06 February 2019

06 February 2019 10:05

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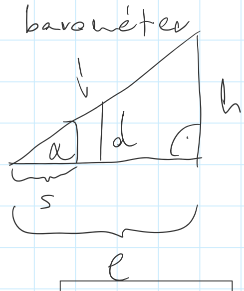
Épületmagasság mérés

1.) $h \approx 100\,m$



$d = 1\,m$ $\quad \Delta d = 2\,mm$

$s = 5\,m$ $\quad \Delta s = 1\,cm$

$l = 500\,m$ $\quad \Delta l = 3\,m$

relatív hiba

$\frac{\Delta d}{d} = 2\cdot 10^{-3}$

$\frac{\Delta s}{s} = 2\cdot 10^{-3}$

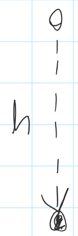
$\frac{\Delta l}{l} = 6\cdot 10^{-3}$

$$\boxed{h = d\cdot\frac{l}{s}}$$

$$\frac{\Delta h}{h} = ? \quad \frac{\Delta d}{d},\ \frac{\Delta s}{s},\ \frac{\Delta l}{l}$$

$$\frac{\Delta h}{h} = \frac{\Delta d}{d} + \frac{\Delta s}{s} + \frac{\Delta l}{l} = 1\%$$

2.)



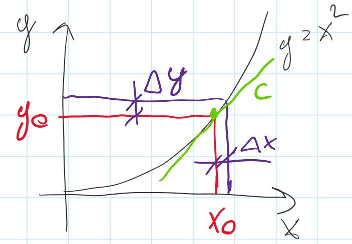
$$h = \frac{g}{2}\cdot t^2$$

$$t = \sqrt{\frac{2h}{g}} \approx 4{,}47\,s \approx 5\,s$$

$\Delta t = 0{,}1\,s$

$\frac{\Delta t}{t} = 2\%$

$$\frac{\Delta h}{h} = \frac{2\Delta t}{t} = 4\%$$



$c = \frac{\partial h}{\partial t}$

$\Delta h = c\cdot\Delta t$

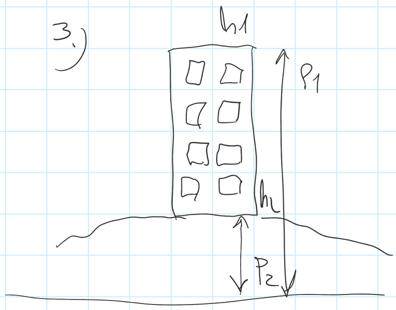
$y_0 = x_0^2$

$$\cancel{y_0} + \Delta y = x^2 = (x_0 + \Delta x)^2 = \cancel{x_0^2} + 2\Delta x\, x_0 + \cancel{\Delta x^2}$$

$$\Delta y \cong 2 x_0 \Delta x$$

$$\frac{\Delta y}{y_0} \cong \frac{2\Delta x}{x_0}$$

3.)



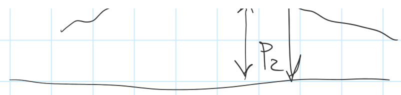
$$p(h) = p_0\cdot e^{-\frac{\rho_0\cdot g}{p_0}\cdot h}$$

$p_0 = 10^5\,Pa$

$\rho_0 = 1{,}29\,\frac{kg}{m^3}$

$$h_1 = -\frac{p_0}{\rho_0\cdot g}\left[\ln p_0 - \ln p_1\right]$$

$$h_2 = -\frac{p_0}{\rho_0\cdot g}\left[\ln p_0 - \ln p_2\right]$$



$\rho_0 = 1{,}29\ \frac{kg}{m^3}$

$g = 9{,}81\ \frac{m}{s^2}$

$$\left\{ h = h_1 - h_2 = \frac{p_0}{\rho_0 \cdot g}\left[\ln p_2 - \ln p_1\right] \right\}$$

$$\frac{\Delta h}{h} = \frac{\cancel{k}\cdot\frac{1}{p_1}\cdot\Delta p_1 \mp \cancel{k}\frac{1}{p_2}\Delta p_2}{\cancel{k}\cdot(\ln p_2 - \ln p_1)} =$$

$$= \frac{1}{\ln p_2 - \ln p_1}\cdot\left(\frac{\Delta p_1}{p_1} \pm \frac{\Delta p_2}{p_2}\right)$$

$$\frac{1}{\ln\frac{p_2}{p_1}} \gg 1$$

$100\,m \rightarrow 1250\,Pa$

$$\frac{1}{\ln\frac{p_2}{p_1}} \approx 80$$

Alapvető mérési módszerek

- közvetlen összehasonlítás
- közvetett összehasonlítás
- helyettesítő módszer
- differencia módszer
- felcserélési (Gauss) módszer

$$m = \sqrt{m_1 \cdot m_2}$$

Mérési hibák

X_h: helyes érték

X_m: mért érték

abszolút hiba: $\Delta X = X_m - X_h$

relatív hiba: $h = \frac{\Delta X}{X_h} \approx \frac{\Delta X}{X_m}$

$h = 1\% = 10^{-2} = 10^4\,ppm$ $\qquad 1\,ppm = 10^{-6}$

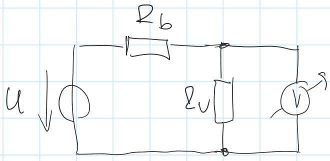
$h = 0{,}01\% = 100\,ppm$

Rendszeres hiba

Véletlen hiba

Hibaszámítás

Rendszeres hiba



$$U_m = U \frac{R_v}{R_v+R_b}$$

$U = \dots = U_m + \Delta U$

$$\hat{U} = U_m \cdot \frac{R_v+R_b}{R_b} =$$

korrekció

$$\Delta U = U_m - U = U \cdot \left[\frac{R_v}{R_v+R_b} - 1\right] =$$

$$= U\left[1 - \frac{R_v}{R_v+R_b}\right]$$

$$h = \frac{\Delta U}{U} = 1 - \frac{R_v}{R_v+R_b} = -\frac{R_b}{R_v+R_b}$$

Fakocka (=?) vaskocka

1kg

$\rho_{fa} = 0{,}8\ g/cm^3$ $\quad\rho_{vas} = 7{,}8\ g/cm^3$

$V = 1250\ cm^3$ $\quad V = 128{,}2\ cm^3$

$\Delta V \cong 1122\ cm^3$

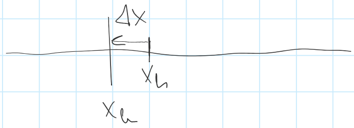
$\rho_{levegő} = 1{,}29 \frac{kg}{cm^3}$

Látszólagos $\Delta m \cong 1{,}44\ g$

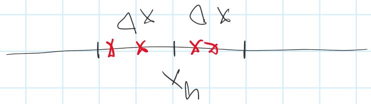
mért érték: K

helyes érték: G

$$\hat{G} = \frac{K}{1 - \frac{\rho_{lev}}{\rho_{fa}}}$$



Véletlen hiba



$X_{ev\,i}$ $\quad i=1,2,\dots$

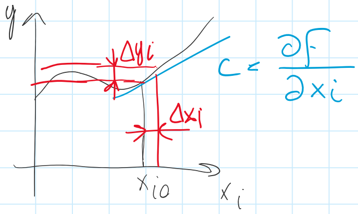
Hibaterjedés

1.) Függvénykapcsolat

$$y = f(x_1, x_2, \dots, x_n) = f(\underline{x})$$

adott $\Delta x_i \Rightarrow \Delta y = ?$

2.) Érzékenység vizsgálat



3.) $\Delta y_i = c \cdot \Delta x_i = \dfrac{\partial f}{\partial x_i} \cdot \Delta x_i \qquad i = 1 \dots N$

4.) Hibaösszegzés

a.) $\Delta y = \sum_{i=1}^{N} \Delta y_i$ előjeles (rendszeresnél használjuk)

b.) $\Delta y = \sum_{i=1}^{N} |\Delta y_i|$ Worst case (véletlennél és rendszeresnél használjuk)

c.) $\Delta y = \sqrt{\sum_{i=1}^{N} (\Delta y_i)^2}$ valószínűségi

5.) Algebrai átalakítás

$$\Delta y_i \Leftrightarrow \Delta x_i \qquad \frac{\Delta y_i}{y},\ \frac{\Delta x_i}{x}$$

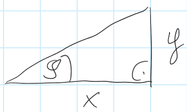
a.) $\Delta y_i = c \cdot \Delta x_i$

b.) $\Delta y_i = c \cdot x_i \dfrac{\Delta x_i}{x_i}$

c.) $\boxed{\dfrac{\Delta y_i}{y} = \dfrac{c \cdot x_i}{f(\underline{x})} \cdot \dfrac{\Delta x_i}{x_i}}$

d.) $\dfrac{\Delta y_i}{y} = \dfrac{c}{f(\underline{x})} \cdot \Delta x_i$

Példa



$\varphi = ?$

$$\varphi = \operatorname{arctg}\Big(\overbrace{\frac{y}{x}}^{r}\Big) = \operatorname{arctg}(r)$$

adott $\dfrac{\Delta x}{x}$, $\dfrac{\Delta y}{y}$, keresett: $\Delta\varphi$

worst case hibaösszegzés

$$\Delta\varphi\Big|_x = \frac{1 \cdot \cancel{x}}{1+r^2} \cdot \Big(\underbrace{-\frac{y}{x^{\cancel{2}}}}_{r}\Big) \cdot \frac{\Delta x}{x} = -\frac{r}{1+r^2} \cdot \frac{\Delta x}{x}$$

$$\Delta\varphi\Big|_y = \frac{1 \cdot y}{1+r^2} \cdot \frac{1}{x} \cdot \frac{\Delta y}{y} = \frac{r}{1+r^2} \cdot \frac{\Delta y}{y}$$

worst case összegzés

$$\Delta\varphi = \underbrace{\frac{r}{1+r^2}}_{g(\varphi)} \cdot \left[\frac{\Delta x}{x} + \frac{\Delta y}{y}\right]$$

Mérőstruktúrák

- kaszkád (soros)
- parallel (párhuzamos)
- visszacsatolt (kör)

Kaszkád struktúra   worst case

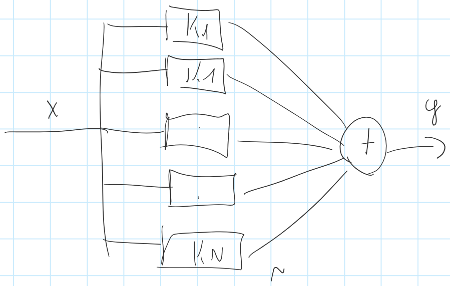
$$X \rightarrow \boxed{K_1} \rightarrow \boxed{K_2} \rightarrow \dots \rightarrow \boxed{K_N} \rightarrow y$$

$$K = \frac{y}{x} = \prod_{i=1}^{N} K_i \qquad \frac{\Delta K}{K} = ? \qquad \frac{\Delta K_i}{K_i} \quad i = 1 \dots N$$

$$\left.\frac{\Delta K}{K}\right|_i = \frac{\prod_{j=1,\, j\neq i}^{N} K_j \cdot K_i}{\prod_{j=1}^{N} K_j} \cdot \frac{\Delta K_i}{K_i} = \frac{\Delta K_i}{K_i}$$

$$\text{w.c.} \quad \frac{\Delta K}{K} = \sum_{i=1}^{N} \left|\frac{\Delta K_i}{K_i}\right|$$

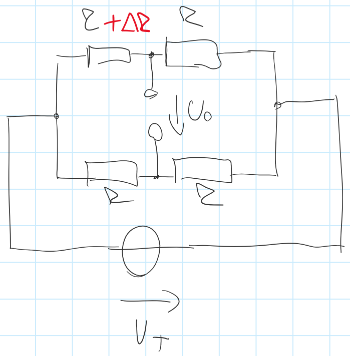
Parallel struktúra



$$K = \frac{x}{y} = \sum_{i=1}^{N} K_i$$

$$\left.\frac{\Delta K}{K_i}\right|_i = \frac{1 \cdot K_i}{\sum_{j=1}^{N} K_j} \cdot \frac{\Delta K_i}{K_i} = \frac{K_i}{\sum_{j=1}^{N} K_j} \frac{\Delta K_i}{K_i}$$

$$\frac{\Delta K}{K} = \sum_{i=1}^{N} \left| \frac{K_i}{\sum_{j=1}^{N} K_j} \frac{\Delta K_i}{K_i} \right|$$



$U_{0,\text{névl}} = 0$

$U_0 \neq 0$!

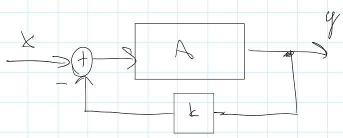
Hibaszámítás

- hibák...
- hibaterjedés...
  - fv kapcsolat
  - érz. számítás
  - hibaösszegzés
    - előjeles
    - worst case
    - valószínűségi
- struktúrák
  - kaszkád
  - parallel
  - visszacsatolt

Struktúrák

- kaszkád $K=\prod_{i=1}^{N} K_i$ (w.c.): $\frac{\Delta K}{K} = \sum_{i=1}^{N} \left|\frac{\Delta K_i}{K_i}\right|$
- parallel $K=\sum_{i=1}^{N} K_i$ (w.c.): $\frac{\Delta K}{K} = \sum_{i=1}^{N} \left|\frac{K_i}{\sum_j K_j}\cdot\frac{\Delta K_i}{K_i}\right|$

- Visszacsatolt struktúra:



$$y = A(x - y\cdot k)$$

$$K = \frac{y}{x} = \frac{A}{1+kA} \underset{A\to\infty}{=} \frac{1}{k}$$

$\frac{\Delta K}{K} = ?$ $\frac{\Delta A}{A}$, $\frac{\Delta k}{k}$

$$\frac{\partial K}{\partial A} = \frac{1+kA-Ak}{(1+kA)^2} = \frac{1}{(1+kA)^2}$$

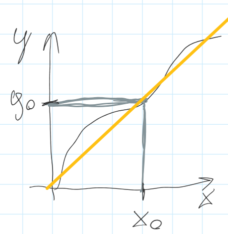
$$\frac{\partial K}{\partial k} = -\frac{A^2}{(1+kA)^2}$$

$$\left.\frac{\Delta K}{K}\right|_A = \frac{1\cancel{A}}{(1+kA)^2}\cdot\frac{\Delta A}{A}\cdot\frac{\cancel{1+kA}}{\cancel{A}} = \frac{1}{1+\underset{H}{kA}}\cdot\frac{\Delta A}{A}$$

$$\left.\frac{\Delta K}{K}\right|_k = -\frac{A^2 k}{(1+kA)^2}\cdot\frac{\Delta k}{k}\cdot\frac{\cancel{1+kA}}{\cancel{A}} = -\frac{kA}{1+kA}\cdot\frac{\Delta k}{k}$$

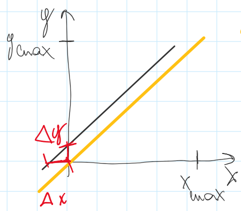
$$\left.\frac{\Delta K}{K}\right|_{A,\ A\to\infty} = 0 \qquad \left.\frac{\Delta K}{K}\right|_{k,\ A\to\infty} = -\frac{\Delta k}{k}$$

Statikus karakterisztikák hibái



$y = x \cdot k$ eredeti görbe

Offset hiba

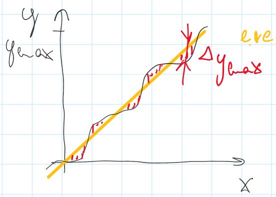


eredeti görbe

$$h = \left|\frac{\Delta y}{y_{max}}\right| \approx \left|\frac{\Delta x}{x_{max}}\right| \qquad \text{(Affin leképezés)}$$

$(ax+b)$ jellegű

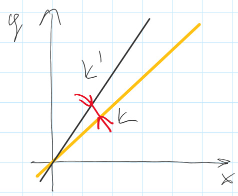
Linearitási hiba



eredeti görbe

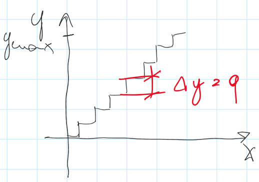
$$h = \left|\frac{\Delta y_{max}}{y_{max}}\right|$$

Erősítési hiba



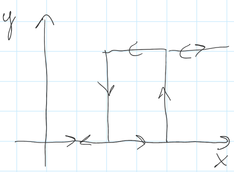
$$h = \left|\frac{k'-k}{k}\right|$$

Kvantálási hiba

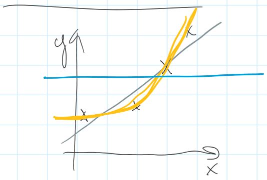


$$h = \left|\frac{q}{y_{max}}\right|$$

Hiszterézis hiba



Görbeillesztés



$(x_i, y_i) \quad i = 1, \dots, N$

$f(x) = ?$

illesztési feladat megoldása:

$\min(C) \Rightarrow f(x)$

1, $f(x)$ meghatározása

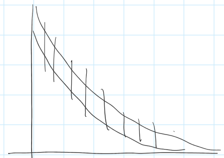
approximáló fv. megválasztása

2, költségfv. meghatározása

$$C = \sum_{i=1}^{N} c\left(f(x_i) - y_i\right)$$

Tipikus választás: négyzetes költségfv

$$C = \sum_{i=1}^{N} \left(f(x_i) - y_i\right)^2$$



lehetséges $f(x)$ függvények:

$$f(x) = ax + b \quad \nearrow ax \quad \searrow b$$

$$f(x) = \sum_{k=0}^{K-1} a_k \cdot x^k$$

$$f(x) = A \cdot e^{-t/\tau}$$

$\vdots$

$f(x) = ax + b \qquad (x_i, y_i) \quad i = 1, \dots, N$

$$C = \sum_{i=1}^{N} \left(y_i - f(x_i)\right)^2 \Rightarrow \min = \sum_{i=1}^{N} \left(y_i - a x_i - b\right)^2 = \sum_{i=1}^{N} \left(y_i^2 + a^2 x_i^2 + b^2 - 2 y_i a x_i - 2 y_i b + 2 a x_i b\right)$$

$$\frac{\partial C}{\partial a} = 0 \quad , \quad \frac{\partial C}{\partial b} = 0$$

$$\frac{\partial C}{\partial a} = \cancel{2} a \overbrace{\sum_{i=1}^{N} x_i^2}^{c_1} - \cancel{2} \overbrace{\sum_{i=1}^{N} y_i x_i}^{c_2} + \cancel{2} b \overbrace{\sum_{i=1}^{N} x_i}^{c_3} = 0$$

$$\frac{\partial C}{\partial b} = \cancel{2} b N - \cancel{2} \overbrace{\sum_{i} y_i}^{c_4} + \cancel{2} a \overbrace{\sum_{i} x_i}^{c_3} = 0$$

$$\left.\begin{aligned} a \cdot c_1 - c_2 + b c_3 &= 0 \\ a \cdot c_3 - c_4 + b N &= 0 \end{aligned}\right\} \quad b = \frac{c_4 - a c_3}{N}$$

$$a\left[c_1 - \frac{c_3^2}{N}\right] = c_2 - \frac{c_4 c_3}{N}$$

$$a = \frac{c_2 - \frac{c_4 c_3}{N}}{c_1 - \frac{c_3^2}{N}} = \frac{N c_2 - c_3 c_4}{N c_1 - c_3^2} = \frac{N \sum_i x_i y_i - \sum_i y_i \sum_i x_i}{N \sum_i x_i^2 - \left(\sum_i x_i\right)^2}$$

$$b = \frac{c_4 - a c_3}{N} = \frac{1}{N} \sum_i y_i - a \frac{1}{N} \sum_i x_i$$

polyfit fv. matlabban

$$f(x) = \sum_{k=0}^{K} a_k x^k$$

$$\boxed{\underline{a} = \underline{\underline{C}}^{-1} \underline{c}} \Leftarrow \boxed{\underline{\underline{C}}\, \underline{a} = \underline{c}}$$

megoldás

egyenletrendszer

K+1 paraméter ⇒ K+1 egyenlet

20 February 2019

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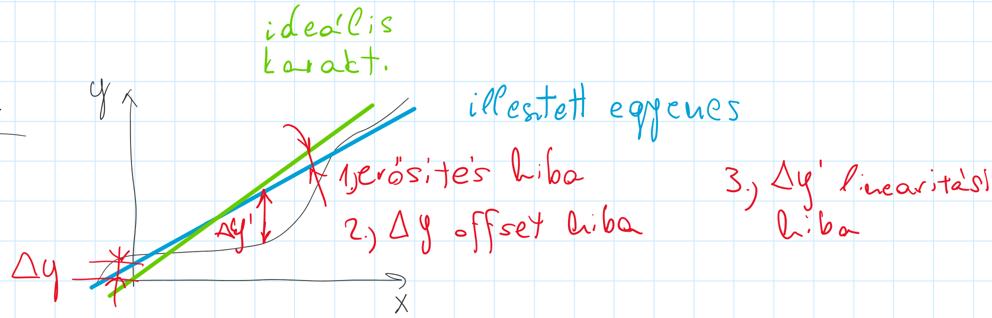
- statikus átalakítók hibái
  - offset
  - linearitási
  - erősítési
  - kvantálási
  - hiszterézis

- görbeillesztés
  - f(x), költségfv.
  - $ax+b$

Általában:

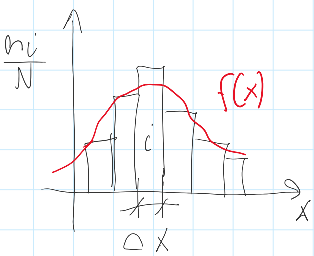


a priori ⇒ egyenes

Valószínűségszámítási áttekintés

Relatív gyakoriság

⇒ hisztogram



valószínűség sűrűség fv.

$$f(x) = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \frac{1}{\Delta x} \frac{n_i}{N}$$

Eloszlásfv.

$$F(x) = \int f(x)\,dx$$

Valószínűség

$$P(x \in [a,b]) = \int_a^b f(x)\,dx$$

Valószínűség

$$p(x \in [a,b]) = \int_a^b f(x)\,dx$$

Tulajdonságok:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad , \quad 0 \leq p \leq 1$$

Definíciók:

várható érték: $\mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

négyzetes várható érték: $\psi^2 = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$

Variancia, szórás négyzet: $\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \cdot f(x)\,dx$

steiner tétel: $\sigma^2 = \psi^2 - \mu^2$

Konvolúció: $y = x_1 + x_2$ (val. vált.)

$$f(x) = f(x_1) * f(x_2)$$

Szórások összegzése:

$x_1, x_2, \ldots$ val. vált. függetlenek!

