IX.28. p.
3.h

PID szabályzók

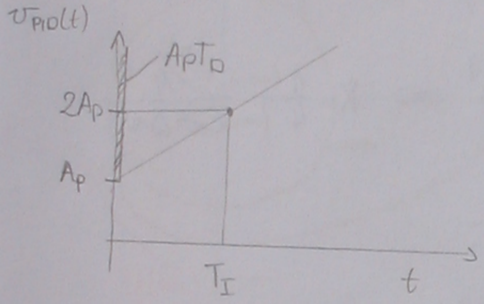
Ideális PID típusú szabályzó:

$$W_{PID}(s) = A_p\left(1+\underbrace{\frac{1}{sT_I}}_{\text{integrátor}} + \underbrace{sT_D}_{\text{diff.}}\right) = A_p + \frac{A_p}{T_I}\frac{1}{s} + A_p T_D s =$$

$$W_{PID}(s) = k_p + k_I\frac{1}{s} + k_D\cdot s$$

$$V_{PID}(s) = W_{PID}(s)\cdot\frac{1}{s} = \frac{A_p}{s} + \frac{A_p}{T_I}\cdot\frac{1}{s^2} + A_pT_D$$

$$\hookrightarrow v_{PID}(t) = A_p + \frac{A_p}{T_I}t + A_pT_D\,\delta(t) \quad ; \quad t \geq 0$$



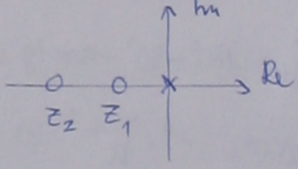
Szabályzó fajták:
- P
- PI
- PD
- PID
- ~~ID~~

Átviteli fv. p-z eloszlása:

A gyök alatt a gyakorlatban valós van!

$$W_{PID}(s) = \frac{A_p}{T_I}\frac{1+sT_I+s^2T_IT_D}{s} \quad ; \quad z_{12} = \frac{-T_I \pm \sqrt{T_I^2-4T_IT_D}}{2T_IT_D}$$

$$T_I^{\cancel{2}} - 4\cancel{T_I}T_D \geq 0 \Rightarrow \underline{T_D \leq \frac{T_I}{4}}$$

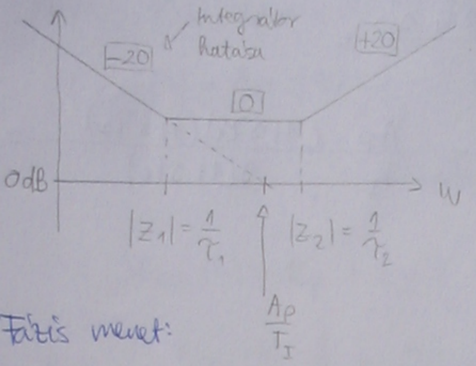


Így mindig negatívok a zérusok!

$$W_{PID}(s) = \frac{A_p}{T_I}\frac{(1+s\tau_1)(1+s\tau_2)}{s}$$

IX.28. P.
3.h.

Bode-diagramja:

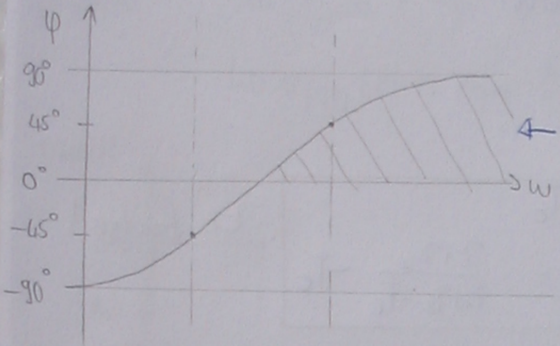


Aplitudó menet

Probléma:

- felerősíti a nagy frekvenciás zajokat
- Induláskor ∞ nagy jelet akar adni

Fázis menet:



← fázis többlet jó irányú javítását okozza majd

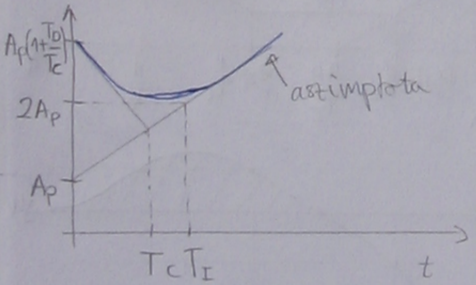
Gyakorlatban: <u>Közelítő PID szabályzó</u> (nem ideális)

$$W_{PID}(s) = A_P\left(1+\frac{1}{sT_I}+\frac{sT_D}{1+s\boxed{T_C}}\right) = A_P + \frac{A_P}{T_I}\cdot\frac{1}{s} + \frac{A_P T_D}{T_C}\,\frac{s}{s+\frac{1}{T_C}}$$

$$V_{PID}(s) = W_{PID}(s)\cdot\frac{1}{s} = \frac{A_P}{s} + \frac{A_P}{T_I}\,\frac{1}{s^2} + \frac{A_P T_D}{T_C}\,\frac{1}{s+\frac{1}{T_C}}$$

$$v_{PID}(t) = A_P + \frac{A_P}{T_I}\cdot t + \frac{A_P T_D}{T_C}\,e^{-\frac{t}{T_C}} \quad ; \quad t \geq 0$$

<u>Átmeneti fv.:</u>



Viselkedés frekvencia tartományban:

$$W_{PID}(s)=\frac{A_P}{T_I}\,\frac{sT_I(1+sT_C)+1+sT_C+sT_I\,sT_D}{s(1+sT_C)}=$$

$$=\frac{A_P}{T_I}\,\frac{1+s(T_I+T_C)+s^2T_I(T_D+T_C)}{s(1+sT_C)}=\frac{A_P}{T_I}\,\frac{(1+s\tau_1)(1+s\tau_2)}{s(1+sT_C)}=$$

$$=\frac{A_P}{T_I}\,\frac{1+s(\tau_1+\tau_2)+s^2\tau_1\tau_2}{s(1+sT_C)}$$

A két tört számlálója egyenlő. Ezért:

$$\tau_1+\tau_2=T_I+T_C$$

$$\underline{\tau_1\cdot\tau_2=T_I(T_D+T_C)}$$

$$\tau_1,\tau_2,T_C\longrightarrow\boxed{\begin{array}{l}T_I=\tau_1+\tau_2-T_C\\ T_D=\dfrac{\tau_1\tau_2}{T_I}-T_C=\dfrac{\tau_1\tau_2}{\tau_1+\tau_2-T_C}-T_C\end{array}}$$

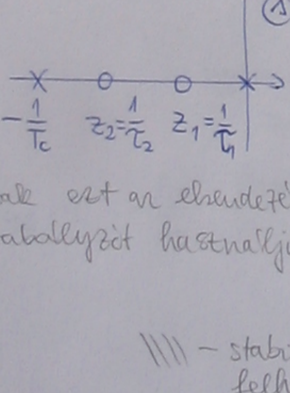
Ez fontos!

Bode:

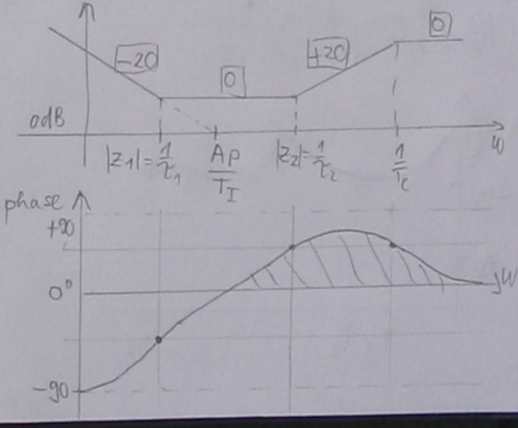
$$z_{12}=\frac{-(T_I+T_C)\pm\sqrt{(T_I+T_C)^2-4T_I(T_D+T_C)}}{2T_I(T_D+T_C)}$$

$$(T_I+T_C)^2-4T_I(T_D+T_C)=T_I^2+2\cancel{T_I}T_C+T_C^2-4T_IT_D-\overset{2}{\cancel{4}}T_IT_C=$$

$$=(T_I-T_C)^2-4T_IT_D\Rightarrow T_D\le\frac{(T_I-T_C)^2}{4T_I}\Rightarrow\text{2 valós zérushely van}$$



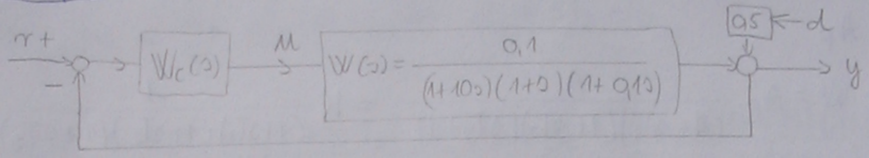
Csak ezt az elrendezésű szabályzót használjuk!



|||| – stabilizálásra felhasználható!

