2018. IV. 19 pótzh. α variáns

1. $\int y\cos 5y\,dy = \int x\sqrt[3]{2+3x^2}\,dx$ (3) $\int y\cos 5y\,dy = \frac{1}{5}y\sin 5y + \frac{1}{25}\cos 5y$ (6)

$\int x\sqrt[3]{2+3x^2}\,dx = \frac{1}{8}(2+3x^2)^{4/3}$ (6) $\boxed{15}$

2. $y' + 2xy = 0$ $\quad y' = -2xy$ (3) $\int \frac{dy}{y} = \int -2x\,dx$ (3) $\ln|y| = -x^2+c$

$y = c(x)e^{-x^2}$ (3) $\quad c'e^{-x^2} - c2xe^{-x^2} + 2xc(x)e^{-x^2} = 2xe^{-x^2}$ (4)

$c'(x) = 2x \quad c(x) = x^2 + c$ (2), $\quad y = (x^2+c)e^{-x^2} \quad (1+c)e^{-1} = 2,\ c=2e$

$y = (x^2 + 2e - 1)e^{-x^2}$ (2) $\quad \boxed{20}$

3. $\lambda^2 + 9 = 0, \quad \lambda = \pm 3i$ (2) $\quad y_h = c_1\cos 3x + c_2\sin 3x$ (3)

$y_p = x(A\cos 3x + B\sin 3x)$ (2)

$y_p'' + 9y_p = -6A\sin 3x + 6B\cos 3x + x(-9A\cos 3x - 9B\sin 3x)$

$+ 9x(A\cos 3x + B\sin 3x) = -6A\sin 3x + 6B\cos 3x = 18\cos 3x$

$A = 0,\ B = 3$ (2) $\quad y = c_1\cos 3x + c_2\sin 3x + 3x\sin 3x$ (2) $\boxed{15}$

4. a) $S_n \to A$ (2) $(n\to\infty)$. Tegyük fel hogy $\sum (a_n + b_n) < \infty$

tehát $\sum_{k=1}^{n}(a_k + b_k) = S_n^1 + S_n^2 \to B\ (n\to\infty)$. Mivel $S_n^1 \to A$

$(n\to\infty)$ akkor $S_n^2 \to B - A\ (n\to\infty)$ és így $\sum b_n$ konvergens

lenne $\boxed{8}$ b) $\sum(-1)^n a_n$ (2), $a_n > 0$, $a_n \to +0$ (2), $a_n \downarrow$ (2) $\Rightarrow$ konv. $\boxed{6}$

5. a) $1 - \cos\frac{1}{n} = 2\sin^2\frac{1}{2n} \le \frac{1}{2n^2}$ (3) és $\sum \frac{1}{k^{4/3}} < \infty$ (4) konv. $\boxed{7}$

b) $\cos(\pi n) = (-1)^n$ (3) $\quad \sum \frac{(-1)^n}{\sqrt{n}}$ + Leibnitz (4) $\Rightarrow$ konv. $\boxed{7}$

c) $\frac{1}{\pi}\arctan \to \frac{1}{2}$ (3) $(n\to\infty)$ + gyök kritérium (4) $\Rightarrow$ konv.

6. a) igen b) nem c) igen d) nem e) nem $\boxed{15}$

IMSC. Jegyzet: 81 oldal, 1.100 példa $f(2018) = -3$

2018. IV. 19 pótzh. β variáns

1. $\int y \sin 5y \, dy = \int x \sqrt[3]{2+x^2} \, dx$ (3) $\int y \sin 5y \, dy = -\frac{1}{5} y \cos 5y + \frac{1}{25} \sin 5y$ (6)

$\int x \sqrt[3]{2+x^2} \, dx = \frac{3}{8} (2+x^2)^{4/3}$ (6)

2. $y' + 2xy = 0$, $y' = -2xy$ (3) $\int \frac{dy}{y} = \int -2x \, dx$ (3) $\ln|y| = -x^2 + c$ (3)

$y = c(x) e^{-x^2}$ (3) $c' e^{-x^2} - c \, 2x e^{-x^2} + 2x c e^{-x^2} = x e^{-x^2}$ (4)

$c'(x) = x$, $c(x) = \frac{x^2}{2} + c$ (2) $y = \left(\frac{x^2}{2} + c\right) e^{-x^2}$ $\left(\frac{1}{2} + c\right) e^{-1} = 1$

$c = e - \frac{1}{2}$ $y = \left(\frac{x^2}{2} + e - \frac{1}{2}\right) e^{-x^2}$ (2) $\boxed{20}$

3. $\lambda^2 + 9 = 0$, $\lambda = \pm 3i$ (2), $y_h = c_1 \cos 3x + c_2 \sin 3x$ (3)

$y_p = x(A \cos 3x + B \sin 3x)$ (2)

$y_p'' + 9 y_p = 18 \sin 3x$, $y_p'' + 9 y_p = -6A \sin 3x + 6B \cos 3x$ (4)

$A = -3$, $B = 0$ (2) $y = C_1 \cos 3x + C_2 \sin 3x - 3x \cos 3x$ (2)

4. ugyan az mint az α variáns

5. a) $1 - \cos\frac{1}{n} \leq \frac{1}{2n^2}$ (3) és $\sum \frac{1}{n^{3/2}} < \infty \Rightarrow$ konv. $\boxed{7}$

b) $(\cos \pi n)^3 = (-1)^{3n} = (-1)^n$ (+) Leibniz $\Rightarrow$ konv. $\boxed{7}$

c) $\frac{1}{\pi} \arctan^2 n \to \frac{1}{\pi} \left(\frac{\pi}{2}\right)^2 = \frac{\pi}{4} < 1$ (+) gyök kr. konv $\boxed{7}$

6. igen nem igen nem nem