$\boxed{11}$

1, a, $\boxed{5}$

$$\left|\frac{2n^3-5n}{n^3+8}-2\right| \overset{1}{=} \left|\frac{\cancel{2n^3}-5n-2(\cancel{n^3}+8)}{n^3+8}\right| = \frac{5n+16}{n^3+8} \le \frac{5n+16n}{n^3} \;(2)\; =$$

$$= \frac{21}{n^2} < \varepsilon, \text{ ha } n > \sqrt{\frac{21}{\varepsilon}} \Longrightarrow \underline{\underline{N(\varepsilon) = \left[\sqrt{\frac{21}{\varepsilon}}\right]+1}} \;(2) \quad \forall \varepsilon > 0 \text{ esetén}$$

b, $\boxed{6}$

$$\lim_{n\to\infty} \sqrt{2n^4+n^2-2}-2n^2 = \lim_{n\to\infty} \frac{2n^4+n^2-2-4n^4}{\sqrt{2n^4+n^2-2}+2n^2} \;(2)\; =$$

$$= \lim_{n\to\infty} \frac{-2n^4+n^2-2}{\sqrt{2n^4+n^2-2}+2n^2} \;(1)\; = \lim_{n\to\infty} \frac{-2n^2+1-\overbrace{\frac{2}{n^2}}^{\to 0}}{\sqrt{2+\underbrace{\frac{1}{n^2}}_{\downarrow\, 0}-\underbrace{\frac{2}{n^4}}_{\downarrow\, 0}}+2} \;(2)\; = \underline{\underline{-\infty}} \;(1)$$

——— o ———

2, $\boxed{13}$

$$f(x) = \frac{x^3}{x-1}\;; \quad D_f = \mathbb{R}\setminus\{1\} \quad (x \ne 1)$$

$$f'(x) = \frac{3x^2(x-1)-x^3}{(x-1)^2} \;(2)\; = \frac{x^2(2x-3)}{(x-1)^2} = 0, \text{ ha } x=0 \lor x=\frac{3}{2}$$

x	x<0	0	0<x<1	1	1<x<3/2	3/2	3/2<x
f'	–	0	–	∄	–	0	+
f	↘		↘	∄	↘	lok. min	↗

(4)

$$f''(x) = \frac{(6x^2-6x)(x-1)^2-(2x^3-3x^2)2(x-1)}{(x-1)^4} \;(3)\; =$$

$$= \frac{6x^3-6x^2-\cancel{6x^2}+6x-4x^3+\cancel{6x^2}}{(x-1)^3} = \frac{2x^3-6x^2+6x}{(x-1)^3} =$$

$$= \frac{2x(x^2-3x+3)}{(x-1)^3}; \;(2)\; x^2-3x+3>0, \text{ mivel } b^2-4ac = 9-12<0$$

x	x<0	0	0<x<1	1	1<x
f''	+	0	–	∄	+
f	∪	infl.	∩	∄	∪

(2)

-2-

3, a, [9]

$$\int_1^9 \frac{1}{e^{\sqrt{x}}}\,dx = \int_{\sqrt{1}}^{\sqrt{9}} \underbrace{e^{-t}}_{u'} \underbrace{2t}_{v}\,dt \overset{\text{parc.}}{=} 2\left[-e^{-t}\cdot t\right]_1^3 - 2\int_1^3 -e^{-t}\,dt =$$ ③ ③

$t=\sqrt{x}$ $\quad u=-e^{-t} \quad v'=1$

$x=t^2;\ dx=2t\,dt$

$$=2\left(-3e^{-3}+e^{-1}\right)-2\left[e^{-t}\right]_1^3 = -6e^{-3}+2e^{-1}-2e^{-3}+2e^{-1}=$$

$$=4e^{-1}-8e^{-3}$$ ③

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b, [9]

$$\frac{1}{x^3+x}=\frac{A}{x}+\frac{Bx+C}{x^2+1} \Rightarrow 1=A(x^2+1)+(Bx+C)x$$ ③

$x=0 \Rightarrow A=1$

$1=x^2+1+Bx^2+Cx \Rightarrow B=-1,\ C=0$ ②

$$\int\frac{1}{x^3+x}\,dx=\int\frac{1}{x}\,dx+\int\frac{-x}{x^2+1}\,dx = \ln|x|-\frac{1}{2}\ln(x^2+1)+C$$ ② ②

f'/f alak

4, [12] (H): $y''-8y'+16y=0 \Rightarrow \lambda^2-8\lambda+16=(\lambda-4)^2 \Rightarrow \lambda_{1,2}=4$ ②