Minta pl.:

példa miatt a nevezőben a 10-es arány! IX.28. p s.h.!



$d(t) = d_0 1(t)$, $|d_0| \leq 10$

$y(\infty) = \frac{0,5 \cdot 10}{1+K} \leq 0,05$ (spec.) $\Rightarrow \frac{5}{1+K} \leq 0.05$

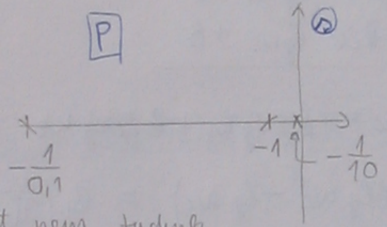
$1+K \geq 100 \Rightarrow \underline{K \geq 99}$

Döntés: P, PD esetén : $\underline{K = 100}$

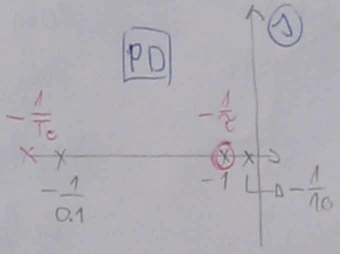
PI, PID —"— : K tetszőleges

Stabilitási tartalék legyen : $\varphi_t \geq 45°$

Időállandók: 0,1 ; 1 ; 10

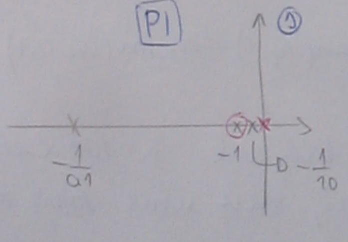


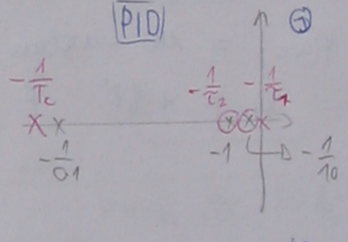
Itt nem tudunk változtatni, csak Ap állítható!



Ezt ajánlatos kiejteni.

2. legnagyobb időállandót álszerű kiejteni!





2 domináns pólust ejtjük ki!

↑ EZ a kompenzálás fő elve! ↑

P szabályozás

$$W_c(s) = A_p$$

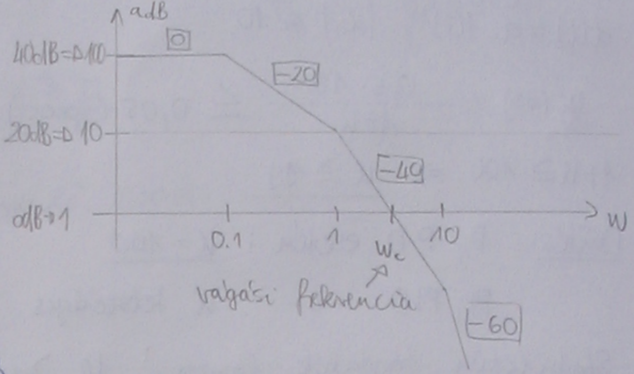
$$W_0 = W_c\, W = A_p \frac{A}{(1+sT_1)(1+sT_2)(sT_3+1)} = K \frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$$

$K = A_p A$;

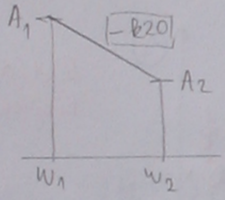
$$s_1 = -\frac{1}{T_1} = -\frac{1}{10} = -0.1$$

$$s_2 = -\frac{1}{T_2} = -\frac{1}{1} = -1$$

$$s_3 = -\frac{1}{T_3} = -\frac{1}{0.1} = -10$$



$$20 \lg |W| = \boxed{-k20} \lg w + B$$



$$\frac{A_1}{A_2} = \left(\frac{w_2}{w_1}\right)^k$$

$$20 \lg A_1 = -k20 \lg w_1 + B$$

$$20 \lg A_2 = -k20 \lg w_2 + B$$

$$20 \lg A_1 - 20 \lg A_2 = -k20 \lg w_1 + k\,20 \lg w_2$$

$$\lg \frac{A_1}{A_2} = k\,\underbrace{(\lg w_2 - \lg w_1)}_{\lg \frac{w_2}{w_1}} = \lg\left(\frac{w_2}{w_1}\right)^k$$

$$\frac{10}{1} = \left(\frac{w_c}{1}\right)^2 \Rightarrow w_c = \sqrt{10} = 3{,}16$$

$$\varphi_t = 180^\circ + \varphi(w_c) = 180^\circ - \arctan(w_c\, 10) - \arctan(w_c\, 1) - \arctan(w_c\, 0.1)$$

$$\varphi_t = +1{,}8^\circ \ngeq 45^\circ$$

$\Rightarrow$ P szabályzónk K=100 esetén nem megfelelő, mert nincs fázis tart.

