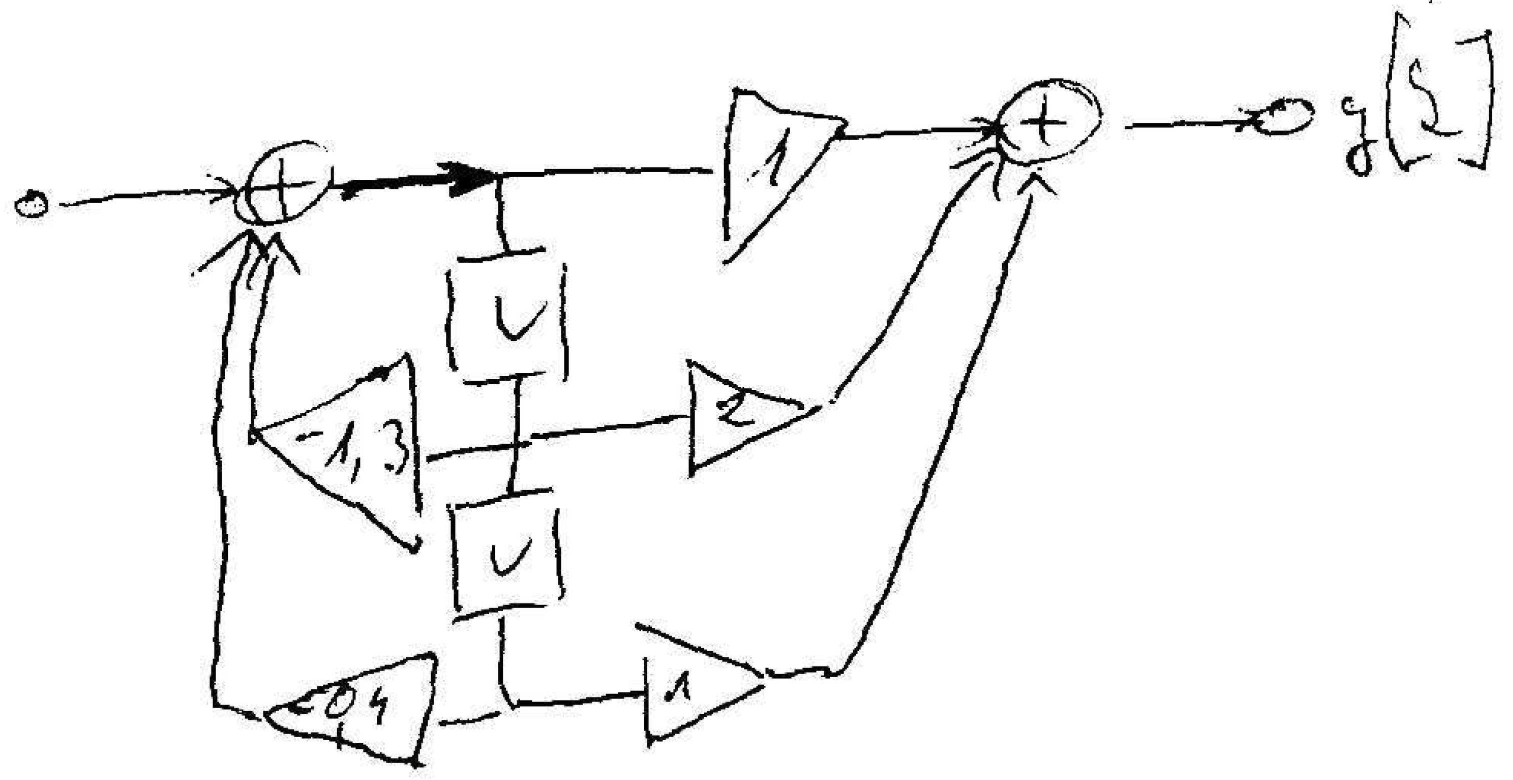
Jéliš Souzi 2012. 11. 30. [redacted] ZH 2012.

2 NP + 10 KP

A csoport

1. NP/ ~~KP~~

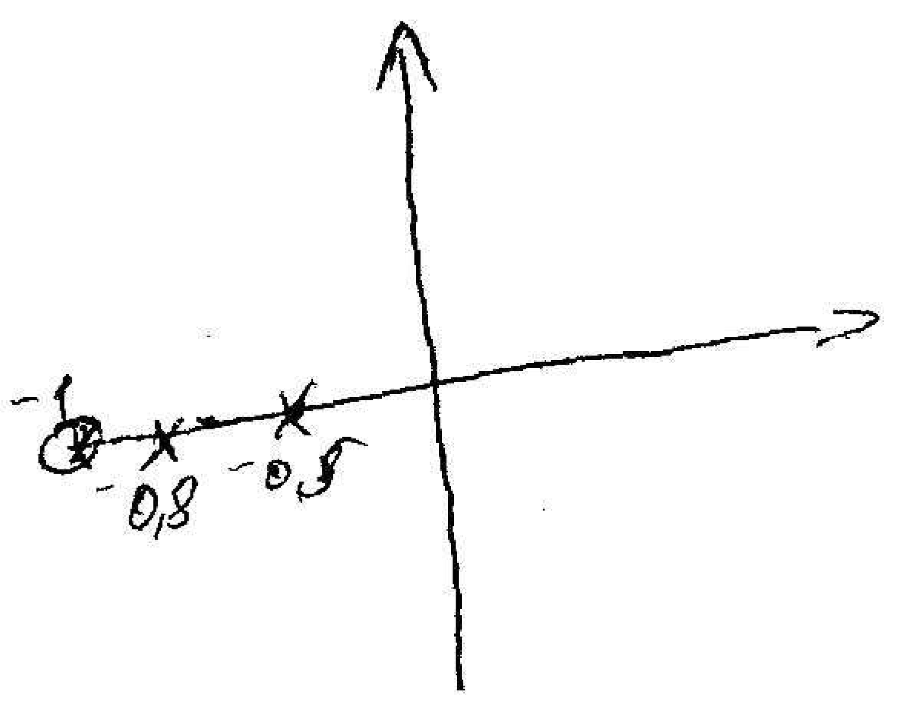
kanonikus alak




5 p) a) $H(z) = \dfrac{1+2z^{-1}+z^{-2}}{\underline{1}+1{,}3z^{-1}+0{,}4z^{-2}} =$

$= \dfrac{z^2+2z+1}{z^2+1{,}3z+0{,}4}$

Ábrázolva




5 p) b) p → z kép.

p. nev. null. h.: $z^2+1{,}3z+0{,}4=0$

$\dfrac{-1{,}3 \pm 0{,}3}{2}$ ↗ $-0{,}5$ ↘ $-0{,}8$

z: nevező null helyei

$z^2+2z+1=(z+1)^2=0$

$z_1=z_2=-1$

c) mivel minden pólus abszolút értéke < 1 ⇒

⇒ Aszimptotikusan stabil ⇒ GV stabil.

