(5.)

$g[k]=(20+8\cdot(-0{,}4)^k)\cdot\varepsilon[k]$

$h[k]=?$

$g[k]=\sum_{i=-\infty}^{k} h[i]$

$h[k]=a\cdot b^k\cdot\varepsilon[k]+c\cdot\delta[k]$

$\hookrightarrow \underline{b=-0{,}4}$

$$\sum_{i=-\infty}^{k}\left(a\cdot(-0{,}4)^k\cdot\varepsilon[i]+c\cdot\delta[i]\right)=\sum_{i=0}^{k}a\cdot(-0{,}4)^i+\underbrace{\sum_{i=-\infty}^{k}c\cdot\delta[i]}_{=c\cdot\varepsilon[k]}$$

$$=a\cdot\frac{1-(-0{,}4)^{k+1}}{1-(-0{,}4)}=\left(\frac{a}{1{,}4}+\frac{0{,}4a}{1{,}4}(-0{,}4)^k\right)\varepsilon[k]$$

$\rightarrow\ g[k]=(20+8\cdot(-0{,}4)^k)\varepsilon[k]$

$g[k]=\left(\frac{a}{1{,}4}+c+\frac{0{,}4a}{1{,}4}(-0{,}4)^k\right)\varepsilon[k]$

$\hookrightarrow \frac{a}{1{,}4}+c=20\rightarrow \underline{\underline{c=0}}$

$\hookrightarrow \frac{0{,}4a}{1{,}4}=8\rightarrow \underline{\underline{a=28}}$

$$\boxed{h[k]=28\cdot(-0{,}4)^k\cdot\varepsilon[k]+0\cdot\delta[k]}$$

(6.)

$h(t)=-18\delta[t]+19e^{-2{,}1t}\cdot\varepsilon(t)$

$g(t)=(a+b\cdot e^{ct})\cdot\varepsilon(t)=?$

$h(t)=g'(t)$

szorzat derivált

$g'(t)=\left((a+b\cdot e^{ct})\varepsilon(t)\right)'=bce^{ct}\varepsilon(t)+a\delta(t)+be^{ct}\delta(t)=\circledast$

$\underline{\underline{c=-2{,}1}}$

csak t=0-ban van értéke $\rightarrow =b\delta(t)$

$\circledast=-2{,}1be^{-2{,}1t}\varepsilon(t)+(a+b)\delta(t)$

$\hookrightarrow -2{,}1b=19$ $\quad\hookrightarrow a+b=-18$

$\underline{\underline{b=-9{,}048}}$ $\quad\underline{\underline{a=-8{,}952}}$

$$\boxed{g(t)=(-8{,}952-9{,}048e^{-2{,}1t})\varepsilon(t)}$$

7.

$$h[k]=\varepsilon[k](-0{,}3)^k\cos(8{,}9k)$$

$$\sum_{i=-\infty}^{\infty}|h[i]|=?$$

$$\sum_{i=-\infty}^{\infty}|\varepsilon[i](-0{,}3)^i\cos(8{,}9i)|\le\sum_{i=-\infty}^{\infty}|\varepsilon[i]\cdot(-0{,}3)^i\cdot 1|=\sum_{i=0}^{\infty}(-0{,}3)^i=\frac{1}{1-(-0{,}3)}=\boxed{0{,}769<\infty}$$

$\Downarrow$

GV-stabil

8.