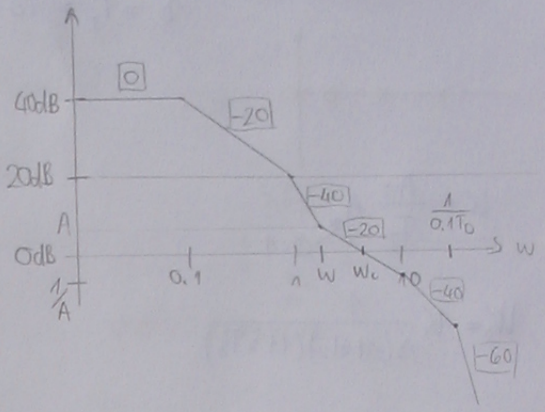
IX. 28 p
3.h

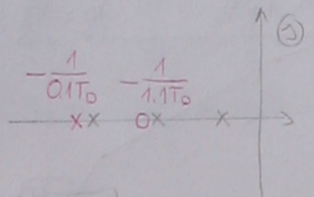
Kompenzálás közelítő PD szabályzóval

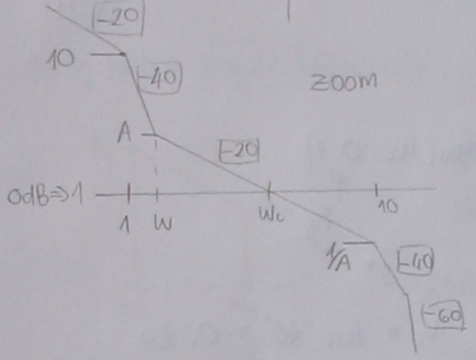
$$W_{PD} = A_p \cancel{\left(1+s\frac{T_D}{T_c}\right)} \quad \left(1+\frac{sT_D}{1+sT_c}\right) = \frac{A_p(1+s(T_D+T_c))}{1+sT_c} = A_p\frac{1+1.1T_Ds}{1+s0.1T_D}$$

Ökölszabály:

Legyen : $T_c = 0.1T_0$







$$\frac{10}{A} + \left(\frac{W}{1}\right)^2 \quad \triangleleft$$

$$\frac{A}{1/A} = \left(\frac{10}{W}\right)^1 \Rightarrow W = \frac{10}{A^2}$$

$$\frac{10}{A} = \left(\frac{10}{A^2}\right)^2 \Rightarrow A^3 = 10 \Rightarrow A = \sqrt[3]{10} =$$
$$A = 2.15$$

$$\frac{A}{1} = \frac{W_c}{W} \Rightarrow W_c = AW = 4.64,$$

$$W = \frac{10}{A^2} = 2.15$$

$$1.1T_D = \frac{1}{W} \Rightarrow T_D = 0.42$$

$$T_c = 0.1T_D = 0.042$$

$$\varphi_t = 180° - \arctan(\underset{\overset{\uparrow}{4.64}}{W_c}\,10) - \arctan(W_c\cdot 1) + \arctan(W_c 1{,}1T_D)$$

$$-\arctan(W_c\cdot 0{,}1) - \arctan(W_c\,\underbrace{0.1T_D}_{T_c}) = \underline{42{,}4° = \varphi_t \approx 45}$$

Ez már jó!

