabil}$$

partikuláris megoldás: $y_p(k) = A\left(\frac{1}{4}\right)^k \rightarrow u(k)$ gerjesztés miatt

$$A\cdot\left(\frac{1}{4}\right)^k - \frac{1}{4}A\cdot\left(\frac{1}{4}\right)^{k-1} - \frac{45}{64}A\cdot\left(\frac{1}{4}\right)^{k-2} = \left(\frac{1}{4}\right)^{k-1} \qquad /:\left(\frac{1}{4}\right)^k$$

$$A = -\frac{16}{15} \rightarrow y_p(k) = -\frac{16}{15}\left(\frac{1}{4}\right)^k$$

$$y(k) = c_1\left(\frac{5}{8}\right)^k + c_2\left(-\frac{3}{8}\right)^k - \frac{16}{15}\left(\frac{1}{4}\right)^k$$

kezdeti feltétel: $y[0] = 0 = c_1 + c_2 - \frac{16}{15}$

$y[1] = 1 = \frac{5}{8}c_1 - \frac{3}{8}c_2 - \frac{16}{15}\cdot\frac{1}{4}$

$c_1 = \frac{5}{3} \qquad c_2 = -\frac{3}{5}$

$$y(k) = \left(\frac{5}{3}\left(\frac{5}{8}\right)^k - \frac{3}{5}\left(-\frac{3}{8}\right)^k - \frac{16}{15}\cdot\left(\frac{1}{4}\right)^k\right)\cdot\varepsilon(k)$$

F/ $y(k) - 0{,}8\,y(k-1) = u(k) - 2u(k-1)$ (n, m)

a, $u(k) = \delta(k) \rightarrow h(k) = ?$

homogén: $\lambda - 0{,}8 = 0$

$\lambda = 0{,}8$

$y_h(k) = c \cdot (0{,}8)^k$

partikuláris: —

kezdeti feltétel: $k \geq m+1-n = 1+1-1 = 1$

$y(0) = 1 \rightarrow$ nem használjuk, mert $k \geq 1$

$y(1) = -1{,}2 = c \cdot 0{,}8^1$

$c = -1{,}5$

$$h(k) = \underbrace{-1{,}5 \cdot 0{,}8^k \cdot \varepsilon(k-1)}_{\text{ez csak a } k\geq 1\text{-től}} + 1 \cdot \delta(k) =$$

$$= -1{,}5 \cdot 0{,}8^k\,\varepsilon(k) + 2{,}5\,\delta(k)$$

b, $u(k) = \varepsilon(k) \cdot 0{,}6^k$ <u>Konvolúcióval</u>

$$y(k) = \sum_{-\infty}^{\infty} u[i]\,h[k-i] = \sum_{0}^{k} 0{,}6^i \cdot \left(-1{,}5 \cdot 0{,}8^{k-i} + 2{,}5 \cdot \delta(k-i)\right) =$$

$$= -1{,}5 \cdot 0{,}8^k \sum_{0}^{k} \left(\tfrac{3}{4}\right)^i + 2{,}5 \sum_{0}^{k} 0{,}6^i \cdot \delta(k-i) =$$

$$= -1{,}5 \cdot 0{,}8^k \cdot \frac{1-\left(\frac{3}{4}\right)^{k+1}}{1-\frac{3}{4}} + 2{,}5 + 0{,}6^k = \left(-6 \cdot 0{,}8^k + (4{,}5+2{,}5)\,0{,}6^k\right)\varepsilon(k)$$

c, hom: $y_h(k) = C \cdot 0{,}8^k$ F/ $u(k) = 2 - \varepsilon(k)$

part: $y_p(k) = A \rightarrow A - 0{,}8 \cdot A = 1 - 2 \cdot 1 \rightarrow A = -5$

$y(k) = c \cdot 0{,}8^k - 5$

kezdeti feltétel: $y[-1] = -5 \cdot 2 = -10$

$y[0] = -8 + 1 - 4 = -11 = C \cdot 0{,}8^0 - 5 \rightarrow c = -6$

$y(k) = (-6 \cdot 0{,}8^k - 5)\,\varepsilon(k) - 10\,(1 - \varepsilon(k))$

F/ $y(k) - y(k-1) + 0{,}5\,y(k-2) = 2u(k) \rightarrow g(k) = ?$

homogén: $\lambda^2 - \lambda + 0{,}5\lambda = 0 \rightarrow \lambda_{1,2} = \frac{\sqrt{2}}{2} e^{\pm j\frac{\pi}{4}}$