$$h(t)=\varepsilon(t)\frac{1}{(t+5{,}5)^2}$$

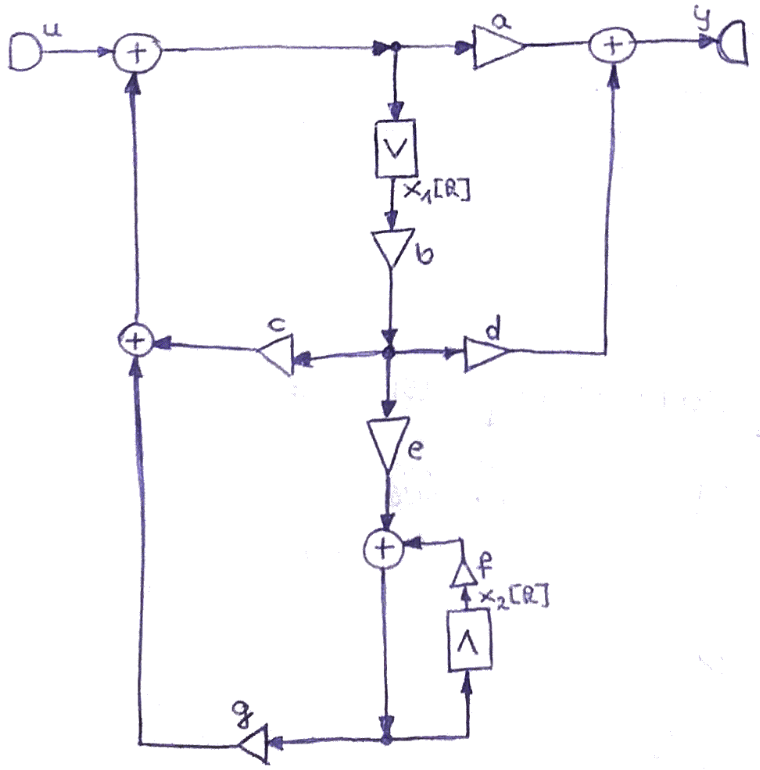
$$\int_{-\infty}^{\infty}|h(t)|dt=?$$

$$\int_{-\infty}^{\infty}\left|\varepsilon(t)\frac{1}{(t+5{,}5)^2}\right|dt=\int_{0}^{\infty}\frac{1}{(t+5{,}5)^2}dt=\left[-\frac{1}{(t+5{,}5)}\right]_0^{\infty}=0-\left(-\frac{1}{5{,}5}\right)=\boxed{0{,}182<\infty}$$

$\Downarrow$

GV-stabil

9.



$a=0{,}2$
$b=0{,}6$
$c=-0{,}6$
$d=0{,}6$
$e=0{,}6$
$f=0{,}9$
$g=0{,}3$

$\underline{\underline{A}}=?$ $\underline{B}=?$ $\underline{C}^T=?$ $D=?$ $\underline{\underline{A}}$ karakterisztikus egyenlete? $\lambda_{1,2}=?$ ASZ-stabil?

$$x_1[k+1] = u[k] + cbx_1[k] + gebx_1[k] + gfx_2[k] = (c \cdot b + g \cdot e \cdot b)x_1[k] + g \cdot f \cdot x_2[k] + u[k]$$

$$x_2[k+1] = bex_1[k] + fx_2[k]$$

$y[k] = ($ [redacted] $+ x_1[k+1]) \cdot a + dbx_1[k] = (ab(c+ge)+db)x_1[k] + agfx_2[k] + (a$ [redacted] $u[k]$

$$\Rightarrow \underline{\underline{A}} = \begin{bmatrix} cb+geb & gf \\ ab(c+ge)+db & agf \end{bmatrix} = \begin{bmatrix} \overbrace{-0{,}36+0{,}108}^{-0{,}252} & 0{,}27 \\ -0{,}018 & 0{,}054 \end{bmatrix} = \begin{bmatrix} -0{,}252 & 0{,}27 \\ -0{,}018 & 0{,}054 \end{bmatrix}$$

$\underline{B} =$

$$\underline{\underline{A}} = \begin{bmatrix} cb+geb & gf \\ be & f \end{bmatrix} = \begin{bmatrix} -0{,}252 & 0{,}27 \\ 0{,}36 & 0{,}9 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\underline{C}^T = [\, ab(c+ge)+db \quad agf \,] = [\, -0{,}018 \quad 0{,}054 \,]$$