$\sigma_1, \sigma_2, \ldots$ szórással

$y = \sum_{i=1}^{N} x_i$ vált $\quad \sigma_y = \sqrt{\sum_{i=1}^{N} \sigma_i^2}$

Centrális határeloszlási tétel

$y = \sum_{i=1}^{N} a_i x_i \qquad a_i = 1 \ldots N \quad N \to \infty$

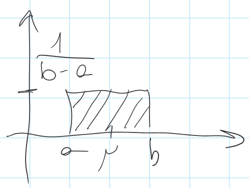
y eloszlása normális függetlenül x_i eloszlásától

$a_i \simeq 1$ $\quad N \gg 1$ y közelítően normális eloszlású

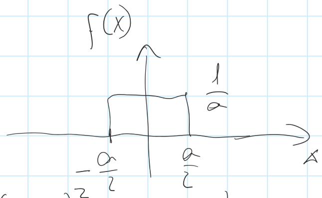
Nevezetes eloszlások:

- egyenletes eloszlás
- binomiális
- Poisson
- exp, geom
- normális (Gauss)

Egyenletes eloszlás:

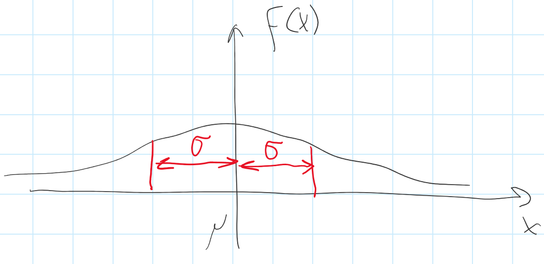


⇒



$$\mu = \frac{a+b}{2}, \quad \sigma^2 = \frac{a^2}{12} = \frac{(b-a)^2}{12} \quad (\mu = 0)$$

Normális (Gauss) eloszlás



$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

F(x): táblázat / számítógép

tulajdonságok:

1.) $f(x) \neq 0$, $-\infty < x < \infty$ elvileg nem korlátos

2.) gyakorlatban korlátos

$$p(|x-\mu| \leq \sigma) = 0{,}68$$

$$p(|x-\mu| \leq 2\sigma) = 0{,}95$$

$$p(|x-\mu| \leq 3\sigma) = 0{,}997$$ praktikus korlát

$$P(|x-\mu|\le 3\sigma) = 0{,}997$$ praktikus korlát

$$P(|x-\mu|\le 4\sigma) = 0{,}9999$$

3.) μ = medián = módusz

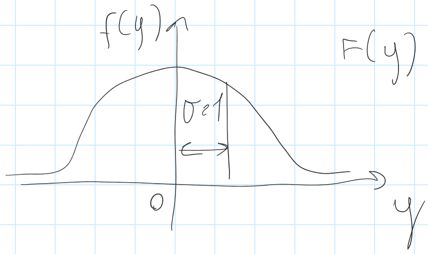
$F(x) = \frac{1}{2}$ $\max[f(x)]$

4.) lineáris (affin) transzformációval szemben invariáns

x: norm. elo., $y = ax + b$ is norm elo.

$$y = [x - \mu(x)]\frac{1}{\sigma(x)}$$ normális elo. $\mu(y) = 0$, $\sigma(y) = 1$

y standard norm. eloszlású



x_1, x_2 norm. elo. $\Rightarrow y = ax_1 + bx_2$ is norm elo

5.) első és másodrendű momentumai egyértelműen meghatározzák:

Ha (μ, σ) adott $\Rightarrow f(x)$ adott

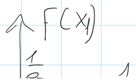
6.) regresszió lineáris

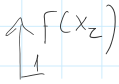
7.) korrelálatlanság $\Leftrightarrow$ függetlenség

Illusztráció: egyenletes eloszlású valváltozók összegzése

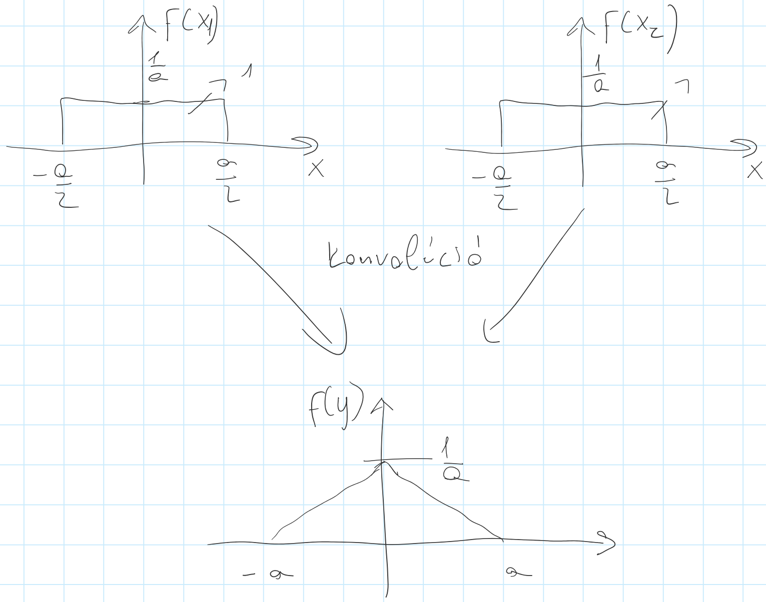
x_1, x_2 egyenletes eloszlású $\left[-\frac{a}{2}; \frac{a}{2}\right]$-ben

$y = ?$

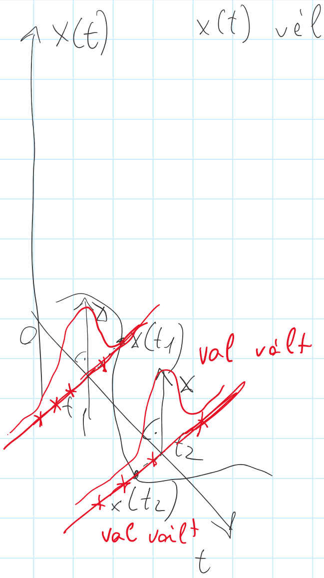




y



Mérési adatok kiértékelése



x(t) véletlen folyamat

x(t) legyen stacionárius

$f(x_1) = f(x_2) = \dots = f(x)$

f(x) nem függ t-től

μ és σ sem változik, nem függ t-től

⇓

Adott $x_i = \mu + n_i$ mérési eredmény

x_i-k egymástól függetlenek

$\boxed{\text{keresett } \mu}$

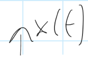
$E\{n_i\} = 0$

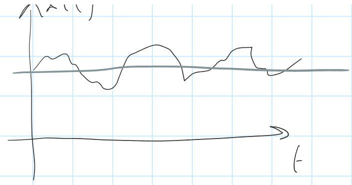
⇓

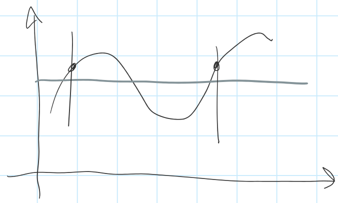
$E\{x_i\} = \mu$

$\mathrm{var}\{n_i\} = E\{n_i^2\} = \sigma^2$ $\qquad$ $\mathrm{var}\{x_i\} = \sigma^2$

$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} x_i \Rightarrow$ folyt köv.

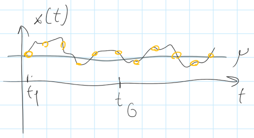



t



- valószín. ismétlés
- mérési adatok kiértékelése

x_i: $i=1 \dots N$ független adatok



stacioner $\mu_1, \sigma_1, \dots, \mu_6, \sigma_6$

$\boxed{\mu, \sigma}$

$x_i = \mu + n_i \qquad E\{x_i\} = \mu \qquad E\{n_i\} = 0$

$\operatorname{var}\{x_i\} = \operatorname{var}\{n_i\} = E\{n_i^2\} = \sigma^2$

$\hat{\mu} = \frac{1}{N} \sum_{i=1}^{N} x_i \qquad E\{\hat{\mu}\} = ?$

$E\{\hat{\mu}\} = E\{\frac{1}{N} \sum x_i\} = E\{\frac{1}{N} \sum (\mu + n_i)\} = \frac{1}{N} E\{\underbrace{\sum \mu}_{N \cdot \mu}\} + \frac{1}{N} E\{\sum n_i\} = \mu$

$\underbrace{\sum E\{n_i\}}_{\emptyset}$

$\operatorname{var}\{\hat{\mu}\} = ?$

$\operatorname{var}\{\hat{\mu}\} = E\{(\frac{1}{N} \sum x_i - \mu)^2\} = E\{(\frac{1}{N} \sum n_i)^2\} = \frac{1}{N^2} E\{(\sum n_i)^2\} = \frac{1}{N^2} E\{\sum_{i=1}^{N} n_i^2 + 2 \sum_{\substack{j=1 \\ j \neq i}}^{N} n_i \cdot n_j\} = \frac{1}{N^2} E\{\sum n_i^2\} + \overbrace{\frac{2}{N^2} E\{\underbrace{\sum n_i}_{0}\} \boxed{i \neq j} E\{\underbrace{\sum n_j}_{0}\}}^{0} =$

$\frac{1}{N} \sum x_i - \mu = \frac{1}{N} \sum (\mu + n_i) - \mu = \underbrace{\frac{1}{N} \sum \mu}_{\cancel{\mu}} + \frac{1}{N} \sum n_i - \cancel{\mu} = \frac{1}{N} \sum n_i$

$= \frac{1}{N^2} \underbrace{\sum \underbrace{E\{n_i^2\}}_{\sigma^2}}_{N \cdot \sigma^2} = \frac{\sigma^2}{N}$

$x_i \Leftrightarrow \mu, \sigma$

$x_i \Rightarrow \hat{\mu}, \hat{\sigma} = ?$

$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$ N független távolság

$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \hat{\mu})^2$ N-1 független távolság

tapasztalati szórásnégyzet
(korrigált)

$E\{\sum_{i=1}^{N} (x_i - \hat{\mu})^2\} = (N-1)\sigma^2 \qquad E\{s^2\} = \sigma^2$

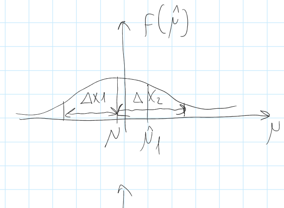
$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \hat{\mu})^2}$

Konfidenciaszámítás

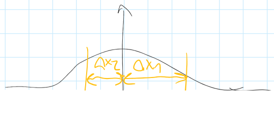
$\hat{\mu} \Leftrightarrow \mu$

val. változó

$P(\underbrace{\mu - \Delta x_1}_{\text{konst}} < \hat{\mu} < \underbrace{\mu + \Delta x_2}_{\text{konst}}) = C$ valószínűségi intervallum, valószínűség

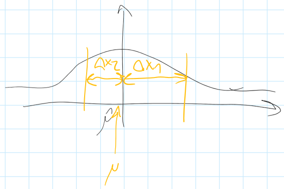


$P(\underbrace{\hat{\mu} - \Delta x_2}_{\text{val. vált}} < \underset{\text{kons}}{\mu} < \underbrace{\hat{\mu} + \Delta x_1}_{\text{val. vált.}}) = C$ konfidencia intervallum, konfidencia szint



val.vált $\uparrow$ val.vált

kons



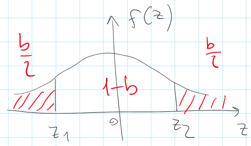
1, Adott x_i, $i=1..N$ független, normális eloszlású mérési adat

adott $\mathrm{var}\{x_i\} = \sigma^2$

Keresett: $\hat{\mu} \pm \Delta x$: C adott érték

$$\hat{\mu} = \frac{1}{N}\sum x_i \qquad \mathrm{var}\{\hat{\mu}\} = \sigma'^2 = \frac{\sigma^2}{N}$$

standard norm. elo. val. vált. $z = \frac{\hat{\mu}-\mu}{\sigma'}$



val. intervallum: $p\left(z_1 < \frac{\hat{\mu}-\mu}{\sigma'} < z_2\right) = C = 1-b$

$z_1 < 0$, $z_2 > 0$

legyen $z_1 = -z_2$

$$p\left(-z_{\frac{b}{2}} < \frac{\hat{\mu}-\mu}{\sigma'} < z_{\frac{b}{2}}\right) = 1-b$$

jelölés

$$z_{\frac{b}{2}} = \int_{z_{\frac{b}{2}}}^{\infty} f(z) = \frac{b}{2}$$

$$\boxed{\text{Konfidenciaintervallum: } p\left(\hat{\mu} - \sigma' \cdot z_{\frac{b}{2}} < \mu < \hat{\mu} + \sigma' \cdot z_{\frac{b}{2}}\right) = 1-b}$$

2, Adott x_i, $i=1\dots N$, független normális eloszlású adatok

σ nem adott

keresett $\hat{\mu} \pm \Delta x$

$$s^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \hat{\mu})^2 \qquad E\{s^2\} = \sigma^2$$

?? $\quad \sigma' \rightarrow s' = \frac{s}{\sqrt{N}}$

s^2 eloszlása ?

$$s^2 = \frac{1}{N-1}\sum (x_i - \hat{\mu})^2$$

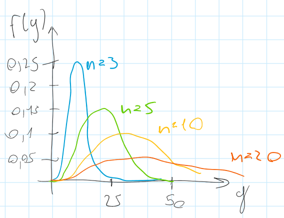
$$y = \sum_{i=1}^{n} z_i^2 \qquad z_i = \text{standard. norm. elo.}$$

χ_n^2: val. vált.: n szabadságfokú χ^2 eloszlású val. vált.

$$f(y) = \left[2^{\frac{n}{2}} \cdot \Gamma\left(\frac{n}{2}\right)\right]^{-1} y^{\left(\frac{n}{2}-1\right)} e^{-\frac{y}{2}}$$

$$\Gamma(x) = \int_0^x e^{-t} t^{x-1} dt$$

$$\Gamma(k+1) = k!$$



27 February 2019

27 February 2019 10:13

Mérési adatok kiértékelése

- $\hat{\mu} = \frac{1}{N} \sum_{N} x_i$ $\quad$ $var\{x_i\} = \sigma^2$

$\boxed{x_i \text{ fgtlen}}$ $\quad$ $var\{\hat{\mu}\} = \frac{\sigma^2}{N}$

- konfidencia számítás

$\boxed{x_i \text{ fgtlen, norm. elo.}}$

1, σ ismert $\longrightarrow$ $P\left(\hat{\mu} - \frac{\sigma}{\sqrt{N}} z_{\frac{b}{2}} < \mu < \hat{\mu} + \frac{\sigma}{\sqrt{N}} z_{\frac{b}{2}}\right) = 1-b$

2, σ nem ismert

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \hat{\mu})^2$$

$$\sigma' = \frac{\sigma}{\sqrt{N}} \longrightarrow \quad s' = \frac{s}{\sqrt{N}} ?$$

$$\frac{\hat{\mu} - \mu}{\sigma'} \longrightarrow \frac{\hat{\mu} - \mu}{s'}$$

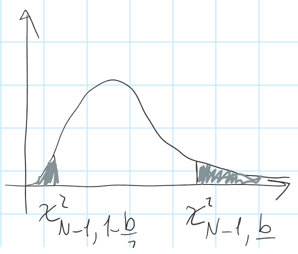
val.vált. $\quad$ val.vált.

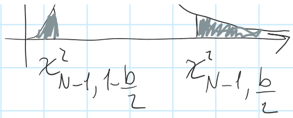
z $\quad$?

$$y = \sum_{i=1}^{n} z_i^2 = \chi_n^2$$

$$s^2 = \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \hat{\mu})^2 = \frac{\sigma^2}{N-1} \cdot \sum_{i=1}^{N} \left(\frac{x_i - \hat{\mu}}{\sigma}\right)^2 = \frac{\sigma^2}{N-1} \chi_{N-1}^2$$

Val. intervallum $\quad$ $P\left[\frac{\sigma^2}{N-1} \cdot \chi^2_{N-1, 1-\frac{b}{2}} < s^2 < \frac{\sigma^2}{N-1} \chi^2_{N-1, \frac{b}{2}}\right] = 1-b$





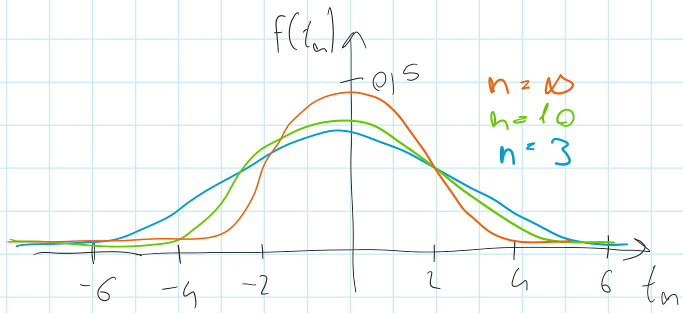
Konfidenciaintervallum:
$$P\left[\frac{s^2(N-1)}{\chi^2_{N-1,\frac{b}{2}}} < \sigma^2 < \frac{s^2(N-1)}{\chi^2_{N-1,1-\frac{b}{2}}}\right] = 1-b$$

$$P\left[\sigma^2 < \frac{s^2(N-1)}{\chi^2_{N-1,1-b}}\right] = 1-b \qquad \sigma^2 \geq 0$$

$$\frac{\hat{\mu}-\mu}{s'} = \frac{\hat{\mu}-\mu}{\frac{s}{\sqrt{N}}} = ?$$

Student-t eloszlás

$$t_n = \frac{z}{\sqrt{\frac{y}{n}}} \qquad t_n = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\sqrt{\pi n}\,\Gamma\left(\frac{n}{2}\right)}\left(1+\frac{t_n^2}{n}\right)^{-\frac{n+1}{2}}$$



$$E\{t_n\} = 0$$

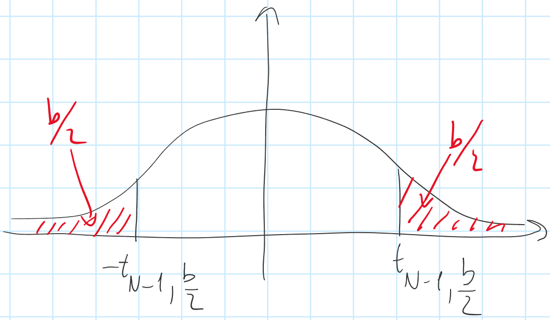
$$\mathrm{var}\{t_n\} = \frac{n}{n-2}$$

$$n > 2$$

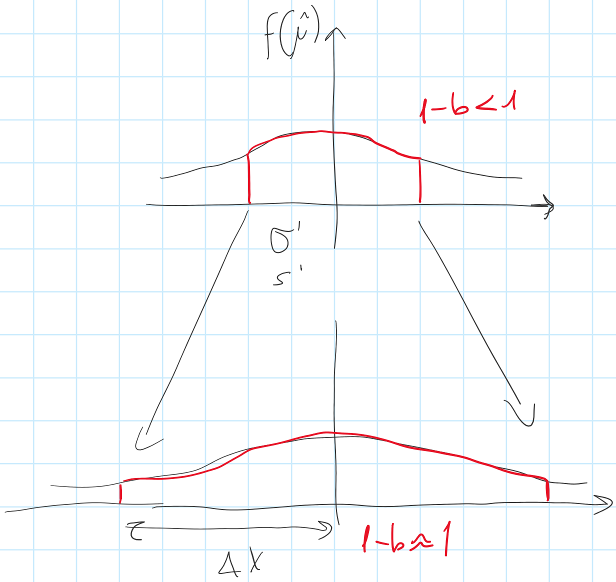
$$\frac{\hat{\mu}-\mu}{s'} = \frac{\hat{\mu}-\mu}{\frac{s}{\sqrt{N}}} = \frac{z\cdot\frac{\cancel{\sigma}}{\cancel{\sqrt{N}}}}{\sqrt{\frac{\cancel{\sigma^2}}{(N-1)\cancel{N}}\chi^2_{N-1}}} = \frac{z}{\sqrt{\frac{\chi^2_{N-1}}{N-1}}} = t_{N-1}$$

val. int.: $P\left[-t_{N-1,\frac{b}{2}} < \frac{\hat{\mu}-\mu}{s'} < t_{N-1,\frac{b}{2}}\right] = 1-b$

$$\text{val. int.: } P\left[-t_{N-1,\frac{b}{2}} < \frac{\hat{\mu}-\mu}{s'} < t_{N-1,\frac{b}{2}}\right] = 1-b$$



$$\text{konf. int.: } P\left[\hat{\mu} - \frac{s}{\sqrt{N}}\, t_{N-1,\frac{b}{2}} < \mu < \hat{\mu} + \frac{s}{\sqrt{N}}\, t_{N-1,\frac{b}{2}}\right] = 1-b$$