$y_h(k) = 2\,\mathrm{Re}\left\{(c_1 + jc_2)\left(\frac{\sqrt{2}}{2}\right)^k e^{j\frac{\pi}{4}k}\right\} = \left(2 \cdot c_1 \cdot \cos\frac{\pi}{4}k + 2c_2 \sin\frac{\pi}{4}k\right) \cdot \left(\frac{\sqrt{2}}{2}\right)^k$

part: $y_p(k) = A \rightarrow A - A + 0{,}5A = 2 \cdot 1 \rightarrow A = 4$ (egységugrás)

$y(k) = 2 \cdot \left(\frac{\sqrt{2}}{2}\right)^k \left(c_1 \cos\frac{\pi}{4}k + c_2 \sin\frac{\pi}{4}k\right) + 4$

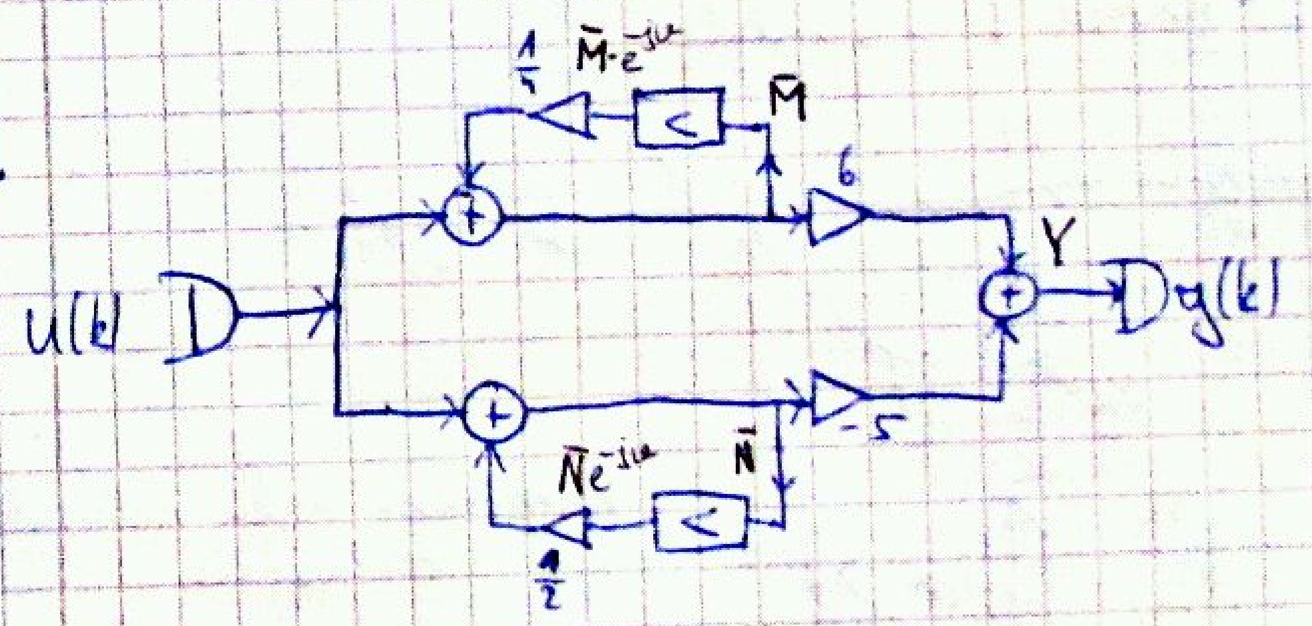
kezd. fel.: $y(0) = 2 \Rightarrow 2 \cdot c_1 + 4 \rightarrow c_1 = -1$

$y(1) = 4 = 2 \cdot \frac{\sqrt{2}}{2}\left(-1 \cdot \frac{\sqrt{2}}{2} + c_2 \cdot \frac{\sqrt{2}}{2}\right) + 4 \rightarrow c_2 = 1$

$$y(k) = \left(2 \cdot \left(\tfrac{\sqrt{2}}{2}\right)^k \left(-\cos\tfrac{\pi}{4}k + \sin\tfrac{\pi}{4}k\right) + 4\right)\varepsilon(k)$$

λ $\quad \vartheta$

4-1/



a, $H(e^{j\vartheta}) = ?$

$\bar{M} = \frac{1}{4}\bar{M}e^{-j\vartheta} + \bar{U} \rightarrow \bar{M} =$

$\bar{N} = \frac{1}{2} \cdot \bar{N}e^{-j\vartheta} + \bar{U} \rightarrow \bar{N} =$

$Y = 6 \cdot \bar{M} - 5\bar{N}$

$$\frac{Y}{U} = \frac{1 - \frac{7}{4}e^{-j\vartheta}}{1 - \frac{3}{4}e^{-j\vartheta} + \frac{1}{8}e^{-j\vartheta 2}} = H(e^{j\vartheta}) = \frac{e^{2j\vartheta} - \frac{7}{4}e^{j\vartheta}}{e^{2j\vartheta} - \frac{3}{4}e^{j\vartheta} + \frac{1}{8}}$$

b, $u(k) = 5\cdot\cos(\frac{\pi}{2}k + 0{,}5) \rightarrow \bar{u} = 5\cdot e^{j0{,}5}$