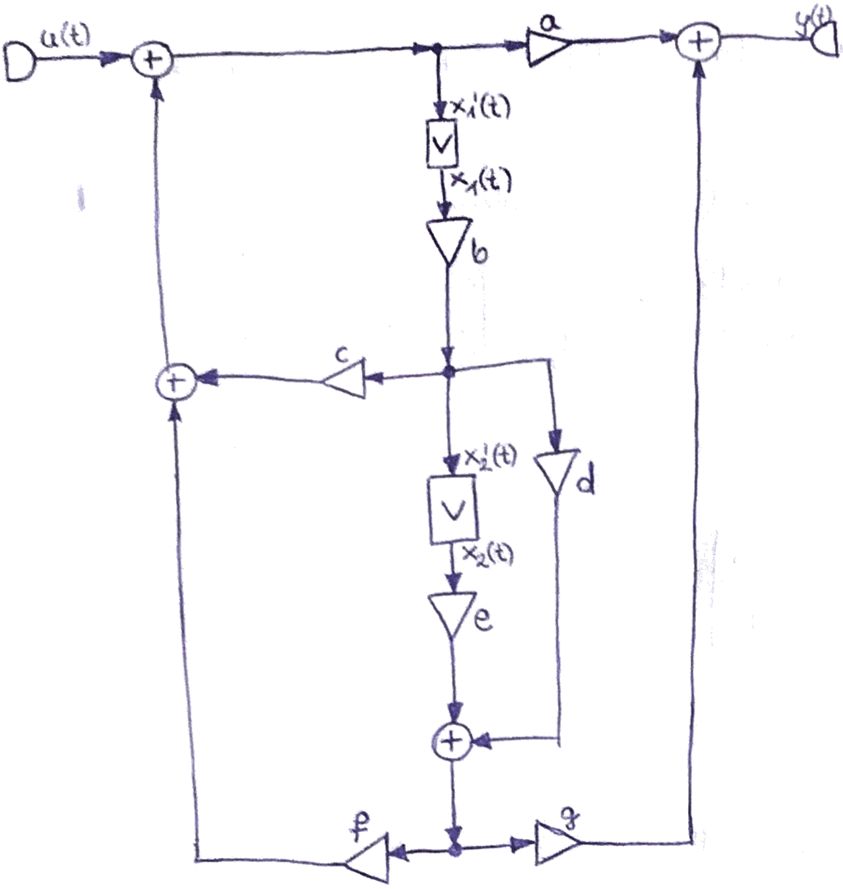
$D = a$ [redacted] $=$ [redacted] $0{,}2$

$$0 = \det(\underline{\underline{A}} - \lambda \underline{\underline{E}}) = \begin{vmatrix} -0{,}252-\lambda & 0{,}27 \\ 0{,}36 & 0{,}9-\lambda \end{vmatrix} = (-0{,}252-\lambda)(0{,}9-\lambda) - 0{,}36 \cdot 0{,}27 =$$

$$= -0{,}227 + 0{,}252\lambda - 0{,}9\lambda + \lambda^2 - 0{,}097 = \lambda^2 - 0{,}648\lambda - 0{,}324$$

$$\lambda_{1,2} = \frac{+0{,}648 \pm \sqrt{0{,}648^2 + 4 \cdot 0{,}324}}{2} \quad \begin{matrix} \nearrow \lambda_1 = \frac{0{,}648+1{,}16}{2} = 0{,}904 \\ \searrow \lambda_2 = \frac{0{,}648-1{,}16}{2} = -0{,}256 \end{matrix} \Bigg\} \rightarrow \text{ASZ-stabil}$$

(10.) ↻



$a = 0{,}1$
$b = 0{,}5$
$c = 0{,}9$
$d = -0{,}3$
$e = 0{,}7$
$f = 0{,}5$
$g = -0{,}2$

$\underline{\underline{A}}, \underline{B}, \underline{C}^T, D = ?$ $\underline{\underline{A}}$ karakterisztikus egyenlete? $\lambda_{1,2} = ?$ ASZ-stabil?