$\Delta x = k \cdot \sigma'(s')$

$k = z_{\frac{b}{2}}$, ha norm elo

$k = t_{N-1,\frac{b}{2}}$ ha student-elo

x_i eloszlása	konf. célja	szórás	eloszlás	megjegyzés
norm	μ	$\frac{\sigma}{\sqrt{N}}$	norm	—
norm	μ	$\frac{s}{\sqrt{N}}$	Student	—
—	μ	$\frac{\sigma}{\sqrt{N}}, \frac{s}{\sqrt{N}}$	norm	$N \gg 1$
norm	x	σ, s	norm	$N \gg 1$, $\hat{\mu} \approx \mu$
—	μ	$\frac{\sigma}{\sqrt{N}}, \frac{s}{\sqrt{N}}$	—	Csebisev-egyenlőtlenség

—	μ	$\frac{\sigma}{\sqrt{N}}$, $\frac{s}{\sqrt{N}}$	—	Csebisev-egyenlőtlenség

$$P\left[|\bar{\mu}-\mu| \le k\cdot\sigma'\right] > 1-\frac{1}{k^2}$$

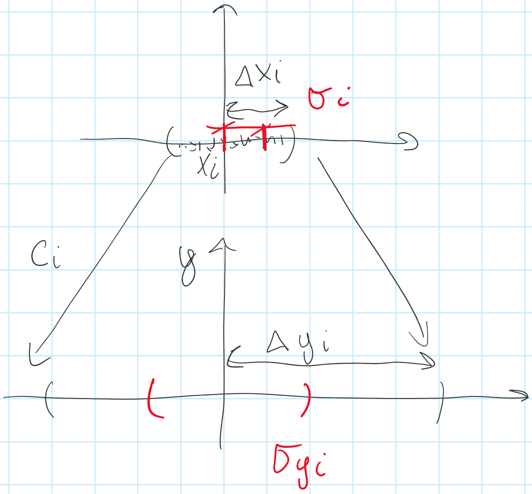
norm. elo. 1-6 ≈ 95%

$$z_{\frac{b}{2}} \cong 2$$

elo nem ismert $\frac{1}{k^2}=\frac{1}{20}$ $\quad k=\sqrt{20}=4{,}5$

$$y=f(x_1,x_2,\dots,x_M)$$

$$\Delta y_i=\underbrace{\frac{\partial f}{\partial x_i}}_{c_i}\cdot\Delta x_i \qquad i=1\dots M$$



$$\sigma_{y_i}=c_i\cdot\sigma_i$$

$$\sigma_y=\sqrt{\sum_{i=1}^{M}\sigma_{y_i}^2}=\sqrt{\sum_{i=1}^{M}c_i^2\cdot\sigma_i^2}$$

Legyen x_i, $i=1\dots M$ eloszlása normális

z_i a konf. int. változója $\left(z_i=z_{\frac{b}{2},i}\right)$

$$\Delta x_i=z_i\,\sigma_i \qquad \sigma_i=\frac{\Delta x_i}{z_i}$$

$$\Delta y=z_y\cdot\sigma_y$$

$$\Delta y=z_y\cdot\sqrt{\sum_{i=1}^{M}c_i^2\cdot\sigma_i^2}=z_y\cdot\sqrt{\sum_{i=1}^{M}c_i^2\cdot\frac{(\Delta x_i)^2}{z_i^2}}=\cancel{z}\cdot\sqrt{\sum_{i=1}^{M}c_i^2\frac{(\Delta x_i)^2}{\cancel{z^2}}}=\sqrt{\sum_{i=1}^{M}c_i^2(\Delta x_i)^2}$$

$$\Delta y = z_y \cdot \sqrt{\sum_{i=1}^{n} c_i^2 \cdot \sigma_i^2} = z_y \cdot \sqrt{\sum_{i=1}^{n} c_i^2 \cdot \frac{(\Delta x_i)^2}{z_i^2}} = \cancel{z} \cdot \sqrt{\sum_{i=1}^{n} c_i^2 \frac{(\Delta x_i)^2}{\cancel{z^2}}} = \sqrt{\sum_{i=1}^{n} c_i^2 (\Delta x_i)^2}$$

legyen $z_1 = z_2 = \ldots = z_n = z_y = z$

$P = U \cdot I$ $\qquad U = U_0(1 \pm h_1)$ $\qquad \Delta U = U_0 \cdot h_1$

$\qquad I = I_0(1 \pm h_2)$ $\qquad \Delta I = I_0 \cdot h_2$

$$U \cdot I = U_0 \cdot I_0 (1 \pm h_1)(1 \pm h_2) = U_0 I_0 (1 \pm h_1 \pm h_2 \pm h_1 \cdot h_2)$$

06 March 2019

06 March 2019 10:12

$$P[\hat{\mu} - \sigma' z_{\frac{b}{2}} < \mu < \hat{\mu} + \sigma' z_{\frac{b}{2}}] = 1-b$$

$$P[\hat{\mu} - s' t_{N-1,\frac{b}{2}} < \mu < \hat{\mu} + s' t_{N-1,\frac{b}{2}}] = 1-b$$

$$\sigma' = \frac{\sigma}{\sqrt{N}}$$

$$s' = \frac{s}{\sqrt{N}}$$

1. „valami" - erre vonatkozik a konf. int
2. valami
3. σ', s': „valami" szórására

$$\Delta y_i = c_i \cdot \Delta x_i$$

$$\Downarrow$$

$$\sigma y_i = c_i \cdot \sigma x_i$$

$$\sigma y = \sqrt{\sum_{i=1}^{M} c_i^2 \cdot \sigma x_i^2}$$

$$\Delta y = z_{\frac{b}{2}} \cdot \sigma y \qquad \sigma x_i = \frac{\Delta x_i}{z'_{\frac{b}{2}}}$$

$$z = z'$$

$$\Delta y = \sqrt{\sum_{i=1}^{M} c_i^2 \Delta x_i^2}$$

GUM (Guide to the Expression of Uncertanity in Measurement)

Mérési bizonytalanság szabványos kifejezése

hiba => bizonytalanság

rendszeres hiba => korrekció

„5"

~~rendszeres~~ hiba ⇒ korrekció

véletlen hiba ⇒ bizonytalanság ⇒ "σ" standard / kiterjesztett } bizonytalanság

nα x^{n}

A típusú bizonytalanság

$$\hat{x}_i = \frac{1}{N_i}\sum_{j=1}^{N_i} x_{ij} \qquad s_i = \sqrt{\frac{1}{N_i-1}\sum_{j=1}^{N_i}\left(x_{ij}-\hat{x}_i\right)^2}$$

A típusú sztenderd bizonytalanság:

$$u_A(x_i) = \frac{s_i}{\sqrt{N_i}}$$

$$N_i = 1 \Rightarrow u_A(x_i) = 0$$

B típusú bizonytalanság

- a priori ismeret
- valószínűség sűrűségfv. ⇒ σ
- műszer specifikációja (adatlapja)

} Δx_i, σ_i

$$u_B(x_i) = \sigma$$

Egyesített standard bizonytalanság

$$u(x_i) = \sqrt{u_A^2(x_i) + u_B^2(x_i)}$$

GUM lépései

1.) $y = g(x_1, x_2, \ldots, x_K)$

2.) $y = f(x_1, x_2, \ldots, x_K, \ldots, x_M)$ $\quad M > K$

2.) $y = f(x_1, x_2, \dots, x_K, \dots, x_M)$ $\quad M > K$

3.) $u_A(x_i)$ $\quad i = 1 \dots M$

4.) $u_B(x_i)$ $\quad i = 1 \dots M$

5.) $u(x_i) = \sqrt{u_A^2(x_i) + u_B^2(x_i)}$ $\quad i = 1 \dots M$, $\quad \hat{x}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} x_{ij}$

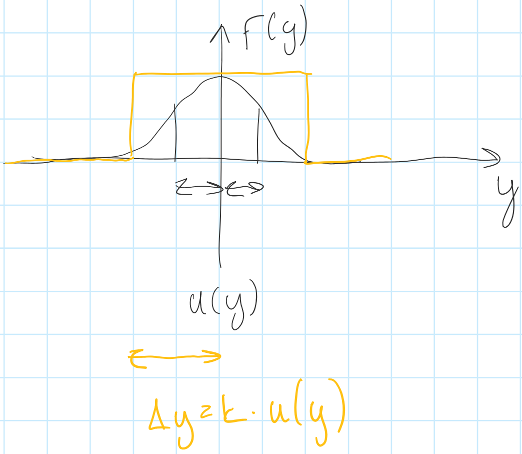
6.) $c_i = \frac{\partial F}{\partial x_i}$ $\quad i = 1 \dots M$

7.) eredő standard bizonytalanság

$$u(y) = \sqrt{\sum_{i=1}^{M} c_i^2 u^2(x_i)}$$

8.) Kiterjesztett bizonytalanság

$\Delta y = k \cdot u(y)$ $\quad k = 2 \dots 3$



9.) Mérési eredmény megadása

$\hat{y} = F(\underline{\hat{X}})$ $\qquad \boxed{y = \hat{y} \pm \Delta y, \; k = \text{konst}}$ (2 v. 3)

$\hat{y} = xxxxx$ $\qquad \Delta y = 0.bb$ $\qquad y = xxxxx.xx\,(bb)$

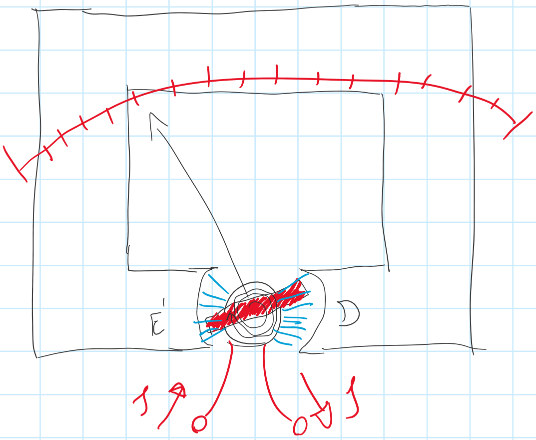
12345.67 $\qquad$ 0.89 $\qquad$ $12345.67 \pm 0,89$

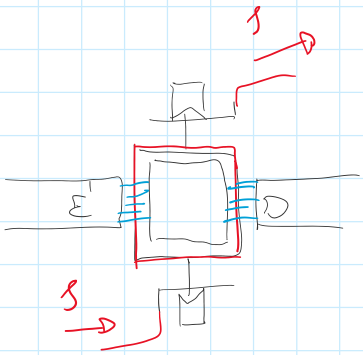
Feszültség és áram mérése

– DC mérés $\qquad$ – analóg

– AC mérés – digitális

Deprez-műszer (állandómágneses forgótekercses műszer)





$$M = B \cdot N \cdot A \cdot J$$

$$M(t) = c \cdot J(t)$$

$$M(t) = \Theta \frac{d^2\varphi}{dt^2} + k \cdot \frac{d\varphi}{dt} + D\varphi(t)$$

$$\frac{M(t)}{\Theta} = \frac{d^2\varphi}{dt^2} + \frac{k}{\Theta}\frac{d\varphi}{dt} + \underbrace{\frac{D}{\Theta}}_{\omega_0^2}\varphi(t)$$

$$\frac{k}{\Theta} = 2 \cdot \underbrace{\frac{k}{2\sqrt{\Theta D}}}_{\xi} \cdot \omega_0$$

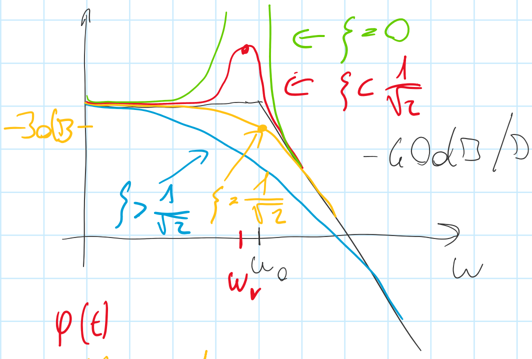
$$\frac{M(t)}{\Theta} = \frac{d^2\varphi}{dt^2} + 2\xi\omega_0\frac{d\varphi}{dt} + \omega_0^2\varphi(t)$$

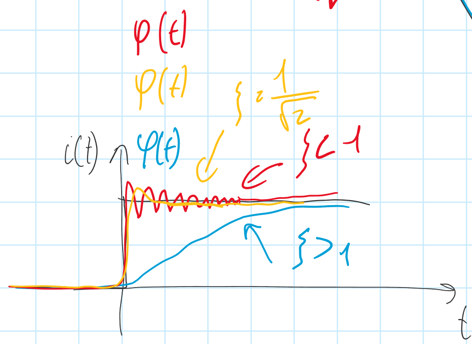
$$\frac{M(s)}{\Theta} = s^2\varphi(s) + 2\xi\omega_0\varphi(s) + \omega_0^2\varphi(s)$$

$$W(s) = \frac{\varphi(s)}{M(s)} = \frac{1}{\Theta}\,\frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

$$W(s) = \frac{\varphi(s)}{M(s)} = \frac{1}{\Theta} \frac{1}{s^2 + 2\xi\omega_0 s + \omega_0^2}$$

$$\omega_r = \omega_0\sqrt{1-2\xi^2}$$





Feszültség és áram mérése

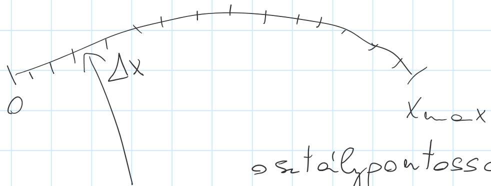
- analóg, digitális
- DC, AC

Deprez műszer

Áramot mér, R_m = tip 1kΩ

$I_{max} \to \varphi_{max}$ $\qquad$ I_{max} tip 100μA

$I_{max} \cdot R_m = U_{max}$ $\qquad$ U_{max} tip 100mV



$x = U, I$

osztálypontosság

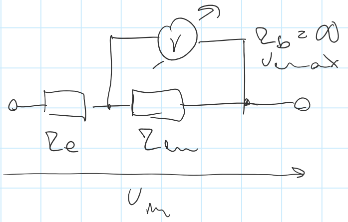
$$h = \frac{\Delta x}{X_{max}} \Rightarrow op = 100 \cdot \frac{\Delta x}{X_{max}} = 100 \cdot h$$

$$\frac{\Delta U}{U}, \frac{\Delta I}{I} \Rightarrow \frac{\Delta x}{x} = \frac{op}{100} \cdot \frac{X_{max}}{X}$$

Méréshatár kiterjesztése

Feszültségmérés

előtétellenállás R_e



R_{be}

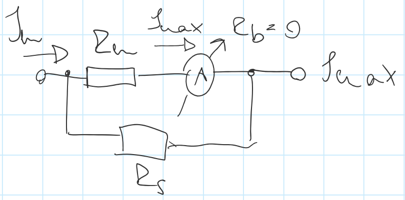
$U_m = k \cdot U_{max}$

$I_m = k \cdot I_{max}$

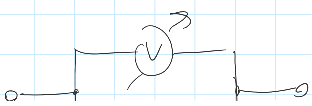
$R_e = (k-1) \cdot R_m$

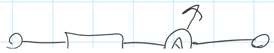
Árammérés

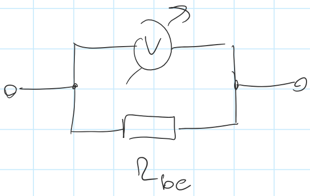
sönt ellenállás R_s

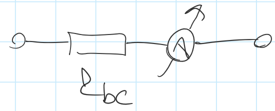


$$R_s = \frac{R_m}{k-1}$$









$R_{be} = k \cdot R_m$

$R_{be} = \dfrac{R_m}{k}$

$U_m = 100\,mV, 300\,mV, 1V, 3V, 10V, \ldots$

$I_m = 100\,\mu A, 300\,\mu A, 1\,mA, 3\,mA, \ldots, 10\,A$

Digitális műszer

U: $R_{be} = $ tip $1\,M\Omega$

I $R_{be} = $ tip $1\,\Omega$

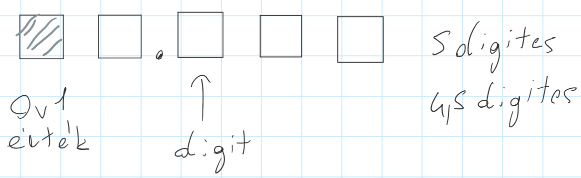
$$\frac{\Delta x}{x} = h_1 + h_2 \frac{X_{max}}{x} + h_3$$

h_1: h of value (mértértékre)

h_2: h of range (végértékre)

h_3: $\frac{1}{N}$ N kijelzett szám tizedespont nélkül (kvantálási hiba)

gépkönyv!



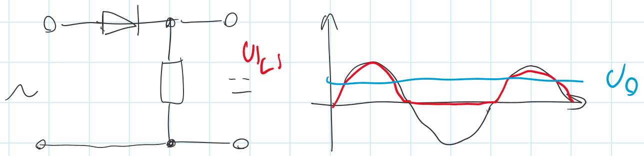
Váltakozó feszültség és áram mérése

AC mérés: mérési elv:
- csúcsérték mérő
- abszolút középérték mérő
- effektív érték mérő

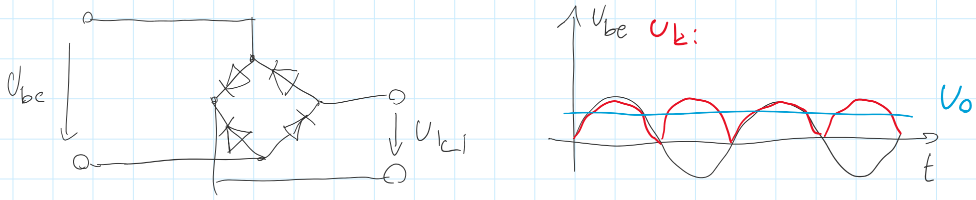
működési mód:
- egyenirányítás + DC mérés
- lágyvasas

- elektrodinamikus
- termoátalakító

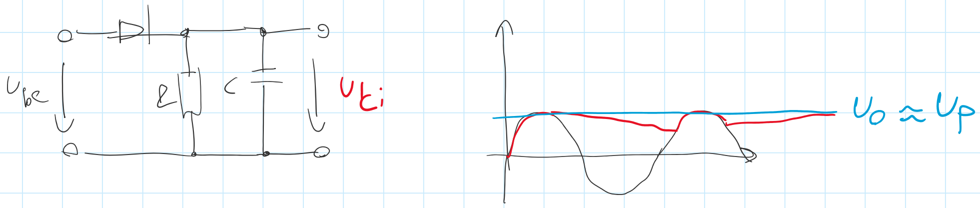
Egyenirányítók



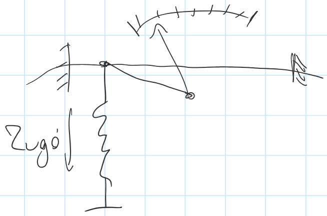
Graetz-híd



Csúcsegyenirányítás



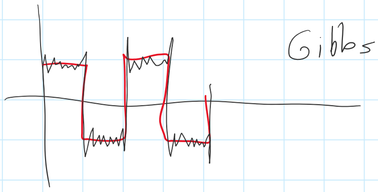
Termoátalakítás



AC jelek ⇒ periodikus jelek

Fourier sor fejtés

$$x(t) \hat{=} x_0 + \sum_{k=1}^{\infty} X_k^A \cos(k\omega t) + \sum_{k=1}^{\infty} X_k^B \sin(k\omega t)$$



Gibbs-oszcilláció

$$X_0 = \frac{1}{T}\int_0^T x(t)\,dt \;,\quad X_k^A = \frac{2}{T}\int_0^T x(t)\cos(k\omega t)\,dt \;,\quad X_k^B = \frac{2}{T}\int_0^T x(t)\sin(k\omega t)\,dt$$

$$x(t) \hat{=} \sum_{k=-\infty}^{\infty} X_k^C e^{jk\omega t} \qquad X_k^C = \frac{1}{T}\int_0^T x(t)\, e^{-jk\omega t}\,dt$$

valós jelre: $X_k^C = \overline{X}_{-k}^C$

Egyszerű középérték

$$X_0 = \frac{1}{T}\int_0^T x(t)\,dt \approx \frac{1}{N}\sum_{n=0}^{N-1} x(n)$$

Abszolút középérték

$$X_{ABS} = \frac{1}{T}\int_0^T |x(t)|\,dt \approx \frac{1}{N}\sum_{n=0}^{N-1} |x(n)|$$