↓

mert a fokszám megegyezik

d) $u[k] = 117 \cdot \varepsilon[k]$ ← $Z\{a^k\} = \dfrac{z}{z-a}$

↓ Z transzformált

$U(z) = 117 \cdot \dfrac{z}{z-1}$

$H(z) = \dfrac{z^2+2z+1}{(z+0{,}5)(z+0{,}8)} \Rightarrow Y(z) = H(z)\cdot U(z) = \dfrac{z^2+2z+1}{(z+0{,}5)(z+0{,}8)} \cdot 117 \cdot \dfrac{z}{z-1} =$

$= 117 \cdot \dfrac{z^3+2z^2+z}{(z-1)(z+0{,}5)(z+0{,}8)}$

probléma: áltört → a fokszám megegyezik → csináljunk belőle valódi törtet sz<n

$Y(z) = \dfrac{z^3+2z^2+z}{z^3+0{,}3z^2-0{,}9z-0{,}4} =$

$= \dfrac{z^3+0{,}3z^2 - 0{,}9z-0{,}4 + 1{,}7z^2+1{,}9z+0{,}4}{(\quad)} =$

$$= 1 + \frac{1{,}7z^2 + 1{,}9z + 0{,}4}{(z-1)(z+0{,}5)(z+0{,}8)} = 1 + \frac{A}{z-1} + \frac{B}{z+0{,}5} + \frac{C}{z+0{,}8}$$

$$A = \left.\frac{1{,}7z^2 + 1{,}9z + 0{,}4}{(z+0{,}5)(z+0{,}8)}\right|_{\substack{z-1=0\\ z=1}} = \frac{1{,}7+1{,}9+0{,}4}{1{,}5\cdot 1{,}8} = \frac{4}{2{,}7} = \frac{40}{27}$$

$$B = \left.\frac{1{,}7z^2 + 1{,}9z + 0{,}4}{(z-1)(z+0{,}8)}\right|_{\substack{z+0{,}5=0\\ z=-0{,}5}} = \dots$$

$$C = \dots$$

$$H(z) = 1 + \left(\frac{Az}{z-1} + \frac{Bz}{z+0{,}5} + \frac{Cz}{z+0{,}8}\right)\cdot z^{-1}$$

probléma: nem z transzformált
mo.: bővítünk z-vel

Késleltetve van! Ha ezt csináljuk akkor mindenhol csináljuk.

$$y[k] = \delta[k] + \varepsilon[k-1]\; A\cdot \underbrace{1^{(k-1)}}_{1} + B\cdot(0{,}5)^{(k-1)} + C\cdot(-0{,}8)^{(k-1)}$$

$$Z\{a^k\} = \frac{z}{z-a}$$

2) $h(t) = \delta(t) + \varepsilon(t) * 4e^{-t}$

a) $H(j\omega) = ?$

átviteli karakterisztika ($j\omega$) a kérdés és nem a komplex frekvencia... 

Kérdés: ... ($s$)

$$\mathcal{L}\{h(t)\} = H(s) = 1 + 4\cdot\frac{1}{s+1} = \frac{s+1+4}{s+1} = \boxed{\frac{s+5}{s+1}}$$

Ez még nem válasz

$$\mathcal{L}\{e^{-\lambda t}\} = \frac{1}{s+\lambda}$$

$$H(j\omega) = \frac{j\omega+5}{j\omega+1}$$

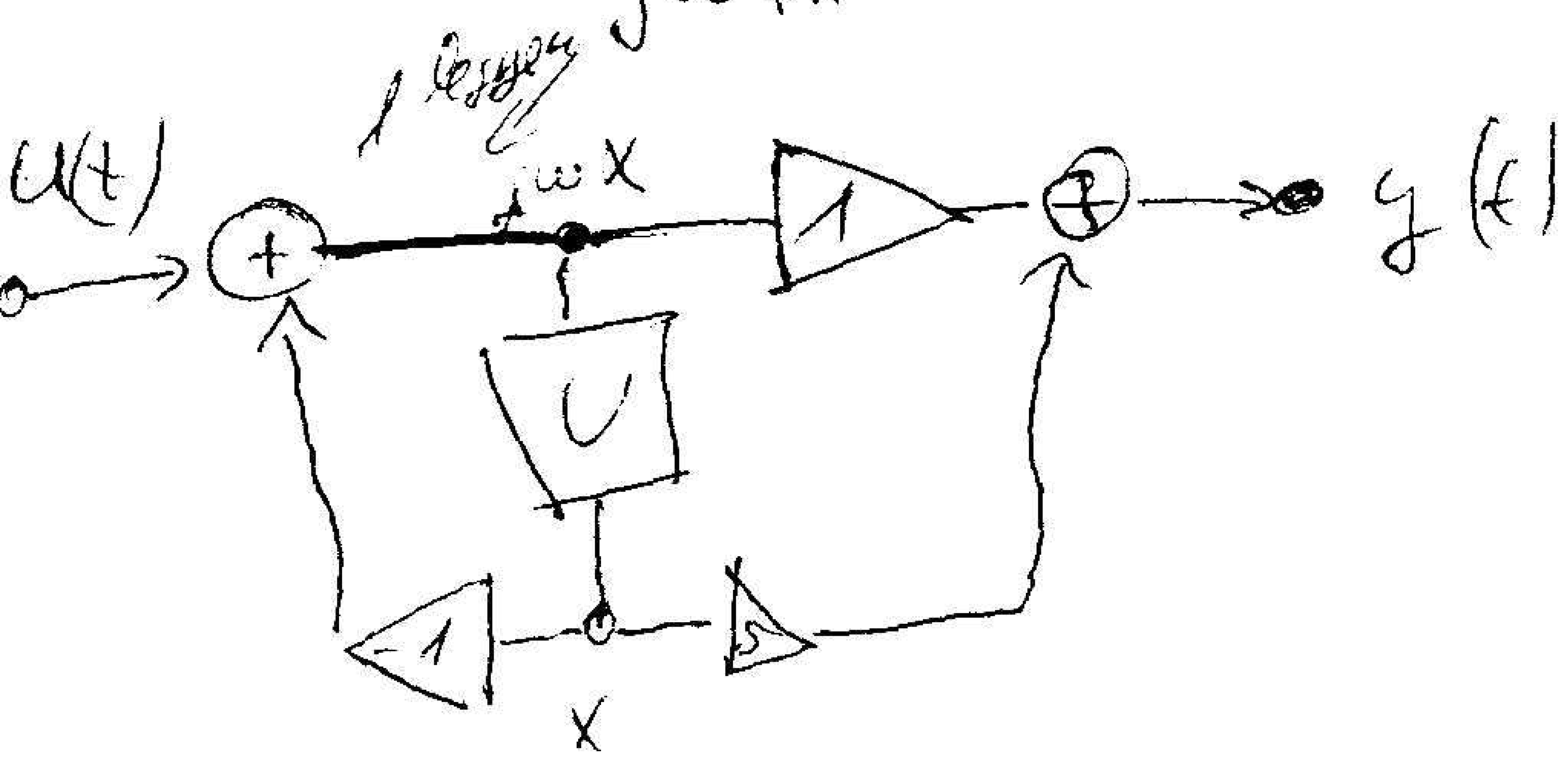
ez már igen

Lelez Sanzi 2012.11.30

2NP

b) kanonikus alak beli realizáció

$$H(j\omega) = \frac{j\omega+5}{j\omega+1} \longleftarrow \frac{\text{polinom}}{\text{polinom}} \text{ alak}$$