$x_1'(t) = u(t) + bcx_1(t) + (bdx_1(t) + ex_2(t))f = (bc + bdf)x_1(t) + efx_2(t) + u(t)$

$x_2'(t) = bx_1(t)$

[redacted]

$y(t) = ax_1'(t) + (bdx_1(t) + ex_2(t))g = a(bc+bdf)x_1(t) + aefx_2(t) + au(t)$

$$\underline{\underline{A}} = \begin{bmatrix} bc+bdf & ef \\ b & \emptyset \end{bmatrix} = \begin{bmatrix} 0{,}05+(-0{,}075) & 0{,}35 \\ 0{,}5 & \emptyset \end{bmatrix} = \begin{bmatrix} -0{,}025 & 0{,}35 \\ 0{,}5 & \emptyset \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 1 \\ \emptyset \end{bmatrix}$$

$$\underline{C}^T = [a(bc+bdf) \quad aef] = [-0{,}0025 \quad 0{,}035]$$

$D = a = 0{,}1$

$$0 = \det(\underline{\underline{A}} - \lambda \underline{\underline{E}}) = \begin{vmatrix} -0{,}025-\lambda & 0{,}35 \\ 0{,}5 & -\lambda \end{vmatrix} = (-0{,}025-\lambda)(-\lambda) - 0{,}35 \cdot 0{,}5 = 0{,}025\lambda + \lambda^2 - 0{,}175 =$$

$$= \lambda^2 + 0{,}025\lambda - 0{,}175$$

$$\lambda_{1,2} = \frac{-0{,}025 \pm \sqrt{0{,}025^2 + 4 \cdot 0{,}175}}{2}$$

$$\lambda_1 = \frac{-0{,}025 + 0{,}837}{2} = 0{,}406$$

$$\lambda_2 = \frac{-0{,}025 - 0{,}837}{2} = -0{,}431$$

$\Rightarrow$ nem ASZ-stabil

11.

$$\underline{x}[k+1]=\begin{bmatrix}-0{,}8415 & 0{,}8944\\ 0{,}0877 & 0{,}4415\end{bmatrix}\underline{x}[k]+\begin{bmatrix}1{,}2\\0{,}8\end{bmatrix}u[k]$$

$$y[k]=[0{,}2\ \ 1{,}5]\,\underline{x}[k]-4{,}4u[k]$$

$\lambda_{1,2}=?$ ASZ-stabil? $\underline{\underline{L}}_{1,2}=?$ $h[k]=?$

$$0=\det(\underline{\underline{A}}-\lambda\underline{\underline{E}})=\begin{vmatrix}-0{,}8415-\lambda & 0{,}8944\\ 0{,}0877 & 0{,}4415-\lambda\end{vmatrix}=(-0{,}8415-\lambda)(0{,}4415-\lambda)-0{,}8944\cdot 0{,}0877=$$

$$=-0{,}3715+0{,}8415\lambda-0{,}4415\lambda+\lambda^2-0{,}0784=\lambda^2+0{,}4\lambda-0{,}4499$$

$$\lambda_{1,2}=\frac{-0{,}4\pm\sqrt{0{,}4^2+4\cdot 0{,}4499}}{2}\nearrow\ \frac{-0{,}4+1{,}3998}{2}=0{,}4999$$

$$\searrow\ \frac{-0{,}4-1{,}3998}{2}=-0{,}8999$$

$\Rightarrow$ ASZ-stabil

$$\underline{\underline{L}}_1=\frac{\underline{\underline{A}}-\lambda_2\underline{\underline{E}}}{\lambda_1-\lambda_2}\qquad \underline{\underline{L}}_2=\frac{\underline{\underline{A}}-\lambda_1\underline{\underline{E}}}{\lambda_1-\lambda_2}=\underline{\underline{E}}-\underline{\underline{L}}_1$$

$$\frac{\begin{bmatrix}-0{,}8415-(-0{,}8999) & 0{,}8944\\ 0{,}0877 & 0{,}4415-(-0{,}8999)\end{bmatrix}}{1{,}3998}=\frac{\begin{bmatrix}0{,}0584 & 0{,}8944\\ 0{,}0877 & 1{,}3414\end{bmatrix}}{1{,}3998}=\begin{bmatrix}0{,}0417 & 0{,}6389\\ 0{,}0627 & 0{,}9583\end{bmatrix}=\underline{\underline{L}}_1$$