Effektív érték

$$X_{eff} = \sqrt{\frac{1}{T}\int_0^T |x(t)|^2\,dt} \approx \sqrt{\frac{1}{N}\sum_{n=0}^{N-1} |x(n)|^2}$$

13 March 2019

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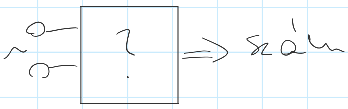
Feszültség és áram mérése

$$X_{eff} = \sqrt{\frac{1}{T}\int_0^T |x(t)|^2 \, dt} \cong \sqrt{\frac{1}{N}\sum_{i=1}^{N} |x_i|^2}$$

$$X_0 = \frac{1}{T}\int_0^T x(t) \, dt$$

$$X_{abs} = \frac{1}{T}\int_0^T |x(t)| \, dt$$



Effektív érték számítása

$$X_{eff} = \sqrt{\sum_{i=1}^{\infty} (X_{i_{eff}})^2} \qquad (X_0 = 0) \qquad X_{ip}: \text{csúcsérték}$$

$$X_{i_{eff}} = \frac{X_{ip}}{\sqrt{2}}$$

$$= \sqrt{\sum_{i=1}^{\infty} \frac{X_{ip}^2}{2}}$$

Torzítási tényező

$$k = \sqrt{\frac{\sum_{i=2}^{\infty} X_{i_{eff}}^2}{\sum_{i=1}^{\infty} X_{i_{eff}}^2}} \cong \sqrt{\frac{\sum_{i=2}^{\infty} X_{i_{eff}}^2}{X_{1_{eff}}^2}}$$

Csúcstényező

$$k_p = \frac{X_p}{X_{eff}}$$

Formatényező

$$k_f = \frac{X_{eff}}{X_{abs}}$$

dB-skála

- relatív/abszolút
- jelszintre/teljesítményre

1.) jelszintekre, relatív

$W_{lin} = \frac{X}{X_{ref}}$ $\qquad W(dB) = 20\lg(W_{lin})$

2.) jelszintekre, abszolút

X_{ref} rögzített $\qquad$ pl.: $U_{ref} = 0{,}775\,V$

$X(dB) = 20\lg\left(\frac{X}{X_{ref}}\right)$ $\qquad$ U_{ref} @ $600\,\Omega \Rightarrow P = 1\,mW$

3.) teljesítményre, relatív

$W_{lin} = \frac{P}{P_{ref}}$ $\qquad W(dB) = 10\log(W_{lin})$

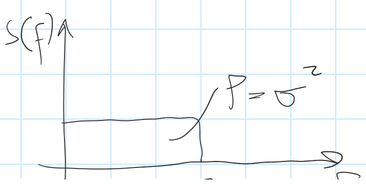
4.) teljesítményre, abszolút

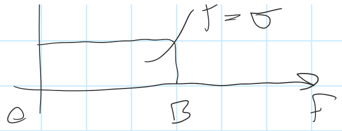
P_{ref} rögzített! $\qquad$ pl.: $P_{ref} = 10^{-12}\,W$ (hangintenzitás)

hallásküszöb

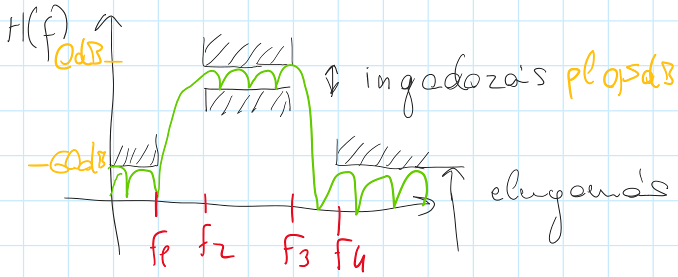
$P(dB) = 10\log\left(\frac{P}{P_{ref}}\right)$

zaj => sávkorlátozott fehérzaj





Szűrés

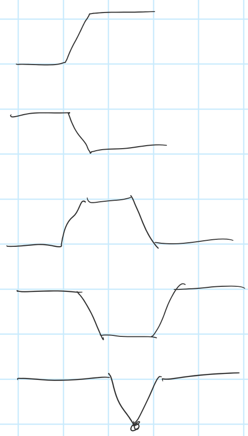


$f_2 \ldots f_3$: áteresztősáv

$\left.\begin{matrix} 0 \ldots f_1 \\ f_4 \ldots \infty \end{matrix}\right\}$ zárósáv

$\left.\begin{matrix} f_1 \ldots f_2 \\ f_3 \ldots f_4 \end{matrix}\right\}$ átmeneti tartomány

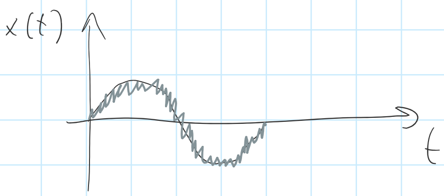
- aluláteresztő, LPF
- felüláteresztő, HPF
- sáváteresztő, BPF
- sávzáró, BSF
- lyukszűrő

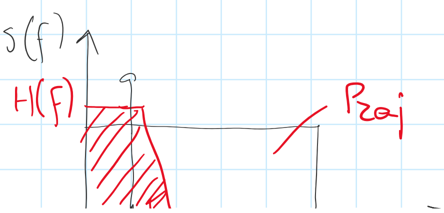


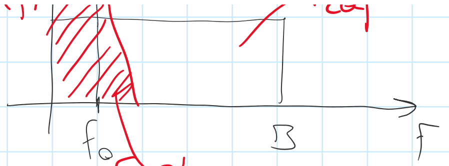
$$x(t) = s(t) + n(t)$$

$$s(t) = X_p \sin(\omega t) \qquad \omega = 2\pi f$$

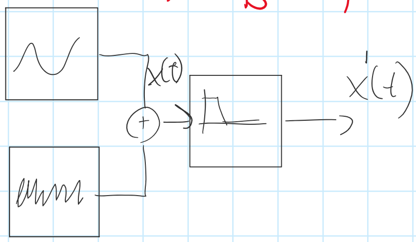
$$n(t) = \text{Gauss-zaj}$$







$f_c P'_{zaj} = \frac{f_c}{B} P_{zaj}$



Jel-zaj viszony, SNR

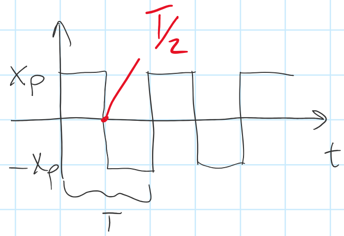
$$SNR = 10 \lg \frac{P_{jel}}{P_{zaj}} \quad [dB]$$

Pl.:

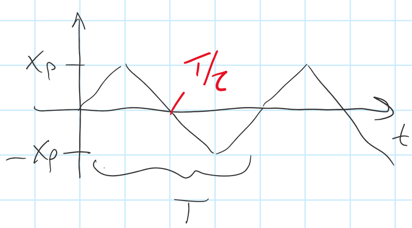
$$SNR' = 10 \lg \frac{P_{jel}}{P'_{zaj}} = 10 \lg \left(\frac{B}{f_c} \frac{P_{jel}}{P_{zaj}} \right) =$$

$$= 10 \lg \left(\frac{B}{f_c} \right) + SNR$$

négyszögjel:



háromszögjel



jel	X_0	X_{abs}	X_{eff}
~	0	$2/\pi$	$1/\sqrt{2}$
⊓⊔	0	1	1
/\/\	0	$1/2$	$1/\sqrt{3}$
Gauss-zaj	0	$\sqrt{\frac{2}{\pi}}\,\sigma$	σ

Kijelzés szinuszos jel effektív értékére vonatkoztatva

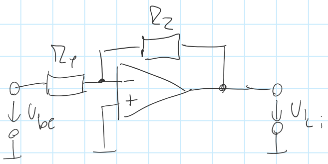
Jel	eff. érték mérő			abs érték mérő			csúcsérték mérő		
	mér	mutat	szorzó	mér	mutat	szorzó	mér	mutat	szorzó
~	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\frac{2}{\pi}$	$\frac{1}{\sqrt{2}}$	k_f 1,11	1	$\frac{1}{\sqrt{2}}$	$\frac{1}{k_p}$ = 0,707
⊓⊔	1	1	1	1	$\frac{\pi}{2\sqrt{2}}$	1,11	1	$\frac{1}{\sqrt{2}}$	0,707
/\/\	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	1	$\frac{1}{2}$	$\frac{\pi}{4\sqrt{2}}$	1,11	1	$\frac{1}{\sqrt{2}}$	0,707

Jelátalakítók

- R, L, C
- feszültségosztó
- kompenzált osztó
- feszültség/áramváltó

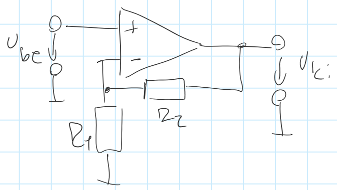
Műveleti erősítős áramkörök

Invertáló alapkapcsolás



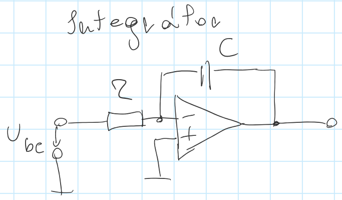
$$A = -\frac{R_2}{R_1} = -\frac{Z_2}{Z_1}$$

Neminvertáló alapkapcsolás

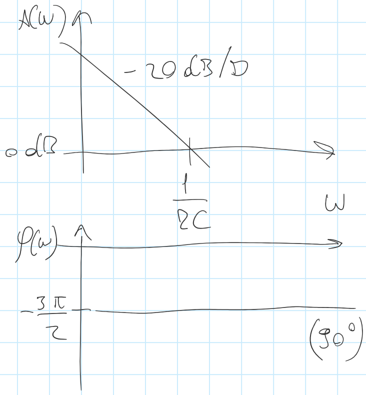


$$A = 1 + \frac{R_2}{R_1} = 1 + \frac{Z_2}{Z_1}$$

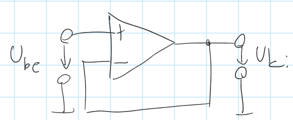
Integrátor



$$A(s) = -\frac{1}{sRC} = -\frac{1}{j\omega RC}$$

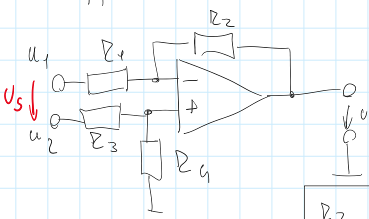


Követő erősítő



$$A = 1$$

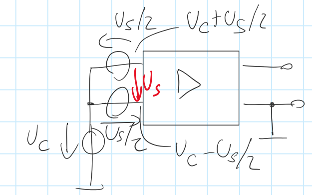
Differencia erősítő



$A_s = -\frac{R_2}{R_1} = \frac{U_{ki}}{U_s}$ (névl)

$A_c ? 0 = \frac{U_{ki}}{U_c}$ (névl.)

$$\boxed{\frac{R_2}{R_1} = \frac{R_4}{R_3}}$$



$U_1 = U_c + \frac{U_s}{2}$

$U_2 = U_c - \frac{U_s}{2}$

$U_s = U_1 - U_2$

$U_c = \frac{U_1 + U_2}{2}$

$$U_{ki} = \left(-\frac{R_2}{R_1}\right)U_1 + \frac{R_2+R_1}{R_1}\cdot\underbrace{\frac{R_4}{R_3+R_4}}_{\frac{R_2}{R_1+R_2}}\cdot U_2 = -\frac{R_2}{R_1}(\underbrace{U_1-U_2}_{U_s})$$

$U_c \neq 0$, felt. nem pontosan teljesül

$$U_{ki} = -\frac{R_2}{R_1}U_1 + \frac{R_2+R_1}{R_1}\cdot\frac{R_4}{R_3+R_4}U_2 =$$

$$= \frac{-R_2(R_3+R_4)+(R_1+R_2)R_4}{R_1(R_3+R_4)}U_c = \frac{R_1R_4 - R_2R_3}{R_1(R_3+R_4)}U_c$$

Közös jelelnyomási tényező

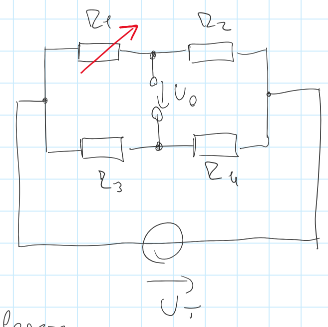
$E = \left|\frac{A_s}{A_c}\right|$, $E_{dB} = 20\lg(E)$

Pl.: $A_s = 40dB$, $E = 80dB$ $\quad U_s = 10mV$, $U_c = 10V$

$A_s = 100$

$A_c = \frac{A_s}{E} = -40dB = 0{,}01$

$U_{ki}\big|_s = A_s \cdot U_s = 100 \cdot 10mV = 1V \qquad U_{ki}\big|_c = 0{,}01 \cdot 1V = 10mV$



Névlegesen:

$R_1 = R_2 = R_3 = R_4 = R$

$U_0 = 0$

$$U_0 = \frac{R_1}{R_1+R_2}U_T - \frac{R_4}{R_3+R_4}U_T =$$

$$= \frac{R_2(R_3+R_4) - R_4(R_1+R_2)}{(R_1+R_2)(R_3+R_4)}U_T =$$

$R_1 = R(1+h)$

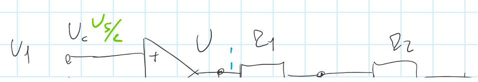
$$= \frac{R_2R_3 - R_1R_4}{(R_1+R_2)(R_3+R_4)}U_T \cong \frac{U_T}{4}\cdot h = U_s$$

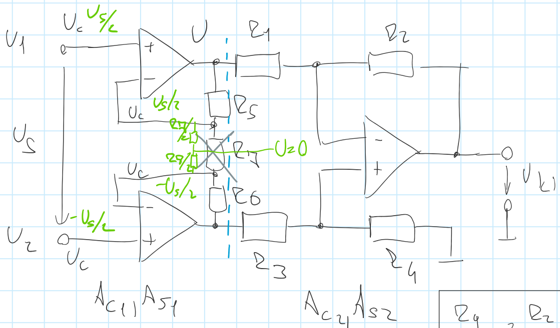
$U_T = 10V$, $h = 10^{-3}$

$U_c = \frac{U_T}{2}$

$U_s \cong 2{,}5mV$, $U_c \cong 5V$

Műszererősítő (3 műveleterősítős mérőerősítő)





$A_S = A_{S1} A_{S2}$

$A_C = A_{C1} A_{C2}$

$A_{S2} = -\frac{R_2}{R_1}$

$A_{C2} = 0$ (ideál.)

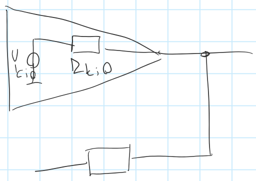
$\boxed{\frac{R_4}{R_3} = \frac{R_2}{R_1}}$

$\boxed{R_5 = R_6}$

$U\big|_{U_S=0,\ U_C\neq 0} = U_C$

$A_{C1} = 1$

$A_{S1} = 1 + \frac{2R_5}{R_7}$



$R_{ki} = R_{ki0}\,\frac{1}{1+H}$ → hurokerősítés

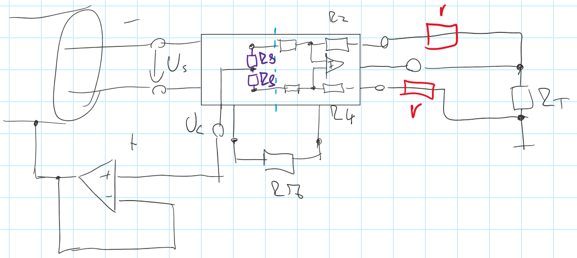
$A = A_0 \cdot \frac{1}{1+k}$ $\qquad H = A_0 \cdot k$

$$A_S = A_{S1} \cdot A_{S2} = \left(1 + \frac{2R_5}{R_7}\right)\left(-\frac{R_2}{R_1}\right)$$

$$A_C = A_{C1} \cdot A_{C2} = 1 \cdot 0 = 0$$

Pl.: $R_1 = R_2 = R_3 = R_4 \Rightarrow A_{S2} = 1$

$$|A_S| = 1 + \frac{2R_5}{R_7}$$



10 April 2019

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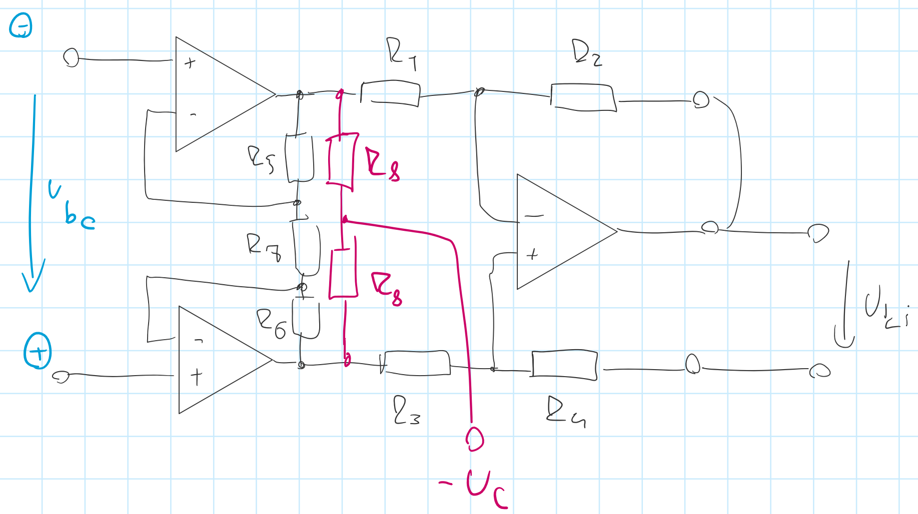
Jelátalakítók

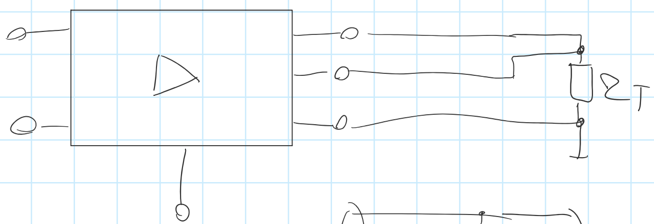
- invertáló
- nem invertáló

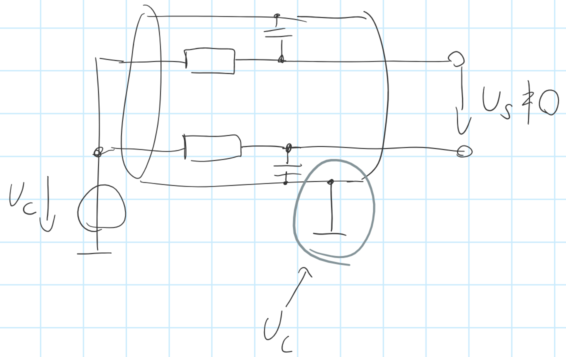
} alapkapcsolás

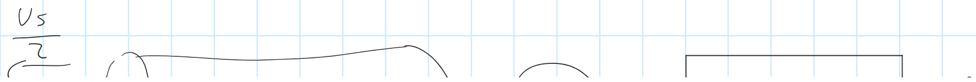
- követő erősítő
- integrátor

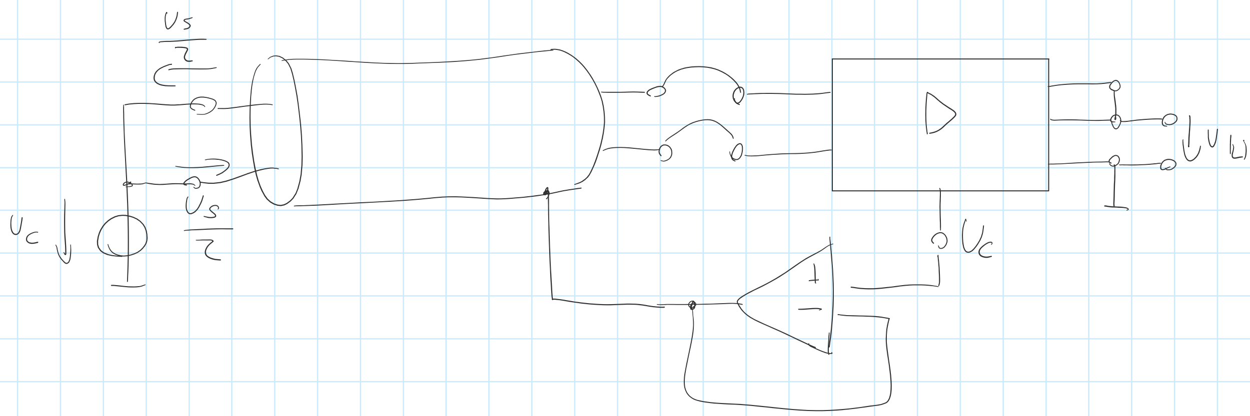
- differencia erősítő
- műszererősítő (3 műv erősítő)