$y_{H,\acute{a}ll}(x)=C_1e^{4x}+C_2xe^{4x}$ ③ belső rezonancia

(I) $y_{I,p}(x)=Ax+B+C\cdot e^{-2x}$ ② $/\cdot 16$

$y'_{I,p}(x)=A+(-2)Ce^{-2x}$ $\quad /\cdot(-8)$

$\oplus\ y''_{I,p}(x)=4Ce^{-2x}$

$$32x+18e^{-2x}=\underbrace{16A}_{32}x+\underbrace{16B-8A}_{0}+e^{-2x}\overbrace{(16C+16C+4C)}^{36C=18}$$

$A=2;\ B=1;\ C=\frac{1}{2};\ y_{I,p}(x)=2x+1+\frac{1}{2}e^{-2x}$ ④

$$y_{I,\acute{a}ll}(x)=y_{H,\acute{a}ll}(x)+y_{I,p}(x)=C_1e^{4x}+C_2xe^{4x}+2x+1+\frac{1}{2}e^{-2x}$$ ①

5, a, $\sum_{n=0}^{\infty} a_n x_0^n$ konvergens $\Longrightarrow \lim_{n\to\infty} a_n x_0^n = 0 \Longrightarrow \{a_n x_0^n\}_{n\in\mathbb{N}}$ korlátos

$\boxed{4}$

b, Legyen a korlát K, azaz $|a_n x_0^n| < K$; $|x| < x_0$.

$\boxed{8}$ $$\sum_{n=0}^{\infty} |a_n x^n| \overset{(2)}{=} \sum_{n=0}^{\infty} \left|\frac{x}{x_0}\right|^n \cdot \underbrace{|a_n x_0^n|}_{<K} \overset{(3)}{<} K \sum_{n=0}^{\infty} \left|\frac{x}{x_0}\right|^n < \infty \ (3) \ ✓$$

$$0 \le q = \left|\frac{x}{x_0}\right| < 1$$

— ∘ —

6*, $f(x,y) = g\left(\frac{3y}{1+x^2}\right)$; a, $f'_x(x,y) = g'\left(\frac{3y}{1+x^2}\right)\cdot \frac{-3y\cdot 2x}{(1+x^2)^2}$ (3)

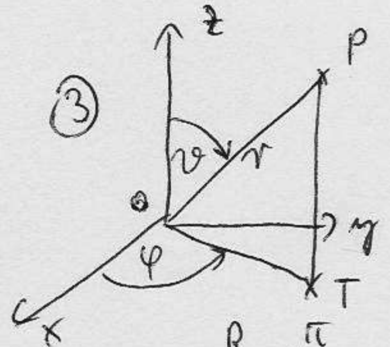
$\boxed{7}$

b, $$f''_{xy}(x,y) = g''\left(\frac{3y}{1+x^2}\right)\cdot\frac{3}{1+x^2}\cdot\frac{-6xy}{(1+x^2)^2} + g'\left(\frac{3y}{1+x^2}\right)\cdot\frac{-6x}{(1+x^2)^2} \quad (4)$$

— ∘ —

7*, a,

$\boxed{6}$ (3)



$$\left.\begin{aligned} x &= r\sin\vartheta\cos\varphi \\ y &= r\sin\vartheta\sin\varphi \\ z &= r\cos\vartheta \end{aligned}\right\} (3)$$

$\boxed{4}$ b, $$V(R) = \int_{r=0}^{R}\int_{\vartheta=0}^{\pi}\int_{\varphi=0}^{2\pi} 1\cdot \underbrace{r^2\sin\vartheta}_{\text{Jac}}\, d\varphi\, d\vartheta\, dr \overset{(2)}{=}$$

$$= \underbrace{\left(\int_{r=0}^{R} r^2\,dr\right)}_{\frac{R^3}{3}}\cdot\underbrace{\left(\int_{\vartheta=0}^{\pi}\sin\vartheta\,d\vartheta\right)}_{[-\cos\vartheta]_0^{\pi}=2}\cdot\underbrace{\left(\int_{\varphi=0}^{2\pi} d\varphi\right)}_{2\pi} = \frac{4\pi}{3}R^3 \quad (2)$$

— ∘ —

8*, a, $\mathcal{F}[f(x-a)](\omega) = e^{-i\omega a}F(\omega)$ (2), mert $\mathcal{F}[f(x-a)](\omega) = \int_{-\infty}^{\infty} e^{-i\omega x} f(\underbrace{x-a}_{y})\,dx$

$\boxed{8}$ $$= \int_{-\infty}^{\infty} e^{-i\omega(y+a)} f(y)\,dy = e^{-i\omega a}F(\omega) \quad (6)$$

$\boxed{3}$ b, $\mathcal{F}\left[f\left(\frac{x}{b}\right)\right](\omega) = |b|\cdot F(b\omega)$ (3)

$\boxed{6}$ c, $\mathcal{F}\left[f\left(\frac{x-3}{2}\right)\right](\omega) = e^{-3i\omega}\mathcal{F}\left[f\left(\frac{x}{2}\right)\right](\omega) \overset{(3)}{=} 2e^{-3i\omega}F(2\omega)$ (3)