átviteli tényező: $H(e^{j\frac{\pi}{2}}) = \dfrac{1+\frac{7}{4}j}{1+\frac{3}{4}j-\frac{1}{8}} = 1{,}74\, e^{j0{,}34}$

$\bar{Y} = \bar{u}\cdot H(e^{j\frac{\pi}{2}}) = 5\cdot 1{,}74\, e^{j(0{,}5+0{,}34)}$

$y(k) = \underbrace{5\cdot 1{,}74}_{\text{erősítés}}\cdot\cos(\frac{\pi}{2}k + 0{,}5 + \underbrace{0{,}34}_{\text{fázistolás}})$

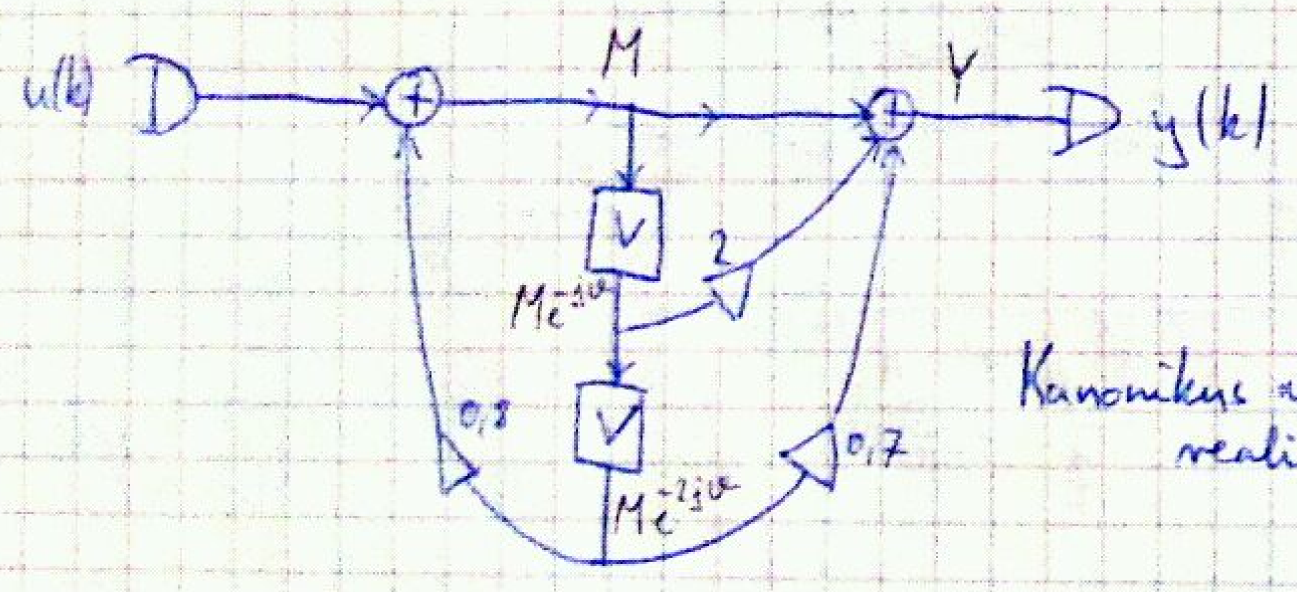
c, rendszeregyenlet:

$\bar{Y}\cdot(1-\frac{3}{4}e^{-j\vartheta}+\frac{1}{8}e^{-2j\vartheta}) = \bar{u}(1-\frac{7}{4}e^{-j\vartheta})$