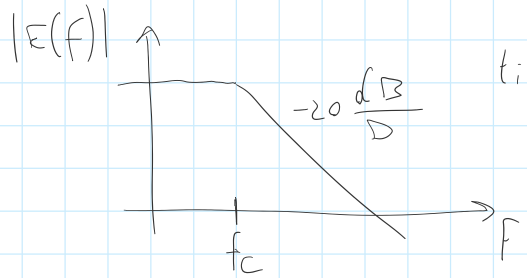












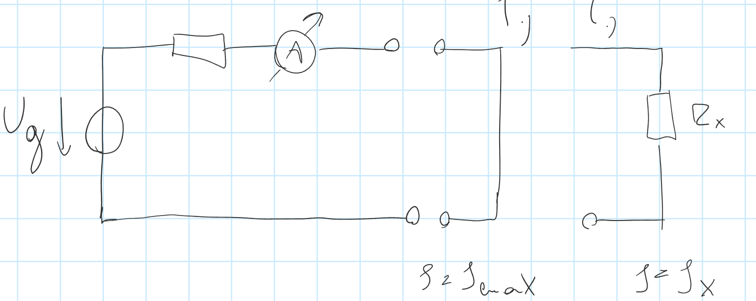
tip. $f_c = 100 \ldots 1000$ Hz

Impedanciamérés

- DC/AC
- kis pontosságú / nagy pontosságú

DC, kis pontosságú módszerek

Soros ohmmérő



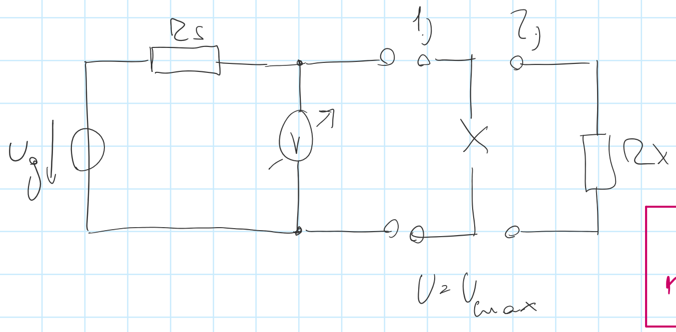
$$\left.\begin{aligned} I_{max} &= \frac{U_g}{R_S} \\ I_x &= \frac{U_g}{R_S + R_X} \end{aligned}\right\} \quad \frac{I_{max}}{I_X} = \frac{R_S + R_X}{R_S}$$

$$R_x = R_S\left(\frac{I_{max}}{I_x} - 1\right)$$

Párhuzamos ohmmérő

R_S 1) 2)

Párhuzamos ohmmérő



$U_{max} = U_g$

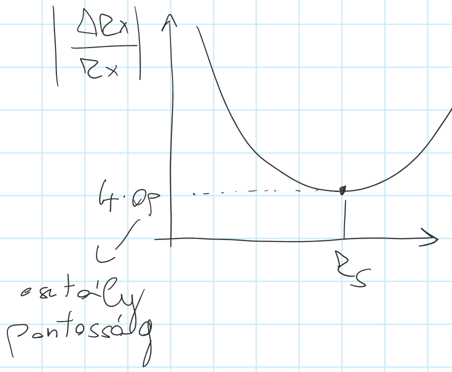
$U_x = U_g \cdot \frac{R_x}{R_s+R_x}$

$\left.\begin{matrix} U_{max} = U_g \\ U_x = U_g \cdot \frac{R_x}{R_s+R_x} \end{matrix}\right\} \frac{U_{max}}{U_x} = \frac{R_s+R_x}{R_x}$

$\boxed{\text{nem lineáris!}}$

$0 \quad R_x \quad \infty$

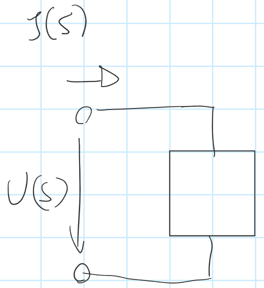
$$R_x = R_s\left(\frac{U_x}{U_{max}-U_x}\right)$$



Impedancia: Z

Immittancia

- impedancia
  - reaktancia
  - rezisztencia
- admittancia
  - konduktancia
  - szuszceptancia



$$Z(s) = \frac{U(s)}{I(s)}\bigg|_{s=j\omega} = \frac{U(j\omega)}{I(j\omega)} = Z(j\omega) = Z(f)$$

$\omega = 2\pi f$

$Z(f) \rightarrow Z(f_0)$

függvény   komplex szám $\rightarrow$ 2 paraméter: - $|Z|$, $\varphi(Z)$

$Re\{Z\}$ $Im\{Z\}$

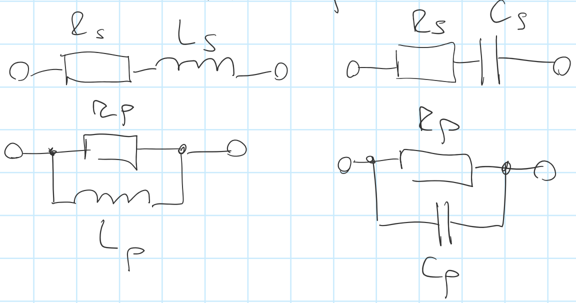
$\downarrow$

- helyettesítő képek

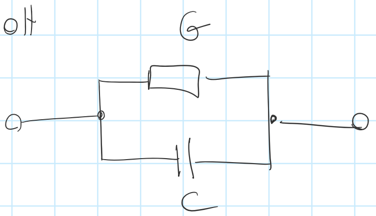
$\downarrow$

sinusos gerjesztés!

- helyettesítőképek
- soros/párhuzamos
- induktív/kapacitív

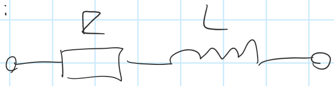


Adott



(valódi!)

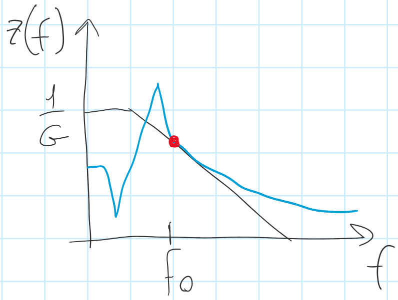
Keresett:



(helyettesítőkép)

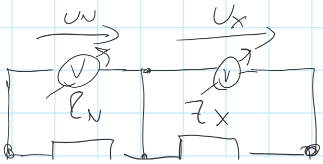
$$Y = G + j\omega C \Rightarrow Z = R + j\omega L$$

$$Z = \frac{1}{Y} = \frac{1}{G + j\omega C} = \frac{G - j\omega C}{G^2 + \omega^2 C^2} = \underbrace{\frac{G}{G^2 + \omega^2 C^2}}_{R > 0} + j\omega \underbrace{\frac{-C}{G^2 + \omega^2 C^2}}_{L < 0}$$

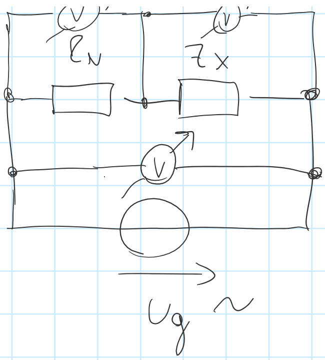


AC kis pontosságú módszerek

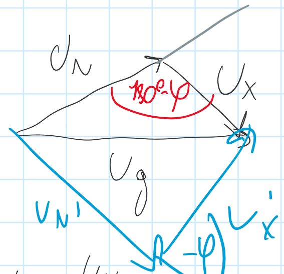
3 voltmérős módszer



$$|Z_x| = R_x \cdot \frac{U_x}{U_N}$$



$$|Z_x| = R_x \cdot \frac{U_x}{U_N}$$



$$U_g^2 = U_N^2 + U_x^2 - 2U_N U_x \cos(180° - \varphi)$$

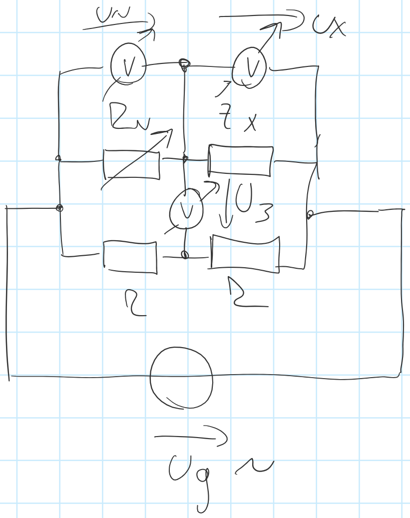
$$U_g^2 = U_N^2 + U_x^2 + 2U_N U_x \cos(\varphi)$$

$$\cos\varphi = \frac{U_g^2 - U_N^2 - U_x^2}{2U_N U_x}$$

$$P = U_x I_x \cos\varphi = U_x \cdot I_N \cos\varphi = U_x \cdot \frac{U_N}{R_N} \cos\varphi =$$

$$= U_x U_N \cdot \frac{1}{R} \cdot \frac{U_g^2 - U_x^2 - U_N^2}{2U_x U_N} = \frac{U_g^2 - U_x^2 - U_N^2}{2R_N}$$

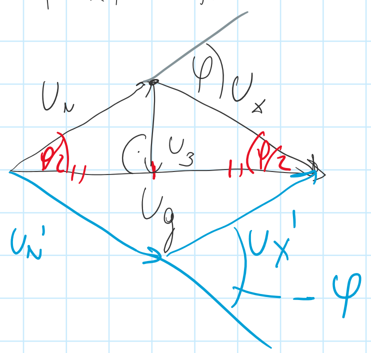
Gritzenacher-híd



1.) $R_N \uparrow : U_N = U_x$

$$|Z_x| = R_N$$

2.)



$$\sin\left(\frac{\varphi}{2}\right) = \frac{U_3}{U_N}$$

$$\Downarrow$$

$$\varphi$$

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Impedanciamérés

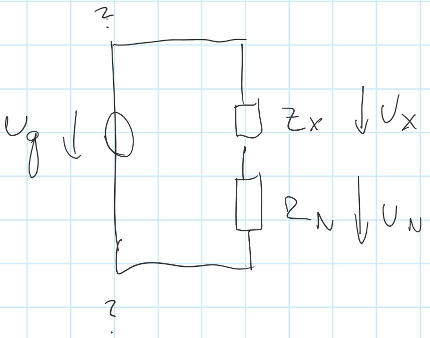
- DC/AC
- kis és nagypontosságú → DC, kisp. soros/párhuzamos ohmmérő
- AC: helyettesítőképek $Z(f_0)$: 2 szabad paraméter
- AC kispontosságú módsz.: három voltmérős, Grützmacher-híd

⇓

teljesítmény mérés

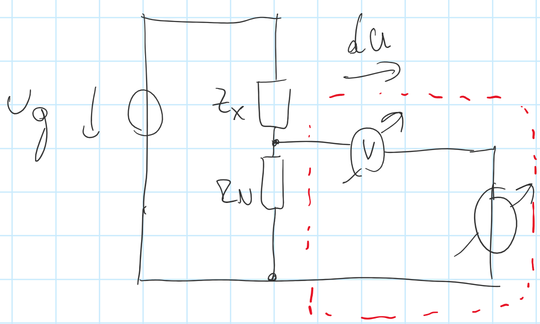
Nagy pontosságú módszerek

Feszültség összehasonlítós módszer



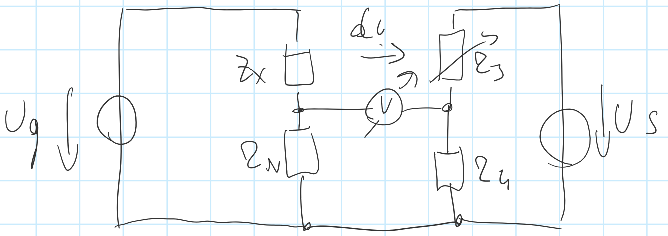
$$Z_x = R_N \cdot \frac{U_x}{U_N} \quad \text{(vektorok!)}$$

Kompenzációs mérés

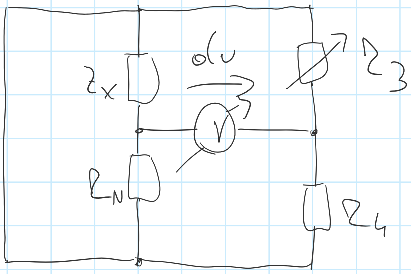


$U_s \nearrow$: $U_s \approx U_N$

$$\boxed{dU \to Z_{be} \to \infty}$$

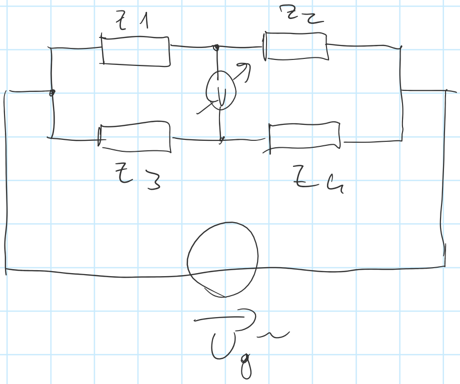


$R_3 \nearrow :\ du \to 0$



$R_3 \nearrow :\ du \to 0$

Wheatstone-Féle híd



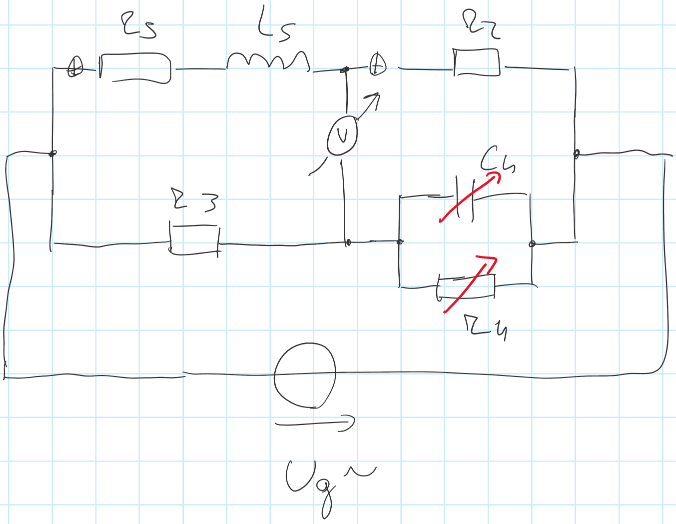
nullindikátor

legyen $Z_1 = Z_x$

$$\frac{Z_x}{Z_3} = \frac{Z_2}{Z_4}$$

$$Z_x = Z_3 \frac{Z_2}{Z_4}$$

Maxwell-Wien híd



kiegyenlítés után

$$\frac{Z_s}{Z_3} = \frac{Z_2}{Z_4} = Z_2 Y_4$$

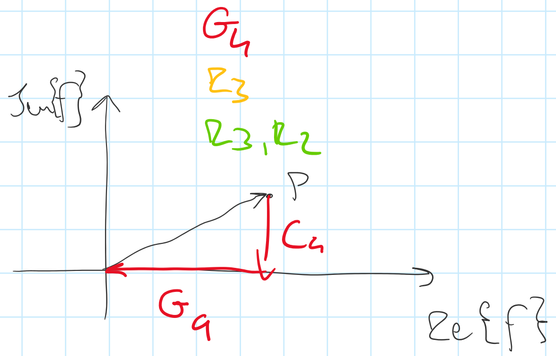
$$\frac{R_s + j\omega L_s}{R_3} = R_2\left(G_4 + j\omega C_4\right)$$

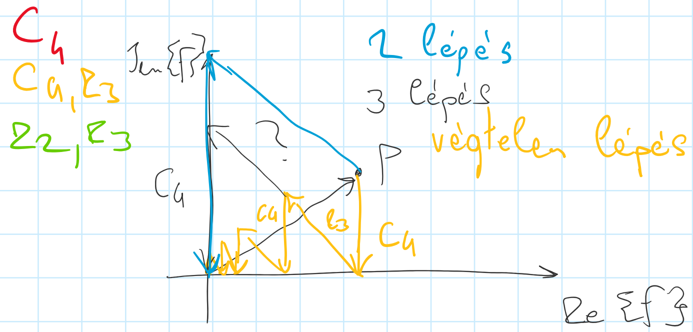
$$R_s + j\omega L_s = R_2 \cdot R_3\left(G_4 + j\omega C_4\right)$$

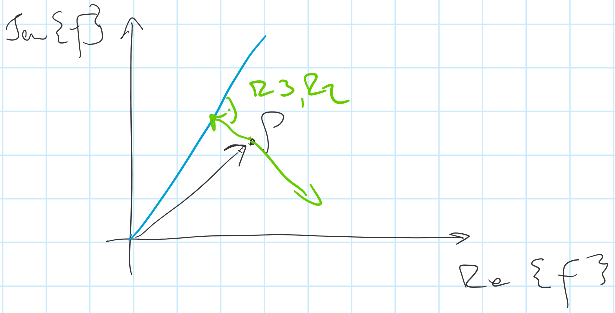
$$R_s = R_2 R_3 G_4 = \frac{R_2 R_3}{R_4}$$

$$L_s = R_2 R_3 C_4$$

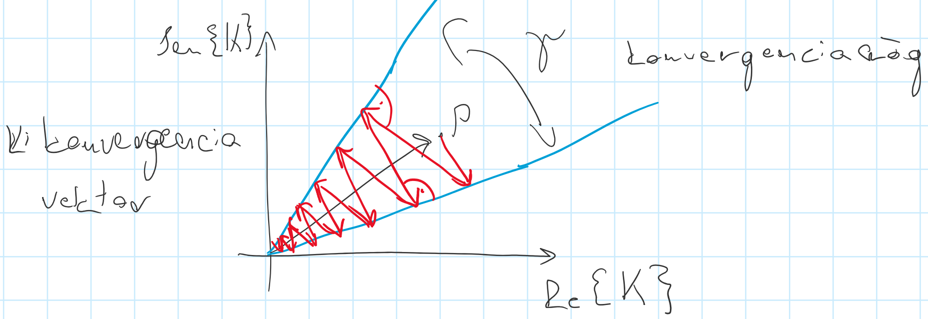
$$f(\,) = [R_5 - R_2 R_3 G_4] + j\omega[L_5 - R_2 R_3 C_4] \to 0$$





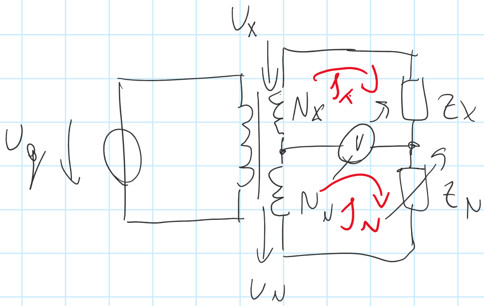


Általában:



ha $\gamma = 90^\circ \Rightarrow$ elvileg 2 lépésben kiegyenlíthető a szög

Aránytranszformátoros mérőhíd



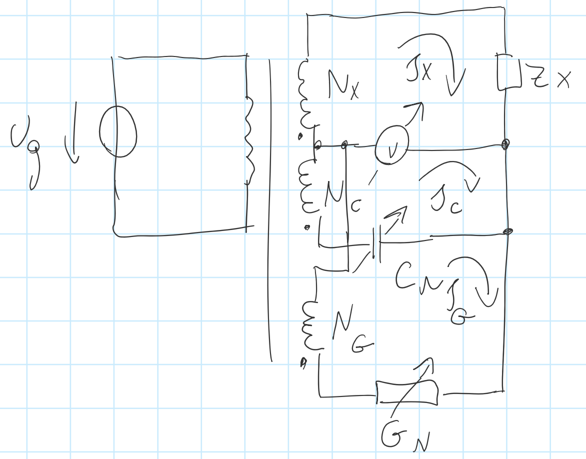
$$I_0 = I_x - I_N \to 0$$

$$I_x = \frac{U_x}{Z_x} \qquad I_N = \frac{U_N}{Z_N}$$

$$\frac{U_x}{Z_x} = \frac{U_N}{Z_N} \to Z_x = Z_N \frac{N_x}{N_N}$$

$U_x \sim N_x$

$U_N \sim N_N$



Kiegyenlítés esetén:

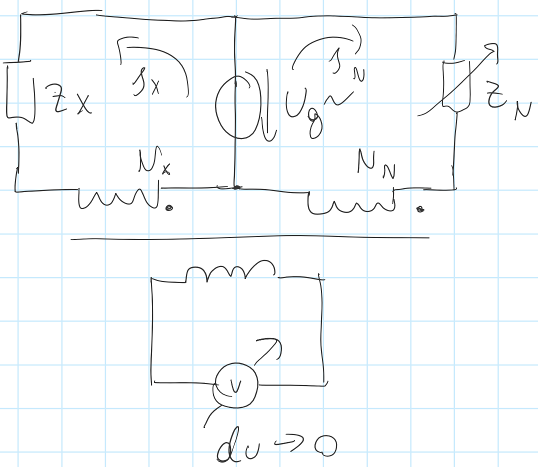
$I_x = I_C + I_G$

$N_x Y_x = N_G \cdot G_N + j\omega C_N \cdot N_C$

$Y_x = \frac{N_G}{N_x} \cdot G_N \pm j\omega \frac{N_C}{N_x} C_N$

$Y_x = \frac{1}{Z_x}$

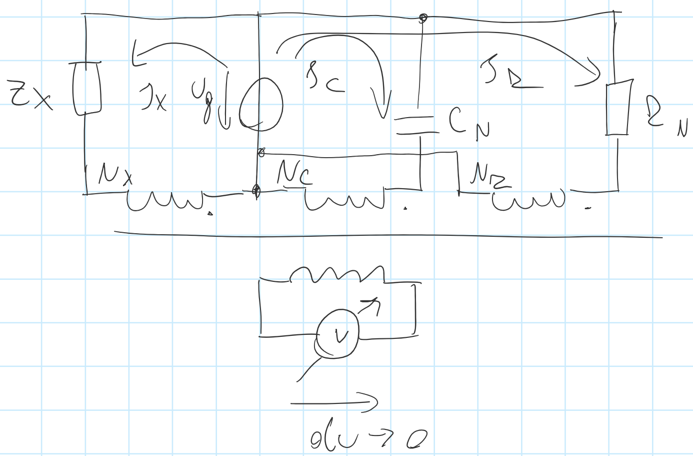
Áramkomparátoros mérőhíd



$I_x N_x = I_N \cdot N_N$

$I_x = \frac{U_g}{Z_x}$, $I_N = \frac{U_g}{Z_N}$

$Y_x = Y_N \cdot \frac{N_N}{N_x}$



$I_x N_x = I_C N_C + I_R N_R$

$$I_x N_x = I_C N_C + I_R N_R$$

$$U_g Y_x N_x = U_g \frac{1}{j\omega C_N} N_C + U_g \frac{1}{R_N} N_R$$

$$Y_x = \frac{N_C}{N_x} j\omega C_N + \frac{N_R}{N_x} \cdot \frac{1}{R_N}$$

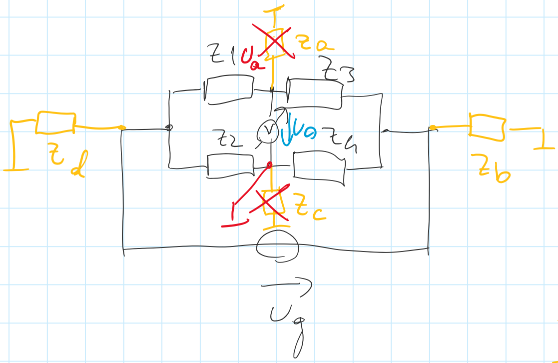
17 April 2019

17 April 2019 10:22

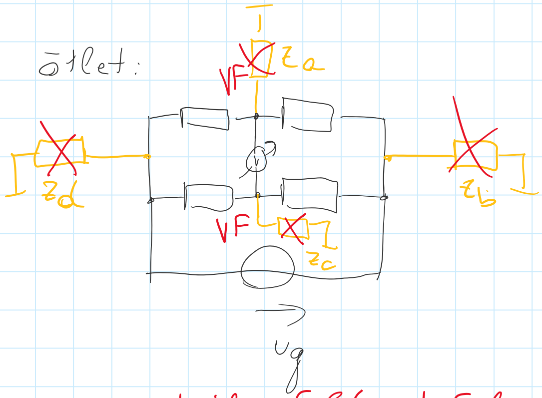
Szórt impedanciák kompenzálása

$\boxed{U_0 \to 0}$ $\boxed{U_a \to 0\ (\perp)}$

Virtuális földpont!