PI kompenzálás

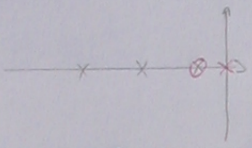
$$W_{PI} = \frac{A_P}{T_I}\,\frac{1+sT_I}{s} = \frac{A_P}{T_I}\,\frac{1+s\tau}{s}$$

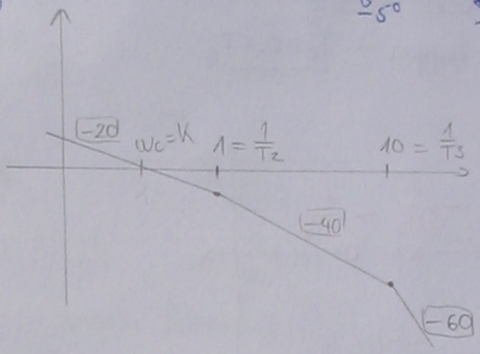
$-\arctan(\omega T)$

$0.1 \rightarrow -5^\circ$

$10 \rightarrow -85^\circ$

$$\tau = T_I = 10$$





$$K = \frac{A_P}{T_I}\,A \leftarrow 0.1$$

$$W_0 = K\,\frac{1}{s(1+sT_2)(1+sT_3)}$$

$$\varphi_t = 180^\circ + \varphi(w_c) =$$

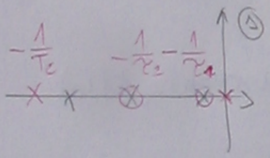
$$45^\circ = 180^\circ - 90 - \arctan(\underset{\underset{T_2}{\uparrow}}{w_c \cdot 1}) \underbrace{- \arctan(w_c\ \underset{\underset{T_3}{\uparrow}}{0.1})}_{-5^\circ}$$

$$\underbrace{90^\circ - 45^\circ - 5^\circ}_{40^\circ} = \arctan(w_c \cdot 1) \Rightarrow w_c = \tan 40^\circ = 0.84$$

$$w_c = K = \frac{A_P}{T_I}\cdot A = \frac{A_P}{10}\cdot 0.1 \Rightarrow \boxed{A_P = 84}$$

$$\boxed{T_I = 10\,\text{sec}}$$

PID szabályozás



$$T_c = 0.1\,T_D$$

$$W_{PID} = \frac{A_P}{T_I}\,\frac{1+s(T_I+\overbrace{0.1T_D}^{T_c}) + s^2 T_I \overbrace{1.1T_D}^{T_D+T_c}}{s(1+s\underbrace{0.1T_D}_{T_c})}$$

$$W_0 = \frac{A_P}{T_I}\,\frac{\cancel{(1+s\tau_1)}\cancel{(1+s\tau_2)}}{s(1+s\,0.1T_D)}\cdot\frac{A}{\cancel{(1+sT_1)}\cancel{(1+sT_2)}(1+sT_3)}$$

$$\tau_1 = T_1,\ \tau_2 = T_2 \Rightarrow (1+s\tau_1)(1+s\tau_2) = 1+s(\underbrace{\tau_1+\tau_2}_{10+1}) + s^2\underbrace{\tau_1\tau_2}_{10\cdot 1} = \rightarrow$$

$$= 1 + 11s + 10s^2$$

IX.28. p.
3.h.

$$T_I + 0.1\,T_D = 11$$

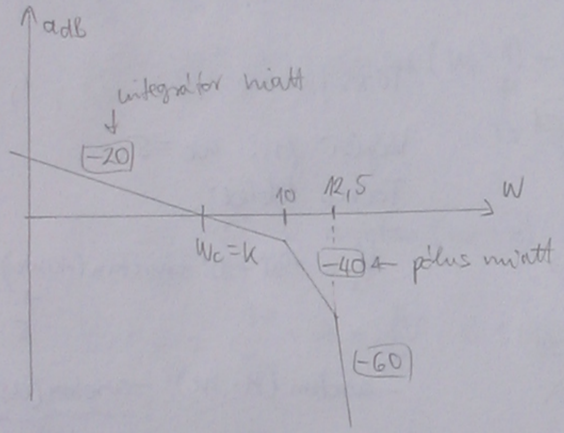
$$T_I\,1{,}1\,T_D = 10 \rightarrow T_I = \frac{10}{1.1T_D}$$

$$\frac{10}{1.1T_D} + 0.1\,T_D = 11 \Rightarrow 10 + \underbrace{0.1\cdot 1{,}1}_{0.11}T_D^2 = \underbrace{11\cdot 1{,}1}_{12.1}T_D$$

$$T_D^2 - 110\,T_D + 90.9 = 0$$

$$T_D \begin{cases} \rightarrow \cancel{109.16} \\ \rightarrow 0.83 \rightarrow T_C = 0.1\,T_D = 0.083 \Rightarrow T_I = \dfrac{10}{1.1\cdot 0.83} = 10{,}92 \end{cases}$$