$y(k) - \frac{3}{4}y(k-1) + \frac{1}{8}y(k-2) = u(k) - \frac{7}{4}u(k-1)$

- nevező | számláló

4-2/



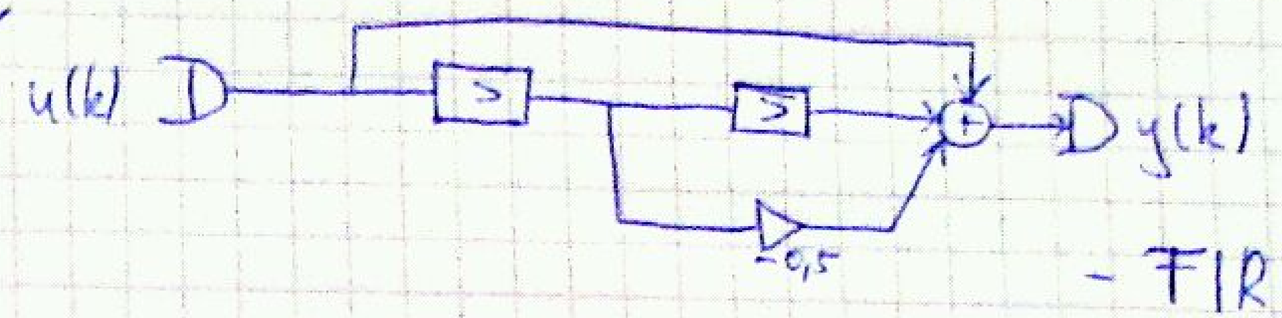
Kanonikus alakú hálózati realizáció

$M = 0{,}8\cdot Me^{-2j\vartheta} + u$

$Y = M(1 + 2e^{-j\vartheta} + 0{,}7e^{-2j\vartheta})$

$H(e^{j\vartheta}) = \dfrac{1+2e^{-j\vartheta}+0{,}7e^{-2j\vartheta}}{1-0{,}8e^{-2j\vartheta}}$

4-3/



- FIR típusú rendszer (véges impulzus válasz)

amplitúdó karakterisztika: $|H(e^{j\vartheta})|$

$H(e^{j\vartheta}) = 1 - 0{,}5e^{-j\vartheta} + e^{-2j\vartheta}$

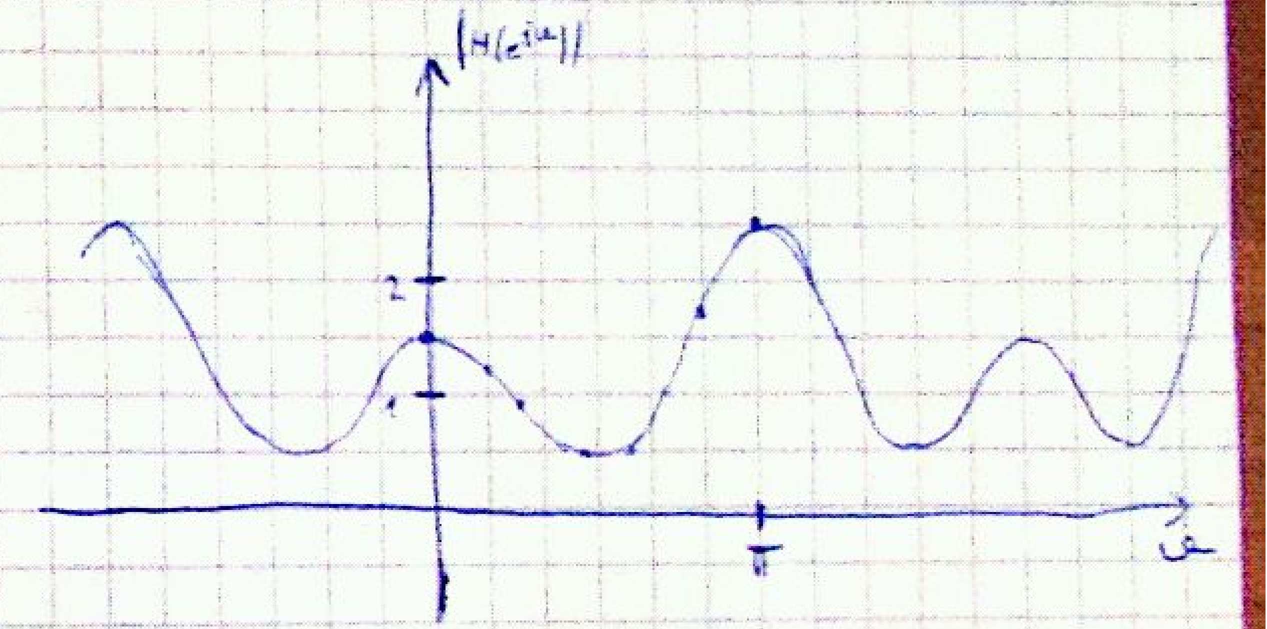
$h(k) = 0$, ha $k > 2$

Nincs visszacsatolás!

korlátosan stabilis

$|H(e^{j\vartheta})| = \sqrt{\underbrace{(1-0{,}5\cos\vartheta+\cos 2\vartheta)^2}_{Re^2} + \underbrace{(0{,}5\cdot\sin\vartheta - \sin 2\vartheta)^2}_{Im^2}}$

ϑ	$\lvert H(e^{j\vartheta})\rvert$
0	1,5
$\frac{\pi}{6}$	1,23
$\frac{\pi}{4}$	0,9
$\frac{\pi}{3}$	0,5
$\frac{\pi}{2}$	0,5
$\frac{3\pi}{4}$	1,5
π	2,5



! páros fv., periodikus (2π)!

kisfrekvencia: 0 környéke

nagyfrekvencia: π környéke

- folytonos fv!!!