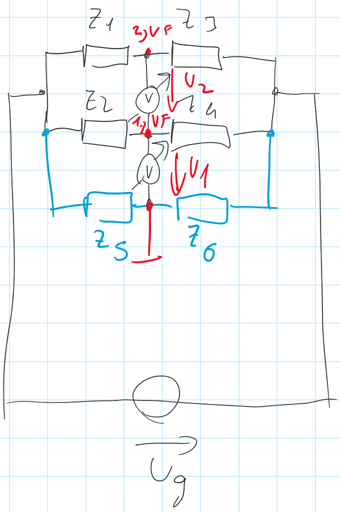


ötlet:



Mert virtuális földpont fele nem folyik áram

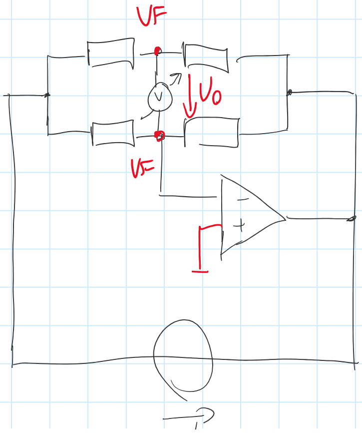
Wagner féle segédhíd



1) $U_1 \to 0$

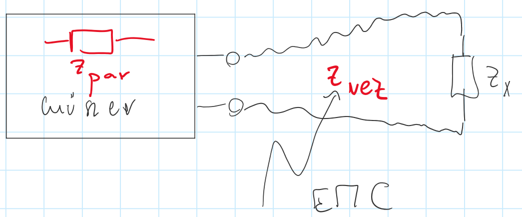
2) $U_2 \to 0$

Műveleti erősítővel:



$U_0 \to 0$

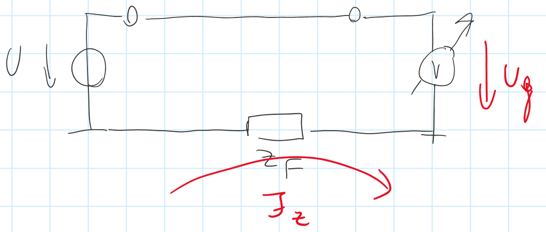
U_g



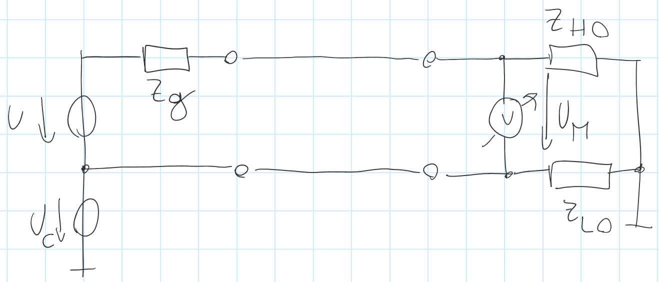
– többvezetékes mérés

– árnyékolás

Mérőhálózatok zavarérzékenysége



Kétvezetékes:

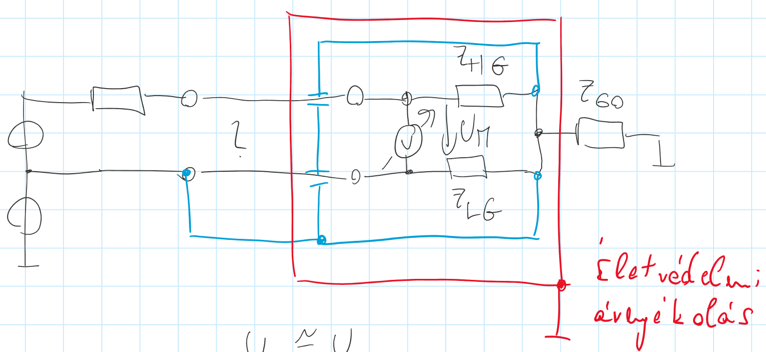


$$U_M = \frac{Z_{HO}}{Z_{HO}+Z_g}\cdot U + U_C\left(\frac{Z_{HO}}{Z_{HO}+Z_g} - 1\right) =$$

$$= \frac{Z_{HO}}{Z_{HO}+Z_g}\,U + \frac{-Z_g}{Z_{HO}+Z_g}\,U_C \cong U - \frac{Z_g}{Z_{HO}}\,U_C$$

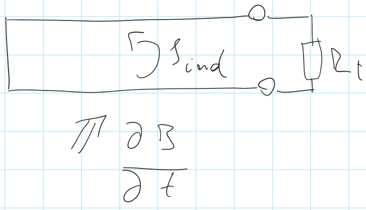
$Z_{HO} \gg Z_g$

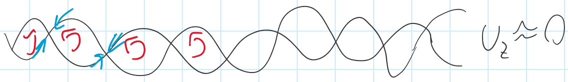
Három vezetékes (guard)



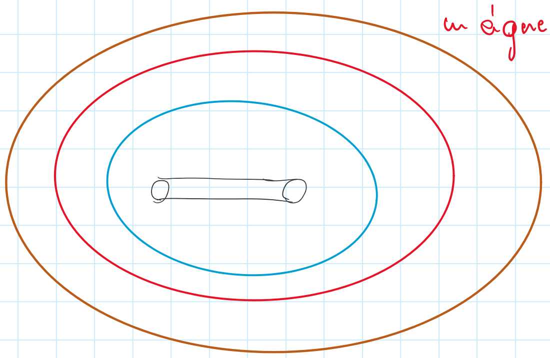
$U_M \cong U$

Árnyékolt + csavart kábel



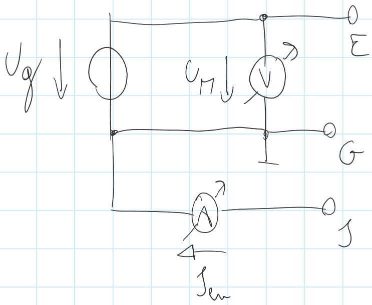


mágneses árnyékolás



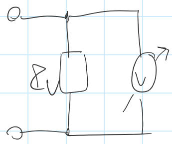
A: árnyékolás nélkül
B: árnyékolással
} 2, 3, 4, 5 vezetékes mérés

Impedanciamérő műszer modellje:

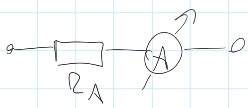


$$\hat{Z}_X = \frac{U_m}{I_m}$$

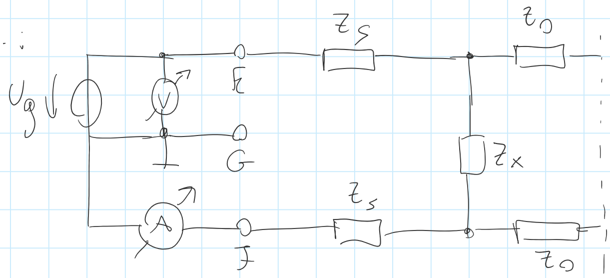
$R_V = \infty$



$R_A = 0$

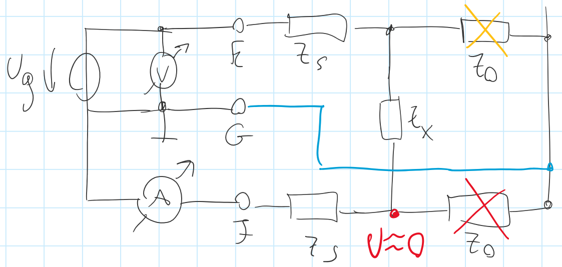


AZ:



$\hat{Z} = Z_x \times 2Z_0 + 2Z_s \neq Z_x$

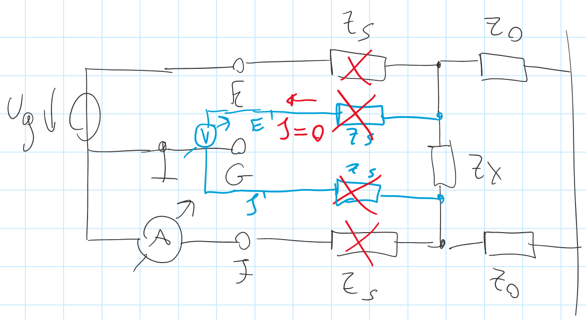
A3



Z_s elhanyagolható

Ez kiküszöböli: Z_0-t és hibát okoz Z_s

A4

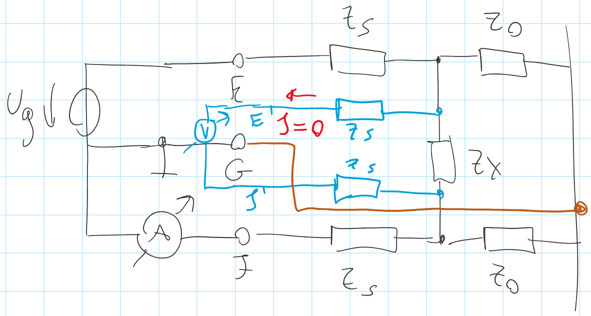


Z_0 elhanyagolható

Kiküszöböli: Z_s-t

hibát okoz Z_0

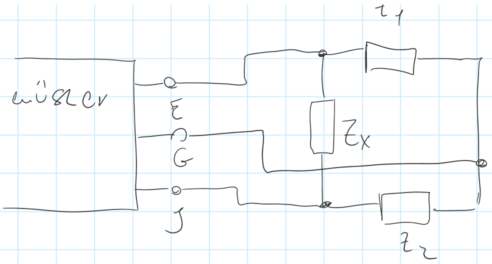
A5:



Kiküszöböli: Z_0, Z_s-t

In-circuit impedancia mérés

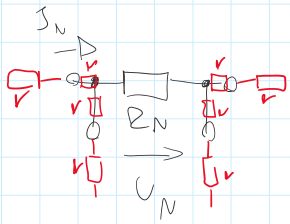
Z_1



Legalább 3 vezetékes mérés

$Z_x \approx Z_1, Z_2 \Rightarrow 5$ vez. mérés

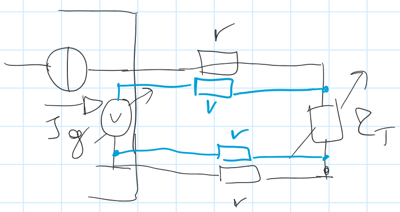
4 vezetékes ellenállás



$U_N = Z_N \cdot J_N$

r: kis értékű ellenállás

Pl.: Hőmérsékletmérés, hőellenállás



24 April 2019

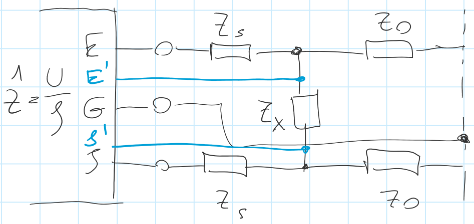
24 April 2019 10:16

- mérőhálózatok zavarérzékenysége
- többvezetékes impedanciamérés
  - árnyékolás nélkül
  - árnyékolással

  } 2, 3, 4, 5 vez. mérés

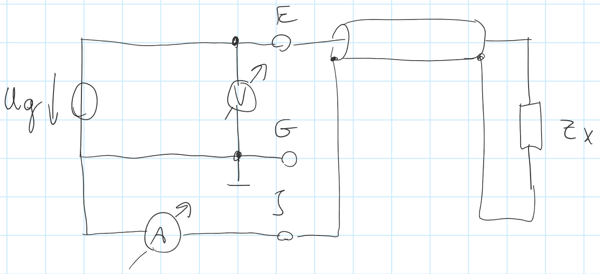
vez	hibát okoz	kiküszöböl
2	Z_0, Z_s	—
3	Z_s	Z_0
4	Z_0	Z_s
5	— ?	Z_0, Z_s

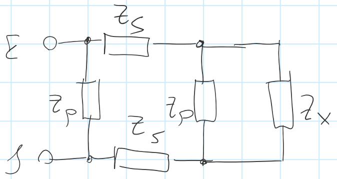
A 2, 3, 4, 5



B 2, 3, 4, 5:

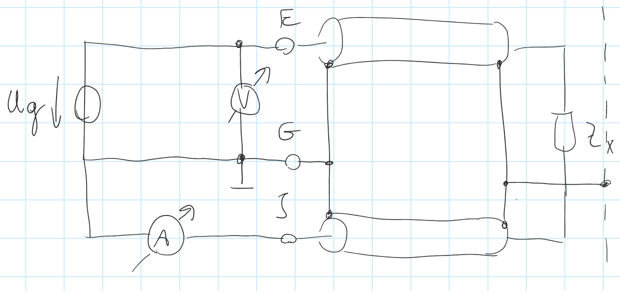
B 2:

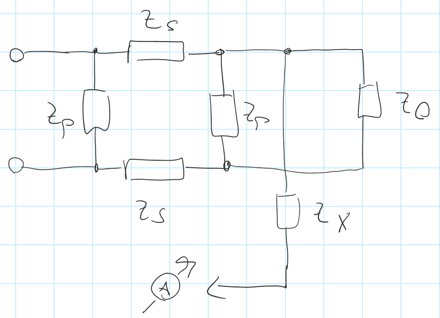




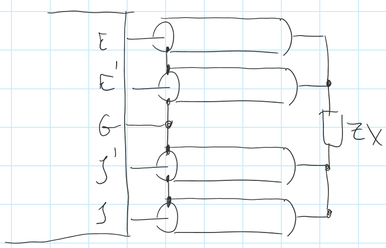
$$\hat{Z} = \left((Z_x \times Z_p) + 2 \cdot Z_s \right) \times Z_p \neq Z_x$$

B 3:

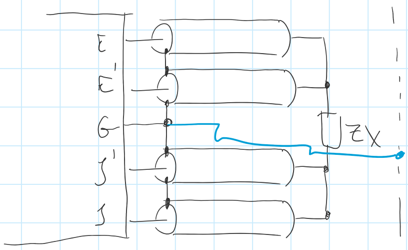


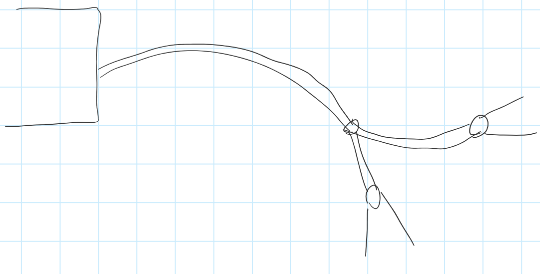


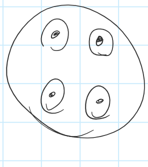
B4:



B5:

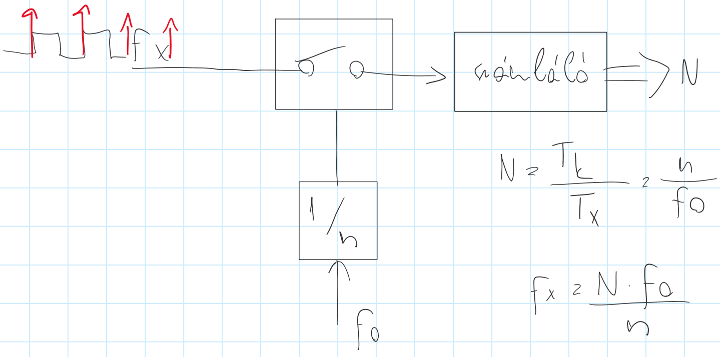






Idő és frekvenciamérés

frekvenciamérés:

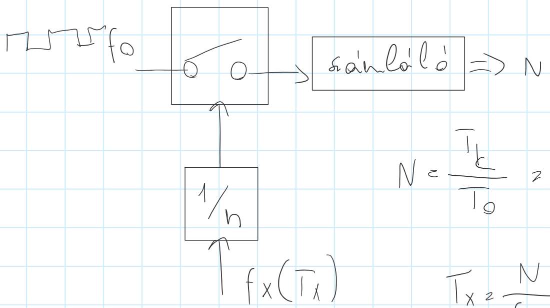


$$N = \frac{T_k}{T_x} = \frac{n}{f_0} \cdot f_x$$

$$f_x = \frac{N \cdot f_0}{n}$$

$$\left|\frac{\Delta f_x}{f_x}\right| = \frac{\Delta N}{N} + \frac{\Delta f_0}{f_0} + \underbrace{\frac{\Delta n}{n}}_{=0} = \frac{1}{N} + \frac{\Delta f_0}{f_0} \qquad \frac{1}{N} = \frac{T_x}{T_k} = \frac{1}{T_k \cdot f_x}$$

(átlag) periódusidő mérés:



$$N = \frac{T_k}{T_0} = n \cdot T_x \cdot f_0$$

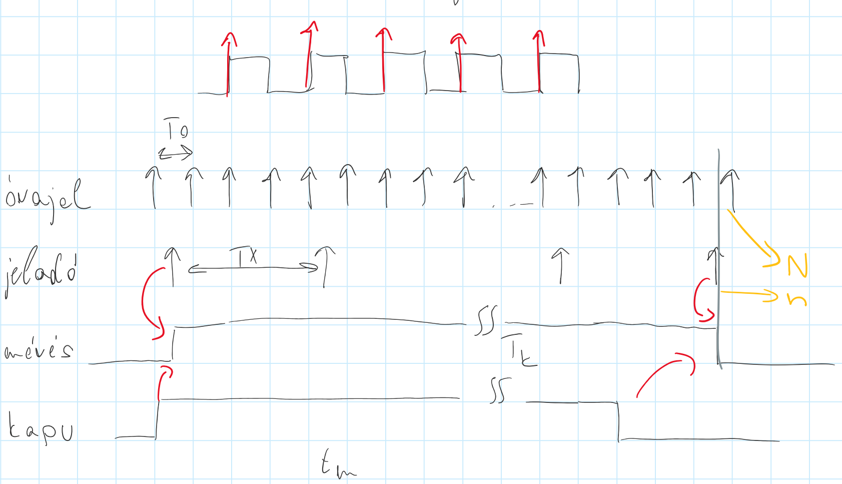
$$T_x = \frac{N}{f_0 \cdot n}$$

$$\left|\frac{\Delta T_x}{T_x}\right| = \frac{\Delta N}{N} + \frac{\Delta f_0}{f_0} + \underbrace{\frac{\Delta n}{n}}_{=0} = \frac{1}{N} + \frac{\Delta f_0}{f_0} \qquad \frac{1}{N} = \frac{T_0}{T_k} = \frac{1}{T_k \cdot f_0} = \frac{1}{n \cdot T_x \cdot f_0}$$

$$f_m \qquad T_m = \frac{1}{f_m} \qquad \frac{\Delta T_m}{T_m} = -\frac{\Delta f_m}{f_m}$$

$$\left|\frac{\Delta f_m}{f_m}\right| = \left|\frac{\Delta T_m}{T_m}\right|$$

Állandó kapuidejű átlag periódusidőmérés



$f_0 = \frac{1}{T_0}$ órajel

t_m: előírt mérési idő (pl.: 0,1 s)