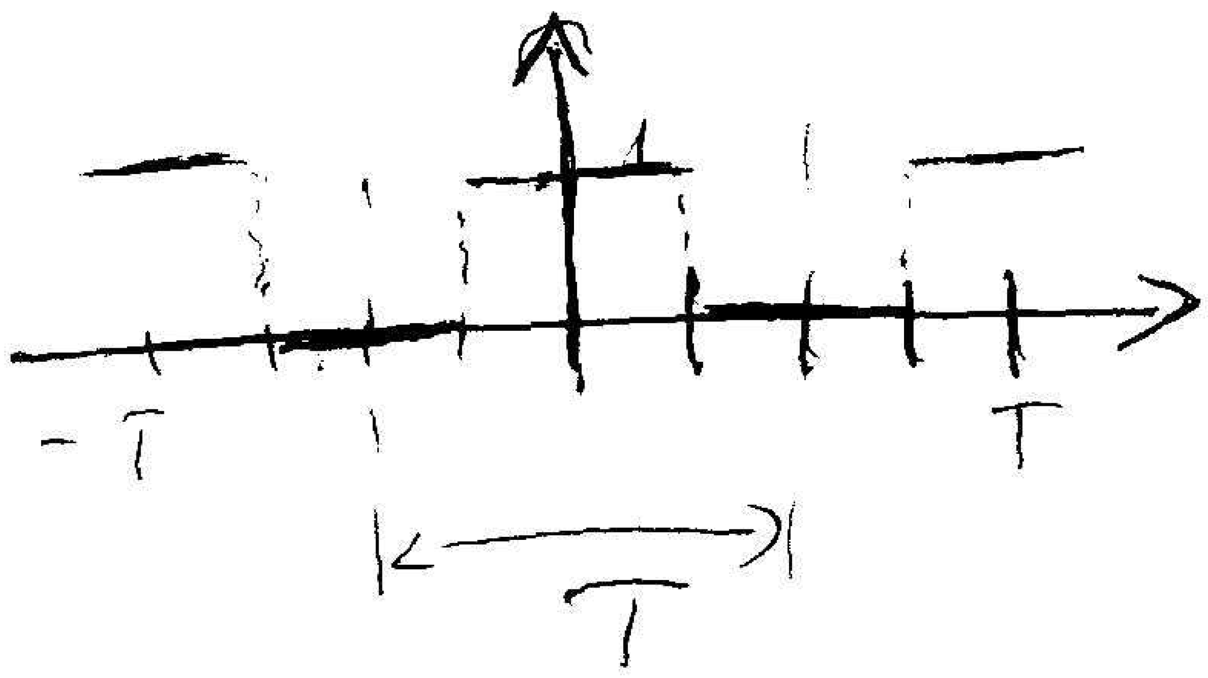



c)

$$u(t) = \begin{cases} 1 & 0 < t < \frac{T}{4} \\ 0 & \frac{T}{4} < t < \frac{3T}{4} \\ 1 & \frac{3T}{4} < t < T \end{cases}$$

$\omega_0 = \frac{2\pi}{T}$, $T = 1$  Négyszögjel lesz!

$\omega_0 = \frac{2\pi}{T} = 2\pi \ \text{rad/s}$




$$U_p^C = ? = \frac{1}{T} \int_{\langle T \rangle} u(t) e^{-jp\omega_0 t} \, dt$$

$p=0$

$$U_0 = \frac{1}{T} \int_{-\frac{T}{4}}^{\frac{T}{4}} 1 \, dt = \frac{1}{T} \cdot [t]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{1}{T}\left(\frac{T}{4} - \left(-\frac{T}{4}\right)\right) = \frac{1}{2}$$ → ez a DC átlag

$p > 0$

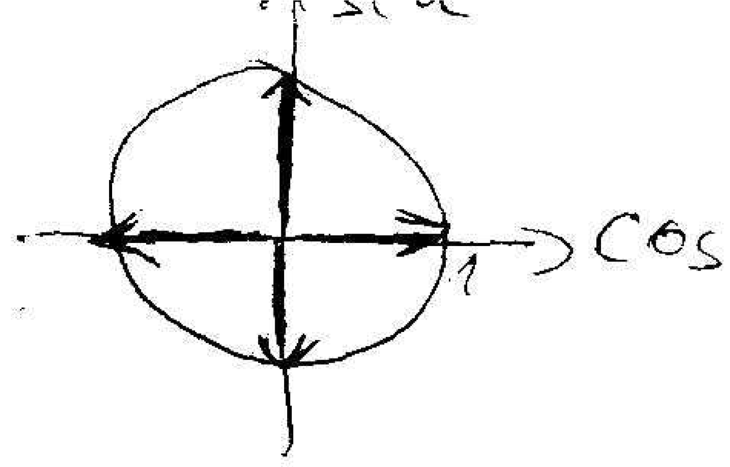
$p \in \mathbb{Z}$

$$U_p^C \ \frac{1}{T} \int_{-\frac{T}{4}}^{\frac{T}{4}} 1 \cdot e^{-jp\omega_0 t} \, dt = \frac{1}{T} \left[ \frac{e^{-jp\omega_0 t}}{-jp\omega_0} \right]_{-\frac{T}{4}}^{\frac{T}{4}} = \frac{1}{T} \cdot \frac{1}{-jp\omega_0} \cdot \left( e^{-jp\omega_0 \frac{T}{4}} - e^{jp\omega_0 \frac{T}{4}} \right) =$$

$$= \frac{1}{T} \frac{2 \cdot 1}{p\omega_0} \cdot \left( \frac{e^{jp\omega_0 \frac{T}{4}} - e^{-jp\omega_0 \frac{T}{4}}}{2j} \right) = \frac{2}{Tp\omega_0} \cdot \sin\left(p\omega_0 \frac{T}{4}\right) \Bigg|_{\substack{\omega_0 = 2\pi \\ T=1}} = \frac{2}{1 \cdot p \cdot 2\pi} \cdot \sin\left(p \frac{2\pi \cdot 1}{4}\right)$$

$$= \frac{1}{p\pi} \cdot \sin\left(p\frac{\pi}{2}\right) \Rightarrow$$

$$\Rightarrow U_1^C = \frac{1}{\pi}\cdot \sin\left(\frac{\pi}{2}\right) = 1$$




$$U_2^C = \frac{1}{2\pi}\cdot \sin 2\frac{\pi}{2} = 0$$

$$U_3^C = \frac{1}{3\pi}\cdot \sin\left(\frac{3\pi}{2}\right) = \frac{-1}{3\pi}$$