$$W_0 = \frac{K}{s(1+\underset{\overset{\uparrow}{T_3}}{s0.1})(1+\underset{\overset{\uparrow}{T_c}}{s\,0.083})}$$



$$45^\circ = 180^\circ - 90^\circ - \arctan(w_c\,0{,}1) - \arctan(w_c\cdot 0{,}083)$$
$$45^\circ = \arctan(w_c\,0{,}1) + \arctan(w_c\cdot 0{,}083)$$

$$\tan(\alpha+\beta) = \frac{\tan(\alpha)+\tan(\beta)}{1-\tan(\alpha)\tan(\beta)} \quad ; \quad \tan(\arctan(x)) = x\,;$$

$$1 = \frac{W_c 0.1 + W_c 0.083}{1 - W_c 0.1 \cdot W_c 0.083} \Rightarrow W_c^2 + 22\,w_c - 120{,}5 = 0 \Rightarrow w_c = 4{,}55$$

$$K = \frac{A_P}{T_I}A = \frac{A_P}{10{,}92}0{,}1 = w_c = 4{,}55 \Rightarrow$$

$$\boxed{\begin{aligned} A_P &= 497 \\ T_I &= 10{,}92\ \text{sec} \\ T_D &= 0{,}83\ \text{sec} \\ T_C &= 0{,}083\ \text{sec} \end{aligned}}$$

Kiegészítés (Extended version)

Segítség a 2. gyakorlat ellenőrző kérdéseihez

②

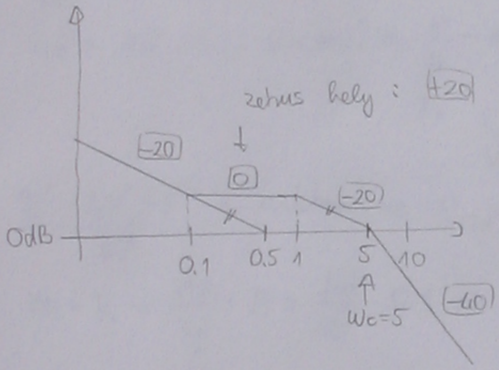
$$W_0 = \frac{25(s+0.1)}{s(s+1)(s+5)}$$ Ez gyöktényezős, ezért át kell írni.

$$W_0 = \underbrace{\frac{25 \cdot 0{,}1}{1 \cdot 5}}_{0{,}5} \cdot \frac{1+10s}{s(s+1)(0{,}2s+1)} = 0{,}5\,\frac{1+10s}{s(1+s)(1+0{,}2s)}$$

$$= \frac{K}{s} W_{01}(s) \quad , \quad W_{01}(1)$$

$K = 0.5$; típus szám $= 1$ (integrátorok száma)

↑ körerősítés



Törés frekvenciák: 0,1 ; 1; 5

Vágási fr.: $w_c = 5$

Fázis többlet!

$$\varphi_t = 180^\circ - 90^\circ + \arctan(10\overset{\uparrow}{\underset{5}{w_c}})$$

$$-\arctan(1 \cdot w_c) \underbrace{-\arctan(0.2 w_c)}_{-45^\circ}$$

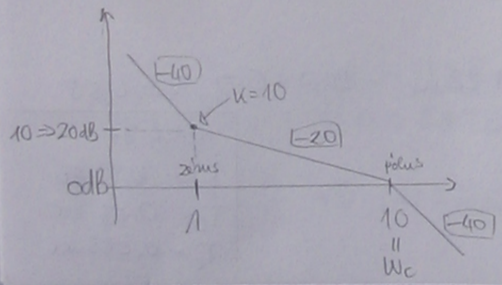
Itt a trükk: $(1+Ts)(..)$ alakra kellett hozni!

④

$$W_0 = \frac{10(1+s)}{s^2(1+0{,}1s)} = \frac{K}{s^2} W_{01}(s) \; , \; W_{01}(0) = 1$$

$K = 10$, típus szám $= 2$ $\left(\frac{1}{s^n} \text{ hatványa}\right)$

$$|W_0| \sim \frac{K}{w^2} \Rightarrow w=1\text{-nél}: \; |W_0| = K$$



$w_c = 10$

$$\varphi_t = \cancel{180^\circ - 90^\circ} - 90^\circ + \arctan(\overset{\uparrow}{\underset{10}{w_c}}\,1) - \arctan(w_c\, 0{,}1) \cong$$

$$\cong 85^\circ - 45^\circ = \underline{40^\circ}$$