ZH!

$$T_x = \frac{N}{f_0 \cdot n} \qquad \left|\frac{\Delta T_x}{T_x}\right| = \frac{1}{T_t \cdot f_0} + \left(\frac{\Delta f_0}{f_0}\right) \cong \frac{1}{t_m f_0} + \left(\frac{\Delta f_0}{f_0}\right)$$

$$n \gg 1 \qquad T_t \approx t_m$$

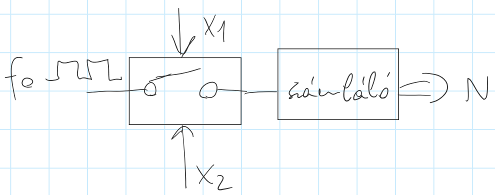
$$h_1 = \frac{1}{1 \cdot T_x \cdot f_0} \quad (1 \text{ periódus})$$

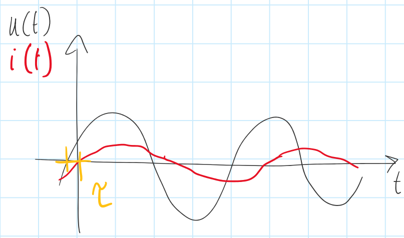
$$h_n = \frac{1}{n \cdot T_x \cdot f_0} \quad (n>1 \text{ periódus})$$

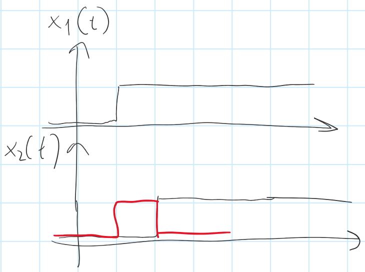
$$h_n = \frac{h_1}{n} \qquad h_N = \frac{h_1}{\sqrt{N}}$$

N: mérések száma

Időintervallum-mérés







$$N = \frac{\tau}{T_0} = \tau f_0$$

$$\tau = \frac{N}{f_0}$$

$$\Delta\varphi = \frac{\tau}{T_x} \cdot 2\pi = \frac{\tau}{T_x} \cdot 360^\circ$$

$$\left|\frac{\Delta\tau}{\tau}\right| = \frac{\Delta f_0}{f_0} + \frac{1}{N} = \frac{\Delta f_0}{f_0} + \frac{1}{\tau \cdot f_0}$$

26 April 2019

26 April 2019 10:15

- frekvenciamérő
- (átlag) periódusidő mérő
- állandó kapuidejű átl. per. mérő
- időintervallum mérő
- fázismérő

$f_x = ?$

$T_x = ?$

$\tau = ?$

$\varphi = ?$

$\Delta f_x / f_x = f(N) \dfrac{\Delta f_0}{f_0}$

$\vdots$

Jelanalízis

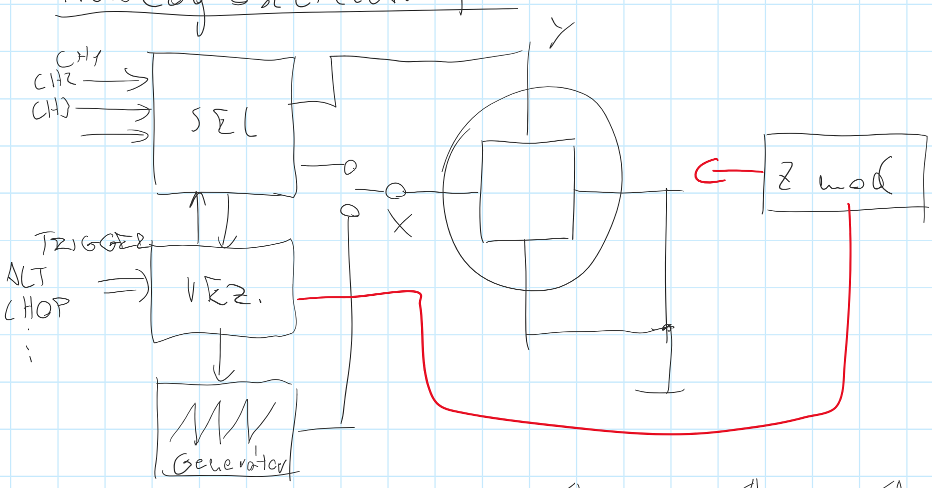
Oszcilloszkópok

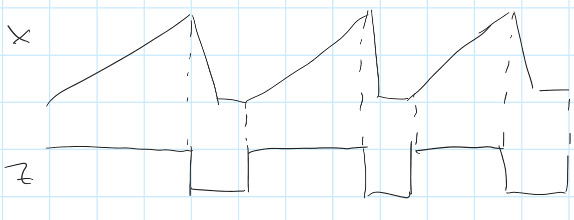
$x(t)$ jel időfüggvényének megjelenítése

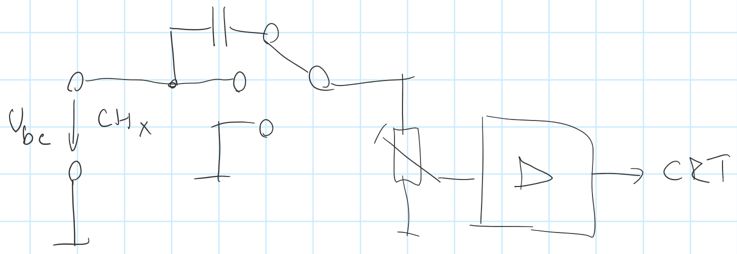
- analóg
- digitális

} oszcilloszkóp

Analóg oszcilloszkóp



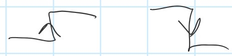




TRIGGER

SOURCE: INT (CH1, CH2, ...)
EXT
LINE

MODE: DC, AC, HF, LF

SLOPE: 

LEVEL:

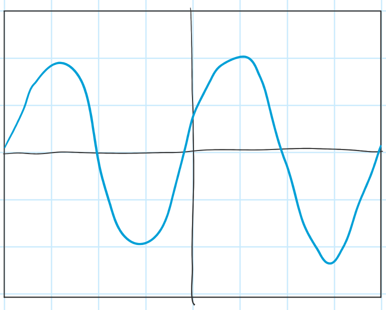
HOLDOFF

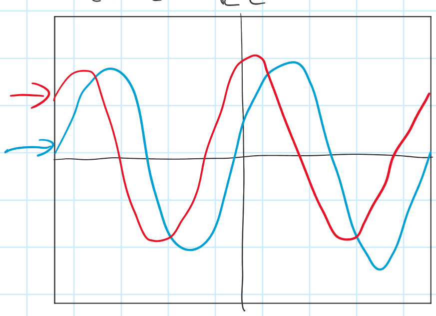
NORMAL / AUTO

Időeltérítés: sec/div (ms, μs, ns)   1s ... 10ns

Feszültség eltérítés V/div (mV)   10V ... 1mV

TRIGGER LEVEL



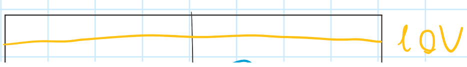


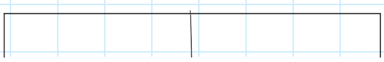
10 cm x 8 cm

TR. SLOPE

DC+AC

pl.:

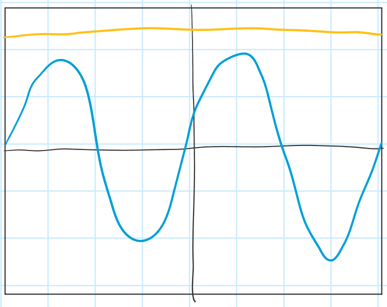




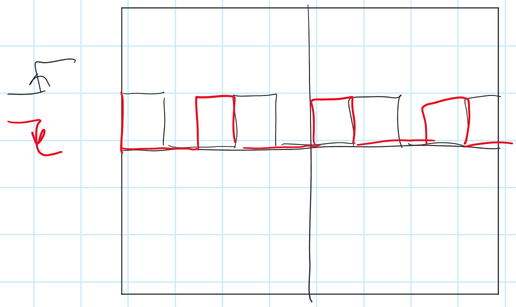
DC+AC

pl.:
10V DC+
10mV AC

AC csat

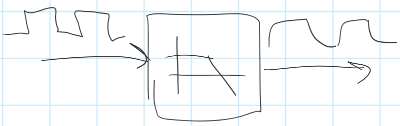


10V



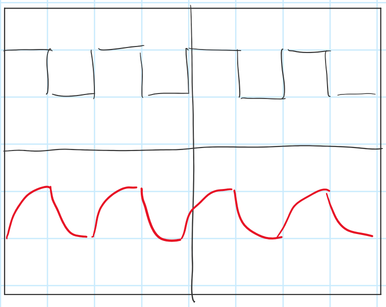
5V/DIV

5mV/DIV



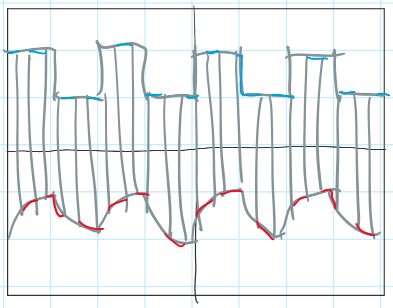
CH1

CH2

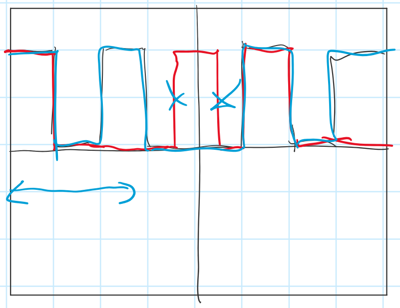


ALTERNATE

CHOPPED



HOLD OFF



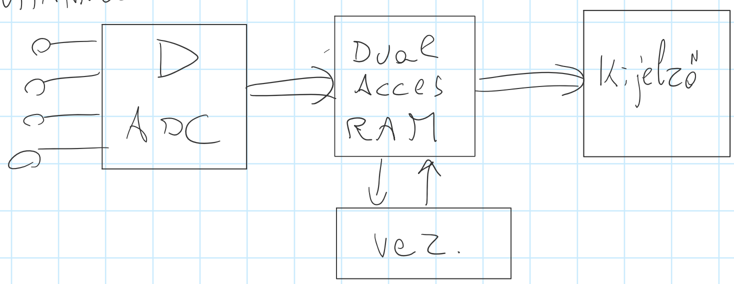
$T_H \approx 0$

?

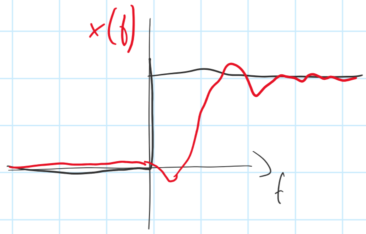
$T_H \neq 0$

Digitális oszcilloszkóp   DSO   digital storage oscilloscope

CHANNELS



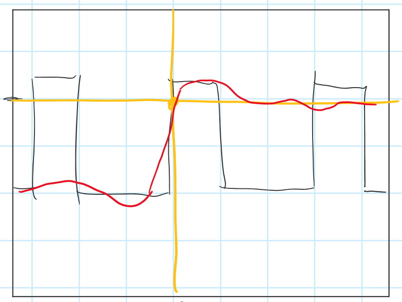
single sweep ⇒ tranziens jelek



Pre trigger

trigger level

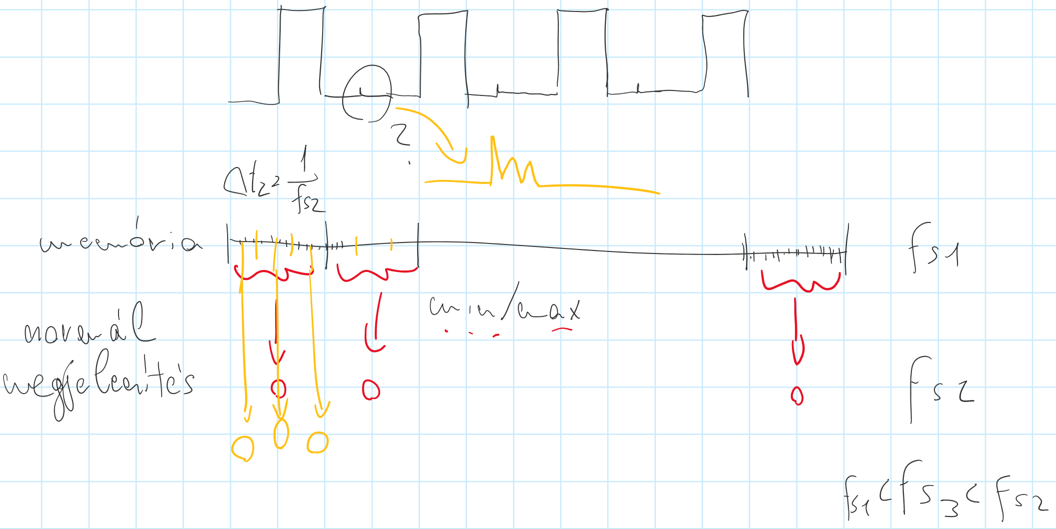
(zoom)



$T_2 < T_1$

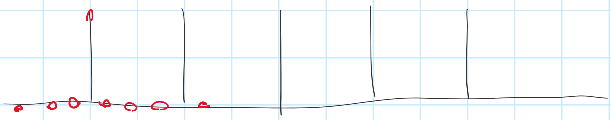
trigger pos

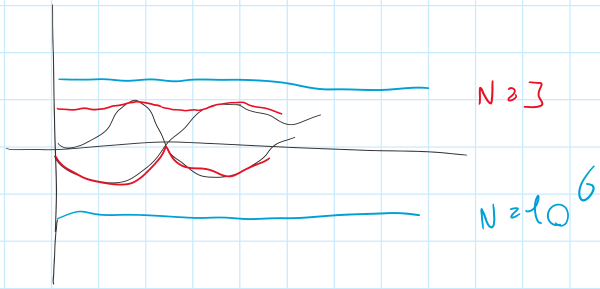
Zoom

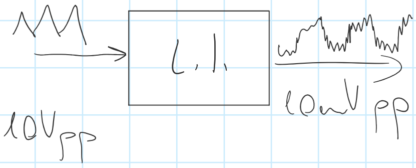


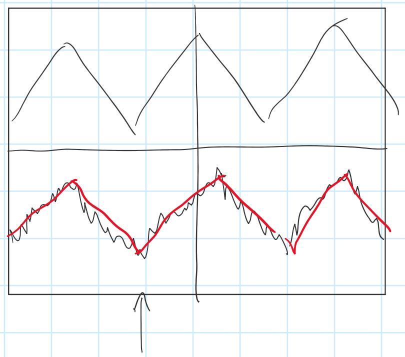
Peak detect $\}$ $f_{s2} \gg f_{s1}$ max/min

Envelope $\}$ $f_{s2} \gg f_{s1}$ max/min hold





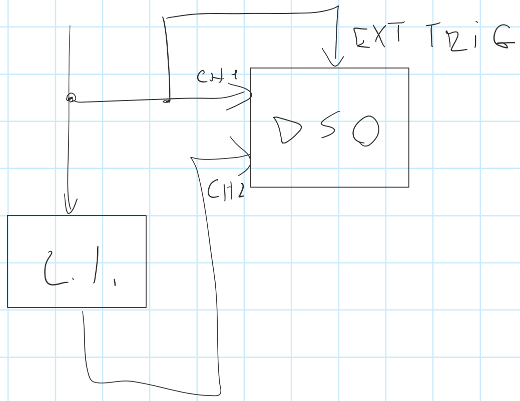




- N db csillapító
- zajmentes triggerforrás

pl.: CH1, EXT TRIG

K.



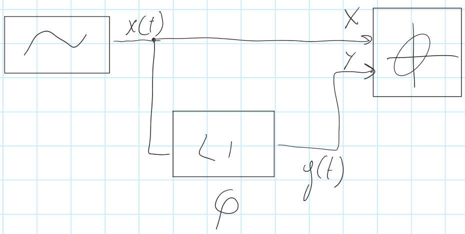
Lissajous-ábrás fázismérés

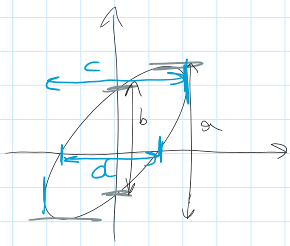
- XY üzemmód

CH1 - X

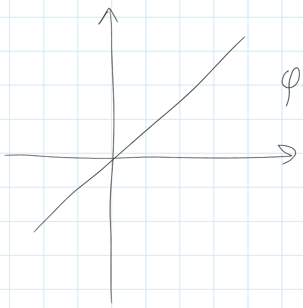
CH2 - Y

- szinuszos gerjesztés

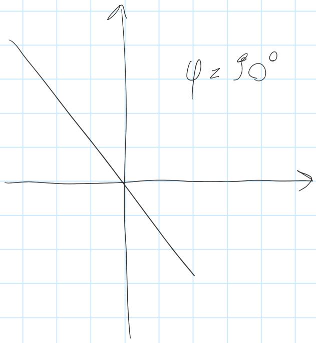




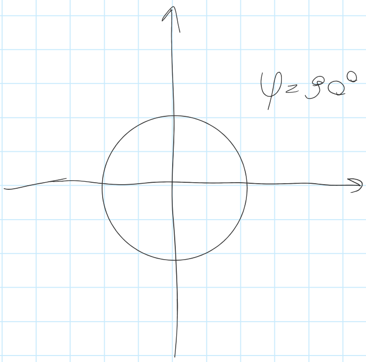
$$\varphi = \arcsin\left(\frac{b}{a}\right) = \arcsin\left(\frac{d}{c}\right)$$



$\varphi = 0^\circ$



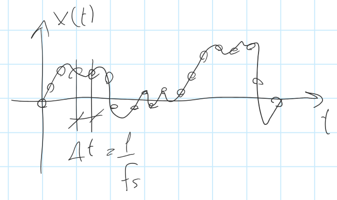
$\varphi = 90^\circ$



$\varphi = 90^\circ$

Példatár 5. X

Mintavételezés

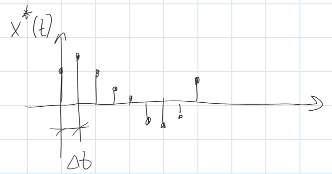


f_s: mintavételi frekvencia

cél az $x(t)$ visszaállítása $x(n \cdot \Delta t)$ mintákból

szorzás $\delta(t)$-val: $\displaystyle\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

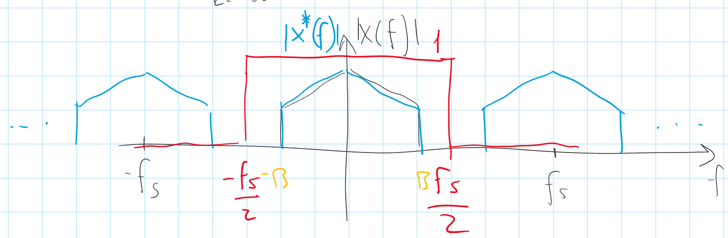
Konvolúció $\delta(t)$-val: $\displaystyle\int_{-\infty}^{\infty} x(\tau)\,\delta(t-t_0-\tau)\,d\tau = x(t-t_0)$



$$x^*(t) = \sum_{n=-\infty}^{\infty} \delta\left(\frac{t-n\cdot\Delta t}{\Delta t}\right) \cdot x(t)$$

$$X(f) = F\{x(t)\}$$

$$X^*(f) = \sum_{k=-\infty}^{\infty} X(f-k\cdot f_s)$$



$$W(f) = \begin{cases} 1 & \text{ha } -\frac{f_s}{2} < f < \frac{f_s}{2} \\ 0 & \text{egyébként} \end{cases}$$

$$w(t) = \frac{1}{\Delta t} \cdot \underbrace{\frac{\sin\left(\pi \frac{t}{\Delta t}\right)}{\pi \cdot \frac{t}{\Delta t}}}_{\text{sinc}} = \frac{1}{\Delta t}\,\text{sinc}\left(\pi \frac{t}{\Delta t}\right)$$

$$x(t) = x^*(t) * w(t) = \sum_{n=-\infty}^{\infty} x(n\Delta t) \cdot \text{sinc}\left[\pi\left(\frac{t}{\Delta t} - n\right)\right]$$

Whittaker-féle interpolációs formula

(1.) mintavételi tétel: Ha $x(t)$ sávkorlátozott, azaz:

$X(f)=0$, ha $|f|>B$, akkor $f_s > 2B$ frekvenciával mintavételezve

$x(t)$, $x^*(t)$-ből helyreállítható

(2.) Frekvenciatartománybeli mintavételi tétel

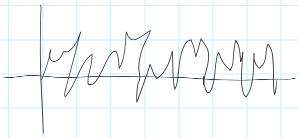
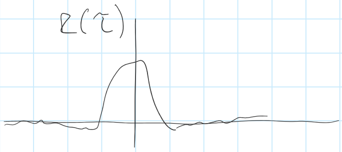
Ha $x(t) \equiv 0$, ha $t \notin [0,T]$ akkor $x(f)$-et $\Delta f < \frac{1}{T}$-vel mintavételezve $X(f)$ a mintáiból helyreállítható.