Fourier közelítés

$$u(t) = u_0 + \sum_{p=1}^{\infty} 2\cdot U_p^C \cdot \cos(p\omega_0 t + \varphi_p)$$

$$\varphi_p = \text{arc}(U_p^C)$$

$$u(t) = \frac{1}{2} + \sum_{p=1}^{\infty} \frac{2}{p\pi}\cdot \sin\left(p\frac{\pi}{2}\right)\cdot \cos(p\cdot 2\pi t) = \cancel{\cos(p\cdot 2\pi\cdot t +}$$

$$= \frac{1}{2} + \underbrace{\frac{2}{\pi}\cos 2\pi t}_{1} + \underbrace{\frac{-2}{3\pi}\cdot \cos(6\pi t)}_{3} + \underbrace{\frac{2}{5\pi}\cos(10\pi t)}_{5} + \underbrace{\dots}_{7}$$

$$p = 0 \Rightarrow \omega = 0 \Rightarrow H(j\omega) = \left.\frac{j\omega + 5}{j\omega + 1}\right|_{\omega=0} = \frac{5}{1} = 5 \rightarrow$$ 5szörös erősítés

$$p = 1 \Rightarrow \omega = 2\pi \Rightarrow H(j2\pi) = \frac{j2\pi + 5}{j2\pi + 1} = \frac{\sqrt{2\pi^2 + 5^2}}{\sqrt{2\pi^2 + 1}}\cdot e^{\left(j\,\text{arctg}\frac{2\pi}{5} - \text{arctg}\frac{2\pi}{1}\right)} = A\cdot e^{j\alpha}$$

nem kell tovább számolni, de lehet

$$p = 3 \Rightarrow \omega = 6\pi \Rightarrow H(j6\pi) = \frac{j6\pi + 5}{j6\pi + 1} = B\cdot e^{j\beta}$$

válaszjel:

$$y(t) = \frac{1}{2}\cdot 5 + \underbrace{\frac{2}{\pi} A\cdot\cos(2\pi t + \alpha)}_{1.\ \text{tag}} + \underbrace{\frac{-2}{3\pi}\cdot B\cdot \cos(6\pi t + \beta)}_{3.\ \text{tag}} + \dots$$

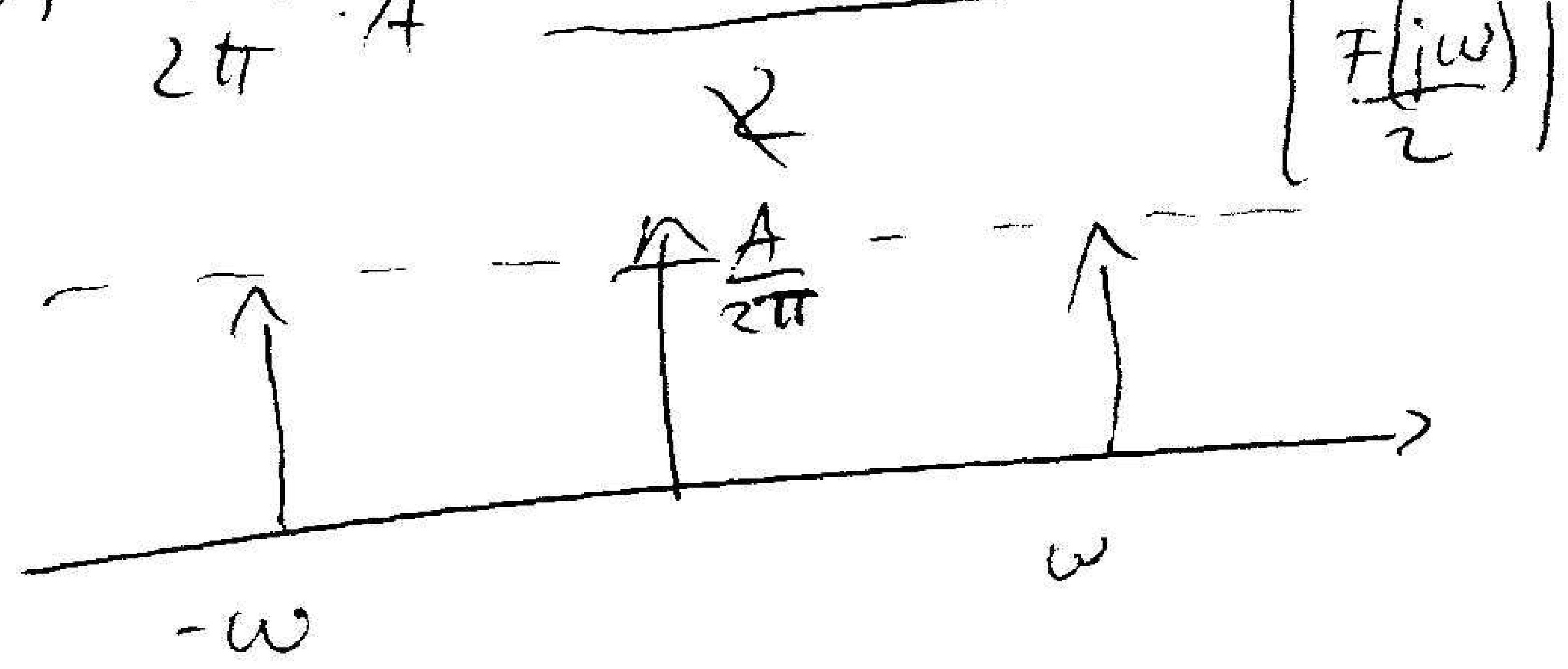
feleZ konzi 2012. 11. 30

1kP)

$f(t) = \frac{2}{2\pi} \cdot A \cdot \cos(\omega t)$ jel spektruma és ábrázold!

$\downarrow$

$f(t) = \frac{\cancel{2}}{2\pi} \cdot A \cdot \frac{e^{j\omega t} + e^{-j\omega t}}{\cancel{2}}$




2kP) $F(j\omega)$ ~~sp~~; hat. meg a jel energiáját

spektrum

Parseval-tétel

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \underbrace{\frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega}_{\text{miatt}}$$ ez a válasz

3.kP) Milyen sávkorlátú jelet lehet 196kHz frekvenciájú jellel mintavételezni?