$$\Rightarrow \underline{\underline{L}}_2=\begin{bmatrix}0{,}9583 & -0{,}6389\\ -0{,}0627 & 0{,}0417\end{bmatrix}$$

$$h[k]=D\cdot\delta[k]+\varepsilon[k-1]\underbrace{(\underline{c}^T\cdot\underline{\underline{A}}^{k-1}\cdot\underline{B})}$$

$$\underline{c}^T\cdot(\lambda_1^k\cdot\underline{\underline{L}}_1+\lambda_2^k\underline{\underline{L}}_2)\cdot\underline{B}=[0{,}2\ \ 1{,}5]\left(\lambda_1^k\underline{\underline{L}}_1+\lambda_2^k\underline{\underline{L}}_2\right)\begin{bmatrix}1{,}2\\0{,}8\end{bmatrix}=\ldots$$

$$\Rightarrow h[k]=-4{,}4\delta[k]+\varepsilon[k-1]\left(0{,}6874\cdot(0{,}4999)^{k-1}-0{,}0584\cdot(-0{,}8999)^{k-1}\right)$$

(12.) $\underline{x}'(t)=\begin{bmatrix}-6,2415 & 3,0277\\ 0,2348 & -1,8585\end{bmatrix}\underline{x}(t)+\begin{bmatrix}-0,1\\ -2,0\end{bmatrix}u(t)$

$y(t)=[0,8\ \ -1,3]\underline{x}(t)-0,6u(t)$

$\lambda_{1,2}=?$ ASZ-stabil? $\underline{\underline{L}}_{1,2}=?$ $h(t)=?$

$0=\det(\underline{\underline{A}}-\lambda\underline{\underline{E}})=\begin{vmatrix}-6,2415-\lambda & 3,0277\\ 0,2348 & -1,8585-\lambda\end{vmatrix}=(-6,2415-\lambda)(-1,8585-\lambda)-3,0277\cdot 0,2348=$

$=11,5998+6,2415\lambda+1,8585\lambda+\lambda^2-0,7109=\lambda^2+8,1\lambda+10,8889$

$\lambda_{1,2}=\dfrac{-8,1\pm\sqrt{8,1^2-4\cdot 10,8889}}{2}$

$\lambda_1=\dfrac{-8,1+4,6962}{2}=-1,7019$

$\lambda_2=\dfrac{-8,1-4,6962}{2}=-6,3981$

$\Rightarrow$ ASZ-stabil

$\underline{\underline{L}}_1=\dfrac{\underline{\underline{A}}-\lambda_2\underline{\underline{E}}}{\lambda_1-\lambda_2}$ $\underline{\underline{L}}_2=\underline{\underline{E}}-\underline{\underline{L}}_1$

$\dfrac{\begin{bmatrix}-6,2415+6,3981 & 3,0277\\ 0,2348 & -1,8585+6,3981\end{bmatrix}}{4,6962}=\begin{bmatrix}0,0333 & 0,6447\\ 0,05 & 4,5396\end{bmatrix}=\underline{\underline{L}}_1$

$\underline{\underline{L}}_2=\begin{bmatrix}0,9667 & -0,6447\\ -0,05 & -3,5396\end{bmatrix}$

$h(t)=D\cdot\delta(t)+\varepsilon(t)\underbrace{(\underline{c}^T e^{\underline{\underline{A}}t}\underline{b})}$

$\rightarrow [0,8\ -1,3]\left(\underline{\underline{L}}_1\cdot e^{\lambda_1 t}+\underline{\underline{L}}_2 e^{\lambda_2 t}\right)\begin{bmatrix}-0,1\\ -2,0\end{bmatrix}=\ldots$

$\Rightarrow h(t)=-0,6\delta(t)+\varepsilon(t)\left(10,7753\cdot e^{-1,7019t}-8,2553\cdot e^{-6,3981t}\right)$