(3.) Közelítő mintavételi tétel:

$x(t) \approx 0$, ha $t \notin [0,T]$ és $X(f) \approx 0$, ha $f \notin [-B,B]$, akkor

$\boxed{N = 2BT}$ mintából $x(t)$ és $x(f)$ közelítőleg helyreállítható

(4.) Mintavételi tétel sztochasztikus jelekre

 $\Rightarrow$ 

autokorrelációs függvény:

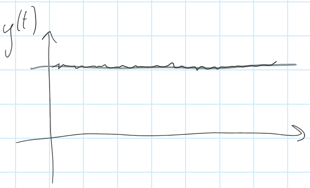
$$R(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} x(t) \cdot x(t+\tau)\, dt$$

$$\approx E\{x(t) \cdot x(t+\tau)\}$$

ergodikus jel

$S(f) = F\{R(\tau)\}$ Teljesítménysűrűség spektrum

Ha $S(f) \equiv 0$, ha $|f| > B$, akkor $R(\tau)$ a mintáiból helyreállítható, $x(t)$ is helyreállítható

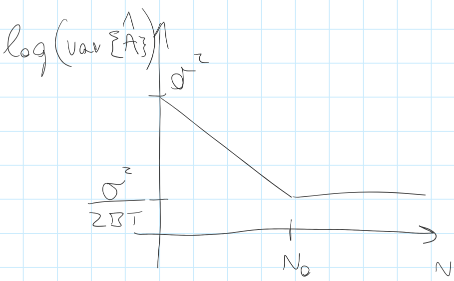


$y(t) = A + x(t)$

$$\hat{A} = \frac{1}{N} \sum_{n=1}^{N} y(n\Delta t)$$

$\mathrm{var}\{y(n\Delta t)\} = \sigma^2$

$\mathrm{var}\{\hat{A}\} = \frac{\sigma^2}{N}$ Feltétel: N független minta



Diszkrét Fourier-transzformáció (DFT)

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} n \cdot k} \qquad x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{+j\frac{2\pi}{N} n \cdot k}$$

$\underbrace{\qquad}_{N} \Leftrightarrow \underbrace{\qquad}_{N}$ $k = 0, \dots, N-1$

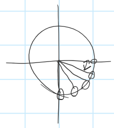
$\underbrace{\qquad}_{N} \Longleftrightarrow \underbrace{\qquad}_{N}$ $\quad k = 0, \dots, N-1$

1.)

$$\underbrace{\begin{bmatrix} N \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{X(k)} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ \vdots & & & & \\ \vdots & & e^{-j\frac{2\pi}{N} n \cdot k} & & \\ \vdots & & n,k = 0, \dots, N-1 & & \\ 1 & & & & \end{bmatrix}}_{N \times N} \underbrace{\begin{bmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}}_{x(n)}$$

1.) $x(n) = 1 \quad , n = 0, \dots, N-1$

$e^{-j\frac{2\pi}{N} n \cdot 1} \longrightarrow$



$\frac{2\pi}{N}$

2.)

$$\underbrace{\begin{bmatrix} 0 \\ N \\ 0 \\ \vdots \\ 0 \end{bmatrix}}_{X(k)} = \underbrace{\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ \vdots & & & & \\ \vdots & & e^{-j\frac{2\pi}{N} n \cdot k} & & \\ \vdots & & n,k = 0, \dots, N-1 & & \\ 1 & & & & \end{bmatrix}}_{N \times N} \underbrace{\begin{bmatrix} e^{+j\frac{2\pi}{N} n} \end{bmatrix}}_{x(n)}$$

2.) $x(n) = e^{j\frac{2\pi}{N} n} \quad n = 0, \dots, N-1$

3.) $x(n) = A \cdot \cos\left(\frac{2\pi}{N} \cdot K \cdot n\right) = \quad n = 0, \dots, N-1$

$$= \frac{A}{2}\left[e^{j\frac{2\pi}{N} K \cdot n} + e^{-j\frac{2\pi}{N} K n}\right] \quad K \text{ egész}$$

$$X(k) = \begin{cases} \frac{A}{2}, & \text{ha } k = K, N-K \\ 0, & \text{egyébként} \end{cases}$$

$$\frac{N-1}{N} \cdot 2\pi \,(\equiv)\, -\frac{2\pi}{N}$$

10 May 2019

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DFT

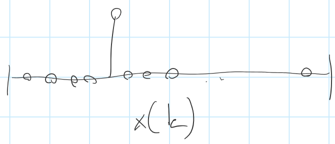
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} \qquad n, k = 0, \dots, N-1$$

$$\underset{\substack{X(k) \\ N\times 1}}{\begin{bmatrix} \\ \\ \end{bmatrix}} = \underset{\substack{N \times N \\ F}}{\begin{bmatrix} e^{-j\frac{2\pi}{N}nk} \end{bmatrix}} \underset{\substack{x(n) \\ N\times 1}}{\begin{bmatrix} \\ \\ \end{bmatrix}} = \underline{X} = \underline{\underline{F}}\,\underline{x}$$

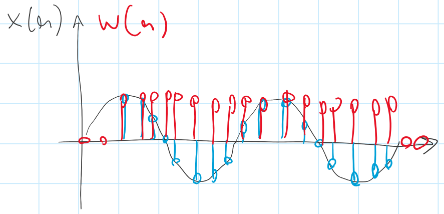
$$x(n) = e^{j\frac{2\pi}{N}k \cdot n} \qquad n = 0, \dots, N-1$$

$k = 0$: DC

$k \neq 0$: AC



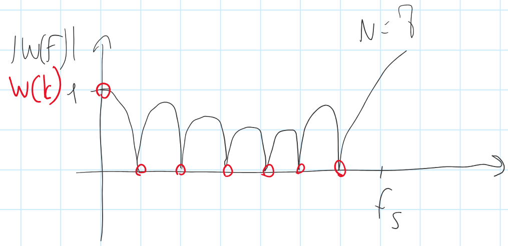
k nem egész?



$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}\cdot n \cdot k} \qquad \sum_{n=0}^{N-1} z^{-n}$$

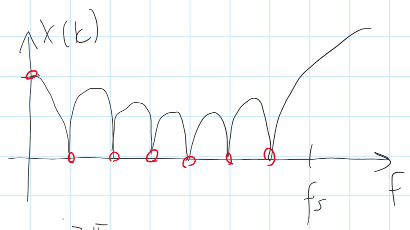
$$w(n) = \begin{cases} 1, \text{ ha } n = 0, \dots, N-1 \\ 0, \text{ egyébként} \end{cases} \qquad z = e^{j\nu}$$

$$W(f) = \frac{\sin\left(N \cdot \pi \cdot \frac{f}{f_s}\right)}{N \cdot \sin\left(\pi \frac{f}{f_s}\right)}$$

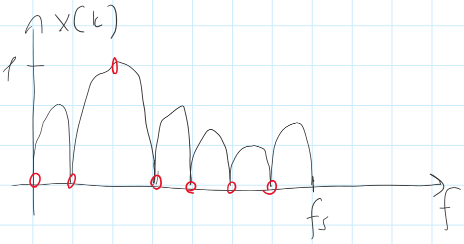


$N = 7 \qquad k \cdot \Delta f = k \cdot \frac{f_s}{N}$

1.) $x(n) = 1, \quad n = 0, \dots, N-1$

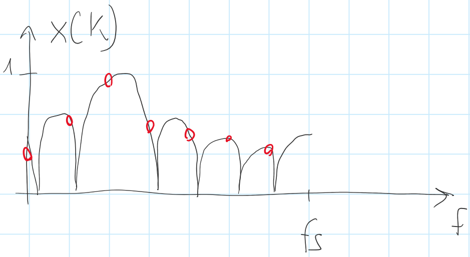


2.) $x(n) = e^{j\frac{2\pi}{N}2 \cdot n} \qquad n = 0, \dots, N-1$

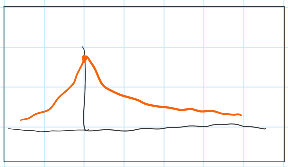


3.) $x(n) = e^{j\frac{2\pi}{N}2{,}3\cdot n}$

K nem egész



N >> 1



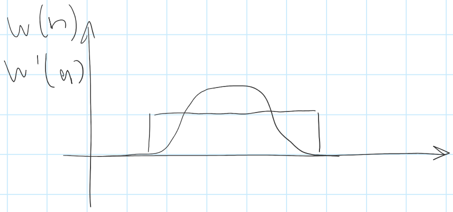
K egész

K nem egész

1.) tetőesés (picked fence)

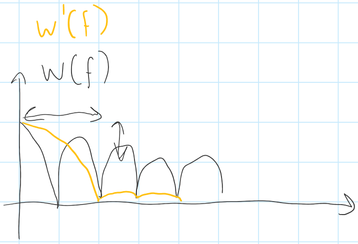
2.) szivárgás (leakage)

Káros hatás csökkentésére: ablakfv. alkalmazása

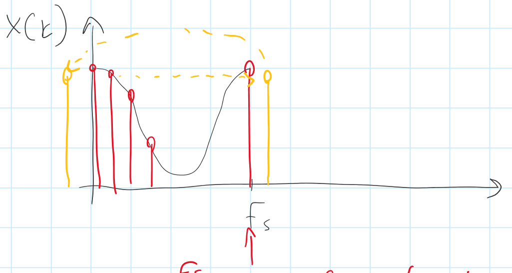


pl Hamming ablak:

$$w'(n) = \frac{1}{2}\left[1-\cos\left(\frac{2\pi}{N}n\right)\right] \qquad n = 0, \dots, N-1$$



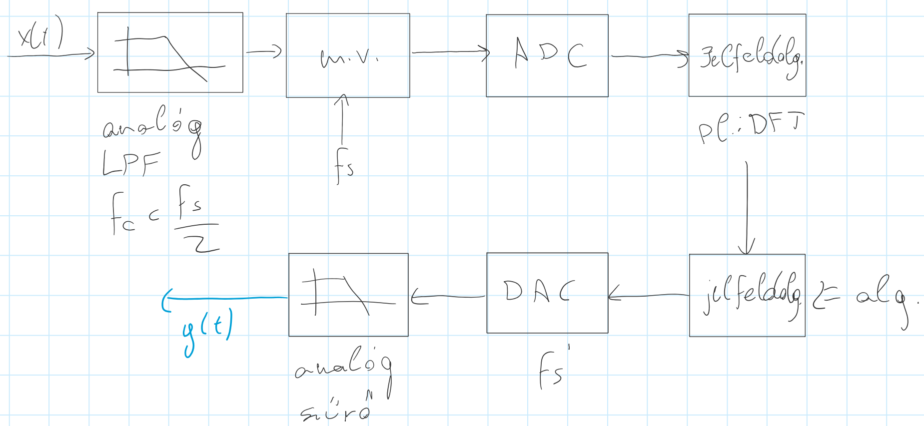
Folytonos (m.v.) jel spektruma



$\Delta f = \frac{f_s}{N}$ N. elem (index k = N − 1)

$$f_k = \frac{N-1}{N} f_s$$

Jelanalízis folyamata

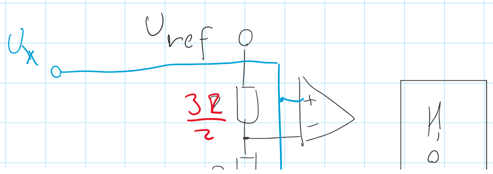


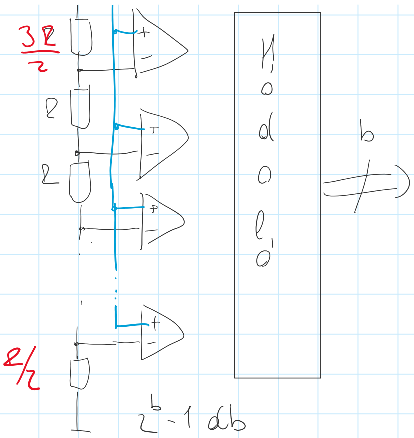
Analóg-digitális / Digitális-analóg } átalakítók

- DAC ⇒ létrahálózatos DA
- ADC: - párhuzamos (flash)
  - SAD (szukcesszív approximációs)
  - kettős meredekségű (dual-slope)
- Δ-Σ AD-DA

1) Flash AD

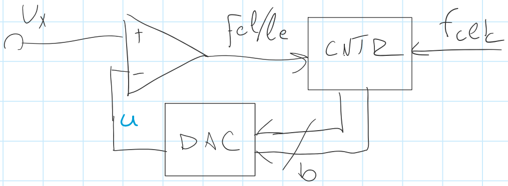
b bit ⇒ 2^b érték

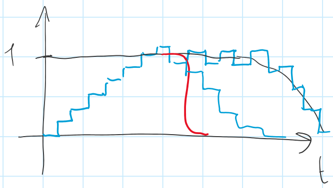




$2^b - 1$ db

Követős számláló AD

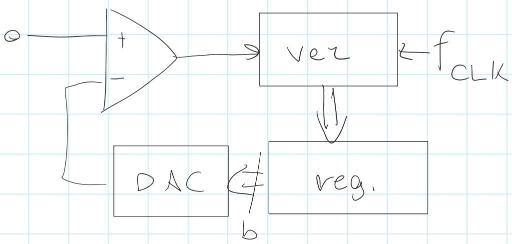




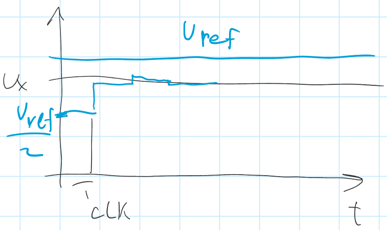
Worst case: $0 \to U_{ref}$, $U_{ref} \to 0$

$t_{átalakítás} = 2^b \cdot T_{CLK}$

Szukcesszív approximációs AD (SAR)

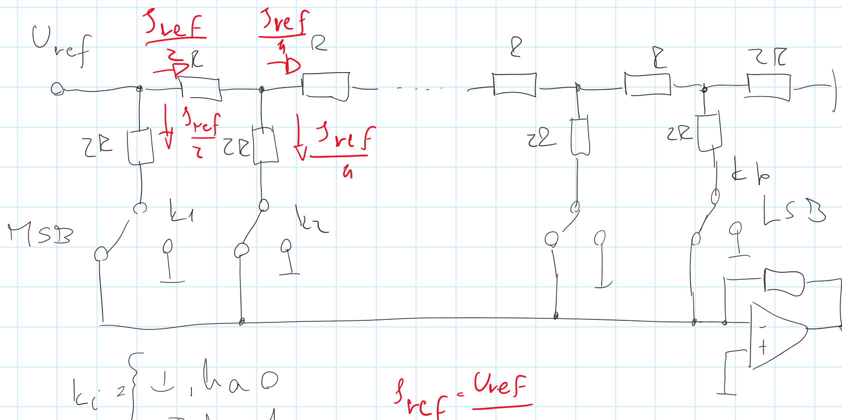


vez: MSB → LSB



WorstCase: $t_{átal} = b \cdot T_{CLK}$

DA-átalakítók ⇒ R–2R létrahálózatos DA



$U_{ki} = \sum_{i=1}^{b} U_{k_i} \cdot \frac{U_{ref}}{2^i}$

$k_i = \begin{cases} \perp, \text{ ha } 0 \\ \rightarrow, \text{ ha } 1 \end{cases}$

$I_{ref} = \frac{U_{ref}}{R}$

$\frac{I_{ref}}{2^i} \quad i = 1 \ldots b$

15 May 2019

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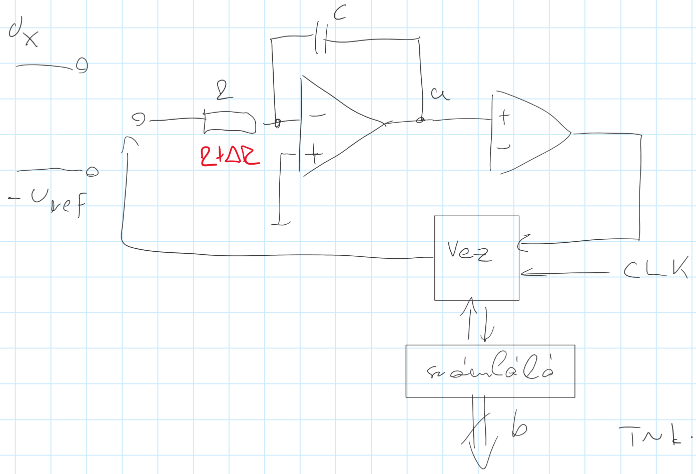
AD/DA

- DA: létrahálózatos DA (R-2R)
- AD: flash
  - SAR
  - Dual-Slope

Dual-slope ADC



$T_0 = RC$

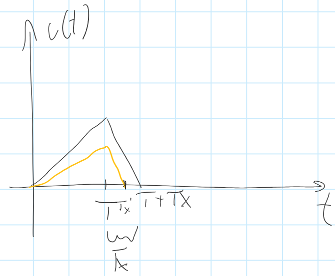
1) $u(t) = \frac{1}{T_0}\int_0^T u_x(t)\,dt$

2) $u(t) - \frac{1}{T_0}\int_0^{T_x} u_{ref}(t)\,dt$

$U_x(t) = U_x = const$

$U_{ref}(t) = U_{ref} = const$

$U_x' < U_x$



T: előírt idő

T_x: $u(t) = 0$

$U_x = U_{ref} \cdot \frac{T_x}{T}$

$T \sim k \cdot 10ms$ (tip)

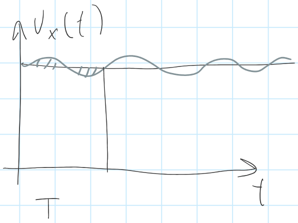
R, C, fo: rövid idejű stabilitást várunk el.

rendszeres órajel hiba kiesik

U_{ref}: hosszútávú stabilitás

$$\left|\frac{\Delta U_x}{U_x}\right| = \frac{\Delta U_{ref}}{U_{ref}} + \underbrace{\frac{\Delta T}{T}}_{0} + \underbrace{\frac{\Delta T_x}{T_x}}_{\frac{1}{N}}$$ kvantálási hiba

$U_x(t) = U_{x0} + U_z(t)$



tipikus beállítás: T = 20ms => 50Hz-es zavarjelet elnyomja

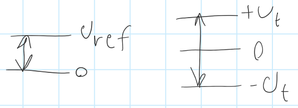
T_z: zavar periódusideje

b: bitszám

Fs: mintavételi frekvencia (elérhető)

B: sávszélesség

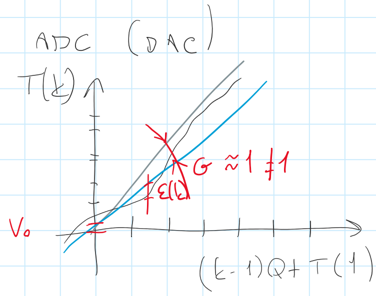
FS: Full Scale



ADC	b	B	Fs
flash	8	1GHz	k·1GHz
SAR	12-16	1MHz	10MHz
Dual Slope	20-24	0	10Hz
Δ-Σ	16-24	100kHz	k·100kHz

(tipikus értékek)

ADC (DAC)



elvi: kvantálási lépcső $q = \frac{FS}{2^b}$

gyakorlat: átalakítási lépcső

$$Q = \frac{T(2^b-1) - T(1)}{2^b - 2}$$

$$(k-1)Q + T(1) = V_0 + G \cdot T(k) + \varepsilon(k)$$

INL: integrális nemlinearitás

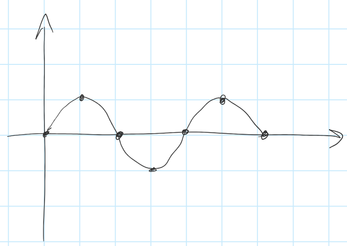
DNL: differenciális nemlinearitás

$$INL(k) = \frac{\varepsilon(k)}{Q}$$

$$INL = \max_{(k)} \left[INL(k) \right]$$

$$DNL(k) = \frac{G\,W(k) - Q}{Q}$$

$$DNL = \max_{k} \left[DNL(k) \right]$$



$$f_x = \frac{f_s}{2}$$

$$\hat{X}_{eff} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} x_i^2} = A\,|\sin(\varphi)|$$

$$0 < \hat{X}_{eff} < A$$

$$x(t) = A \sin(2\pi f_x t_n + \varphi) = A \sin(n\pi + \varphi)$$

$$X_{eff} = \frac{A}{\sqrt{2}}$$

$t_n = n \cdot 1$

$x(t) = A \sin(2\pi f_x t_n + \varphi) = A \sin(n\pi + \varphi)$

$t_n = n \cdot \frac{1}{f_s}$

$x_{eff} = \frac{A}{\sqrt{2}}$

$f_x \neq \frac{f_s}{2}$, $f_x \approx \frac{f_s}{2}$

$$\hat{x}_{eff} = \begin{cases} A, \text{ ha } \alpha(n) \approx \pm\frac{\pi}{2} \\ 0, \text{ ha } \alpha(n) \approx 0 \end{cases}$$