Shannon tétel: $B_{max} \leq \frac{f_s}{2}$

$B_{max} \leq \frac{196\,kHz}{2}$

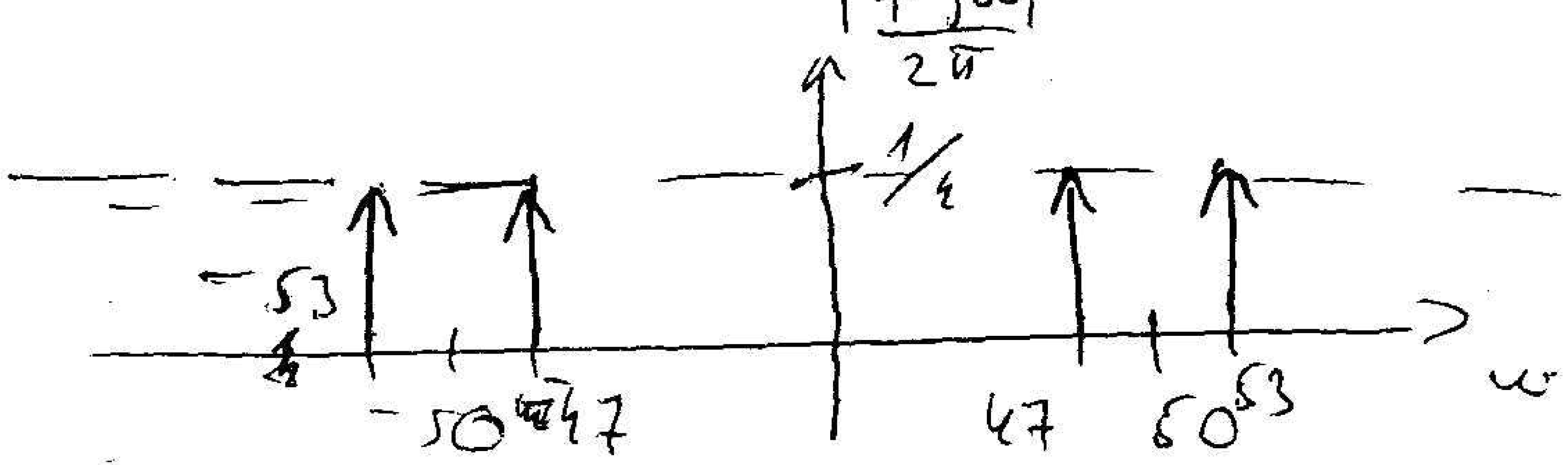
4kP)

$f(t) = \cos(3t)\cos(50t)$ spektruma ábra! körforgó vektor spektrumát kell rajzolni

$\cos\alpha \cdot \cos\beta = \frac{1}{2}\cos(\alpha - \beta) + \frac{1}{2}\cos(\alpha + \beta)$

Euler alak így jön ki a $\frac{1}{4}$

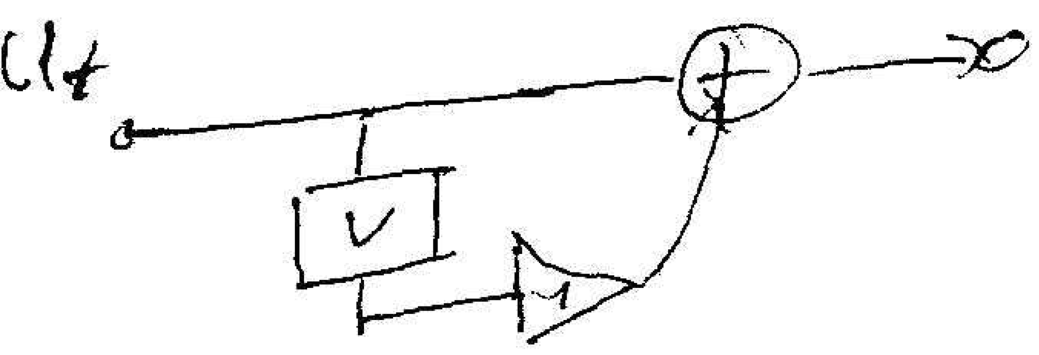
mert $\frac{1}{2} \cdot \frac{e^{\cdots} + e^{-\cdots}}{2} = \frac{1}{4} \cdot e^{\cdots}$

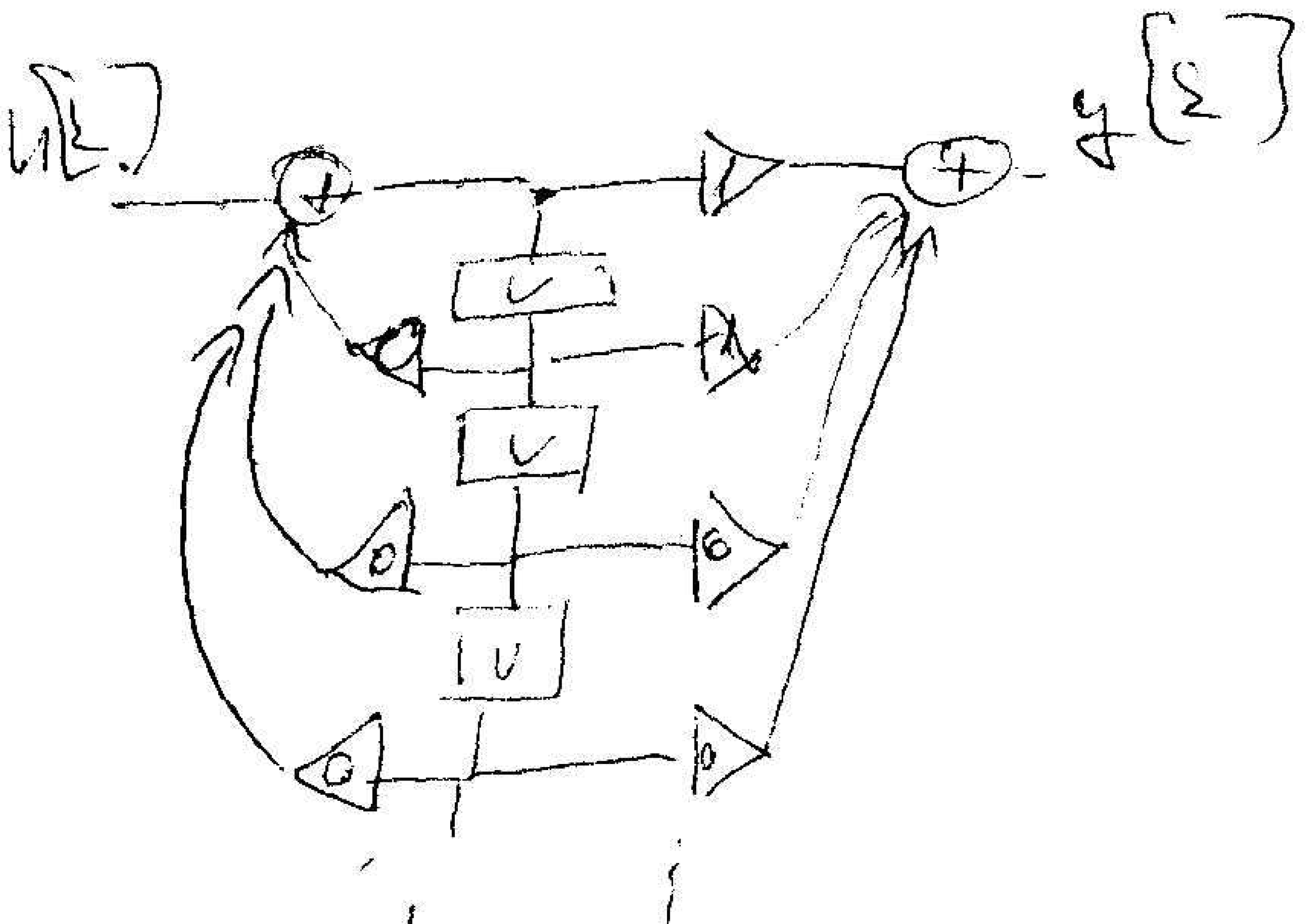


5 KP/ Adja meg a $H(z) = \frac{1-z^{-1}}{1}$ kanonikus alakjával!

→ 0. fokú a nevező → nincs visszacsatolás

DI:






6 KP/ $\mathcal{F}\{x(t-t_0)\} = X(j\omega)\cdot e^{-\omega t_0}$

$\mathcal{F}\{x(t)\} = X(j\omega)$

7. KP) Hat. meg. á f. fgv. normál alakját.

$u[k] = \delta[k]$

$y[k] = \varepsilon[k]\cdot 0{,}5^k$ $\Big\}$ $H(z) = \frac{Y(z)}{U(z)} = \frac{\frac{z}{z-0{,}5}}{1}$

8 KP) ps fgv $\mathcal{F}\{\}$ tisztán valós képz.

ptlen.

9 KP) $H(s) = \frac{1}{s+3} \rightarrow H(j\omega) = \frac{1}{j\omega+3} \Big| = \frac{1}{j4+3} = \frac{1}{5}e^{-j\,\mathrm{arctg}\frac{4}{3}}$

$u(t) = \cos(4t) \rightarrow \omega = 4$

$y(t) = ? = \frac{1}{5}\cdot\cos 4t - \mathrm{arctg}\frac{4}{3}$

10 kp)

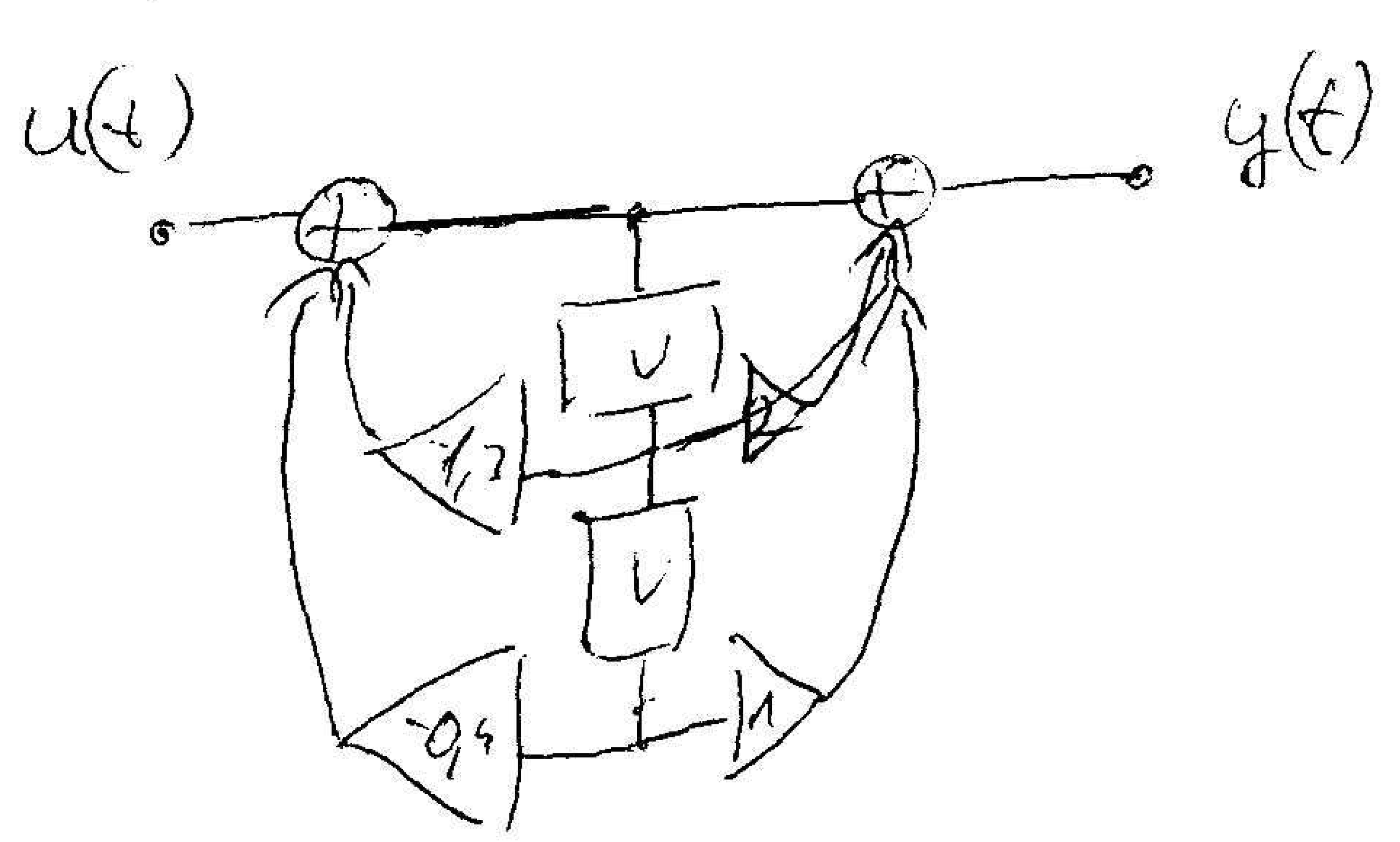
Kezdeti érték tétel:

$H(+0) = \lim_{t\to+0} h(t) = \lim_{s\to\infty} s\,H(s)$ ha $H(s) = \mathcal{L}\{h(t)\}$

$\lim h(t) = \lim s\,H(s)$ ha —"—

Jelek Zárthelyi 2012. 11. 30.

B 1NP)




a) $H(s)=? = \dfrac{s^2+2s+1}{1\cdot s^2+1{,}3s+0{,}4}$

b) $z_1=z_2=-1$

$p_1=-0{,}5$

$p_2=-0{,}8$

egyenletmegoldás

c) $\forall\ \mathrm{Re}\{p_i\}<0 \Rightarrow$ asz stabil $\rightarrow$ GV stabil

Re: valós része

d) $u(t)=2\cdot\varepsilon(t)$ $\quad \mathcal{L}\{e^{-\alpha t}\}=\dfrac{1}{s+\alpha}$

$U(s)=\dfrac{2}{s} \rightarrow \alpha=0$

$Y(s)=H(s)\,U(s)=2\cdot\dfrac{s^2+2s+1}{s(s+0{,}5)(s+0{,}8)}=2\left(\dfrac{A}{s}+\dfrac{B}{s+0{,}5}+\dfrac{C}{s+0{,}8}\right)$

Letakarásos módszer

$A\Big|_{s=0}=\dfrac{s^2+2s+1}{(s+0{,}5)(s+0{,}8)}=2{,}5$

$B\Big|_{s=-0{,}5}=\dots$

$C\Big|_{s=-0{,}8}=\dots$

Válaszjel:

$\underline{y(t)}=\varepsilon(t)\left(2A+2B\cdot e^{-0{,}5t}+2C\cdot e^{-0{,}8t}\right)$

B: 2NP)

$h[k]=2\delta[k]+\varepsilon[k]\cdot 0{,}1^k$

$\mathcal{Z}\{h[k]\}=2+\dfrac{z}{z-0{,}1}=\dfrac{z-0{,}1+z}{z-0{,}1}=\dfrac{3z-0{,}2}{z-0{,}1}$

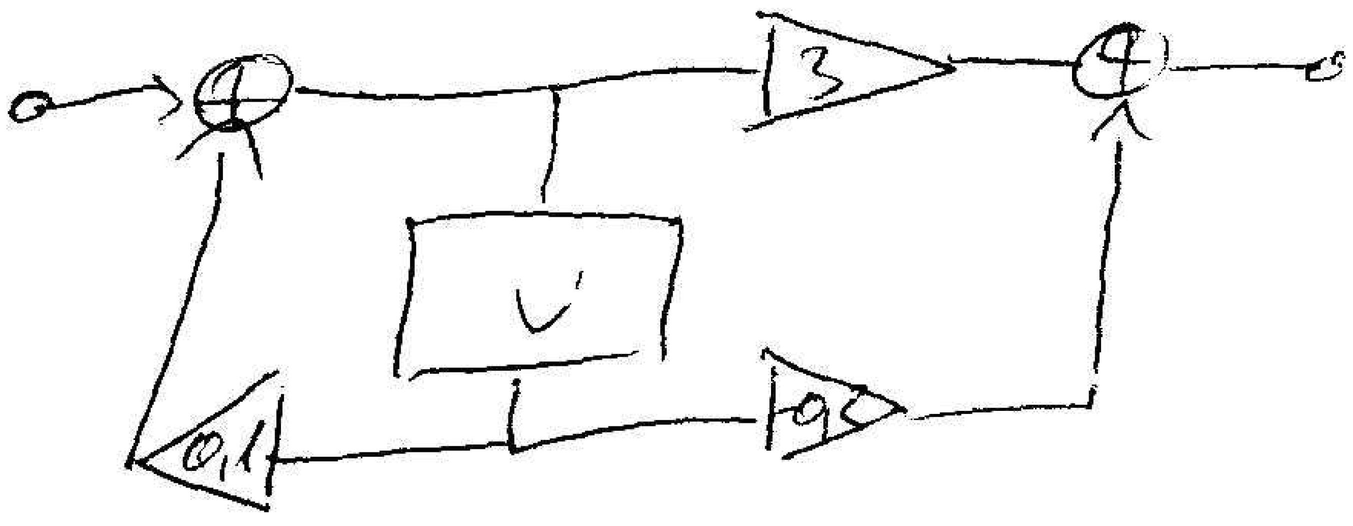
a) Normál alak

$H(e^{j\vartheta})=?$

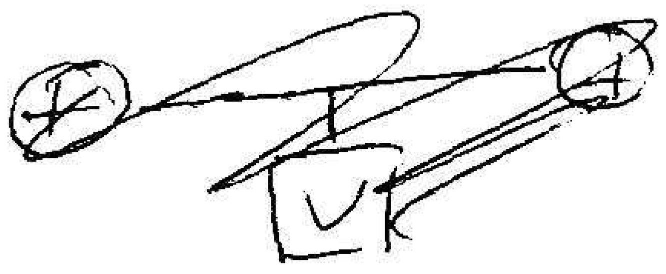
$H(e^{j\vartheta})=\dfrac{3\cdot e^{j\vartheta}-0{,}2}{e^{j\vartheta}-0{,}1}$

kanonikus realizáció

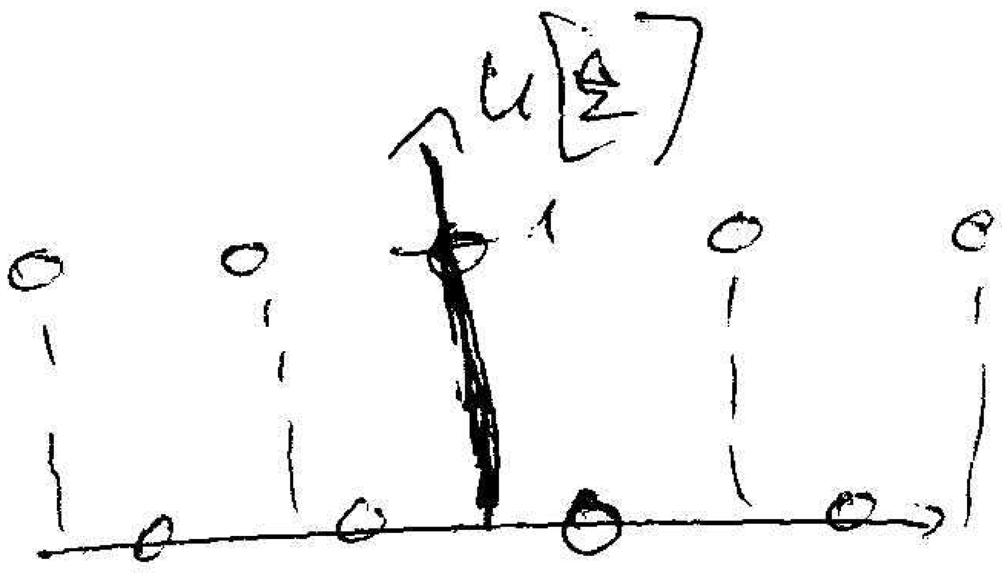
$$H(z) = \frac{3 - 0,2 z^{-1}}{1 - 0,1 z^{-1}}$$




mivel elsőfokú ezért 1 db késleltető van.




c)




$u[0] = 1$

$u[1] = 0$

$L = 2$

$$U_0 = \frac{1}{L} \sum_{k=0}^{L-1} u[k] = \frac{1}{2}(1+0) = \frac{1}{2}$$

$$U_p^c = \frac{1}{L} \sum_{k=0}^{L-1} u[k] \cdot e^{-jp\vartheta k} = \frac{1}{2}\, 1 + 0 \cdot e^{j p \pi} = \frac{1}{2}$$

$$\vartheta = \frac{2\pi}{L} = \frac{2\pi}{2} = \pi$$

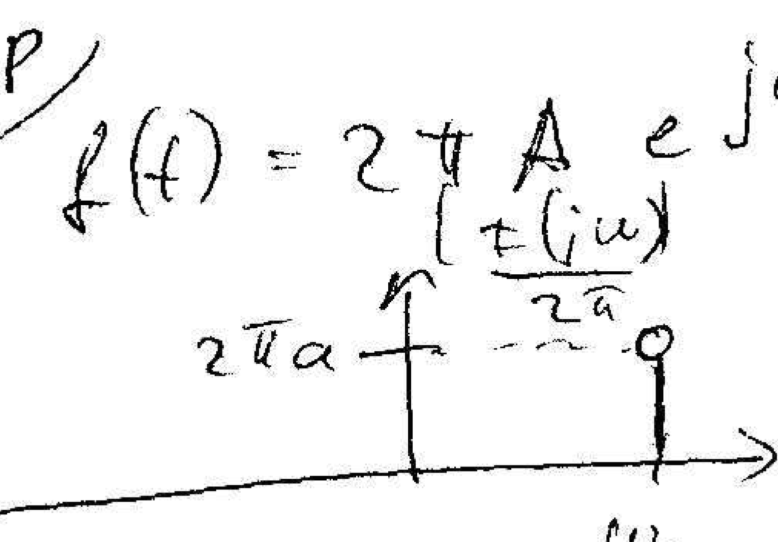
$$u[k] = \frac{1}{2} + \frac{1}{2} \cdot \cos \pi k$$

d)

$$H(e^{j\vartheta}) = \frac{3 e^{j\vartheta} - 0,2}{e^{j\vartheta} - 0,1}$$

$$H(e^{j0}) = \frac{3 - 0,2}{1 - 0,1} = \frac{2,8}{0,9}$$

$$H(e^{j\pi}) = \frac{3 \cdot e^{j\pi} - 0,2}{e^{j\pi} - 0,1} = \frac{-3 - 0,2}{-1 - 0,1} = \frac{3,2}{1,1}$$




valós

0° való tolódás

$$y(t) = \frac{1}{2} \cdot \frac{2,8}{0,9} + \frac{1}{2} \cdot \frac{3,2}{1,1} \cdot \cos(\pi k + 0^\circ)$$

B: 1 KP/

$f(t) = 2\pi A\, e^{j\omega_0 t}$ spektrum ábrája

Jelek kanzi 2012.11.30.

2 $f(t) \to E$

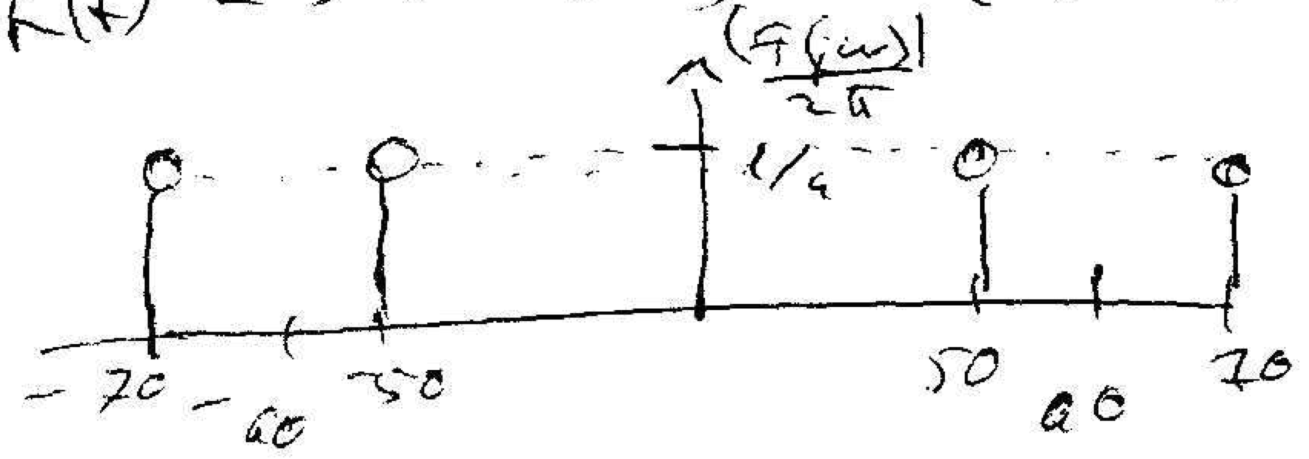
parseval

3/ $B_{max} = 3000\,Hz$

$F_{s\,min} = 2 \cdot B_{max} = 6\,kHz$

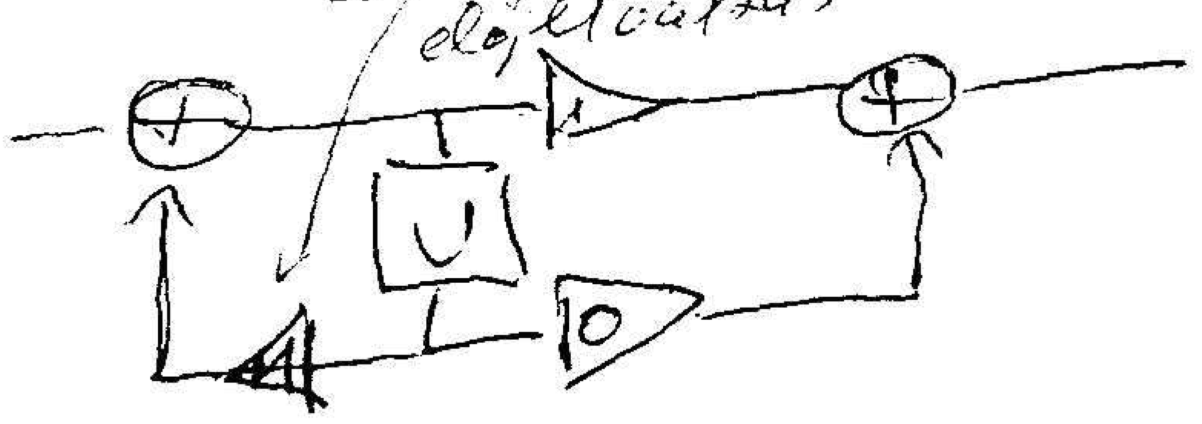
4.

$f(t) = \sin(10t)\,\sin(60t)$




5/ $H(z) = \frac{1}{1-z^{-1}}$ kanonikus

= előjelváltás




6./ $\mathcal{L}\{\varepsilon(t-t_0)\cdot y(t-t_0)\} = y(s)\cdot e^{-st_0}$

$\mathcal{L}\{y(t)\} = y(s)$

7. $u[k] = 5\,\varepsilon[k]$ $\qquad Z\{a^k\} = \frac{z}{z-a}$ $\qquad$ a lehet 0, 1, ...

$y[k] = \varepsilon[k]\cdot 0{,}6^k$

$U(z) = 5\cdot\frac{z}{z-1}$

$y(z) = \frac{z}{z-0{,}6}$ $\qquad H(z) = \frac{y(z)}{u(z)} = \frac{\cancel{z}}{z-0{,}6}\cdot\frac{1}{5}\cdot\frac{z-1}{\cancel{z}} = \frac{1}{5}\cdot\frac{z-1}{z-0{,}6}$

8. PTLW...

9) $H(s) = \frac{1}{s+6} \to H(j\omega) = \frac{1}{\dots}$ $\qquad u(t) = 3\sin(8t) \to \omega = 8$ $\Rightarrow H(j8) = \frac{1}{6+j8} = \frac{1}{10}\cdot e^{-j\,\mathrm{arctg}\,\frac{8}{6}}$