gyakorlás 1. kisZH-ra

Orvosbiológiai méréstechnika

2015. ~~09.~~ 10. 23.

Képletek:

$$\Delta R = \frac{\partial R}{\partial \rho}\cdot\Delta\rho + \frac{\partial R}{\partial l}\cdot\Delta l + \frac{\partial R}{\partial A}\cdot\Delta A \Rightarrow \frac{\Delta R}{R} \triangleq \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A}$$

$$G = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta l}{l}\right)} = 2 \text{ (Gauge-faktor)}$$

$$U_R^2 = 4kTR\cdot\Delta f \text{ (ellenállás zaja)}$$

$$U_S = U_1 - U_2 \qquad E_{KU} = \frac{A_{UK}}{A_{US}}$$

$$U_K = \frac{U_1 + U_2}{2}$$

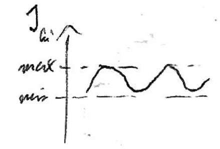
~~U_{K}~~ A_{UK} mérése: $U_{be_1} := U_{be_2} \Rightarrow \frac{U_{ki}}{U_{be}}$?

A_{US} mérése: szuperpozícióval könnyű

$$SpO_2 = \frac{C_{HbO_2}}{C_{HbO_2} + C_{Hb}};$$

$$I_{ki\,max} = I_0\, e^{-(\Delta d + d)\cdot(a_{HbO_2}\cdot C_{HbO_2} + a_{Hb_1}\cdot C_{Hb_1})}$$

$$I_{ki\,min} = I_0\, e^{-(\Delta d \cancel{+} d)\cdot(a_{HbO_2}\cdot C_{HbO_2} + a_{Hb}\cdot C_{Hb})}$$



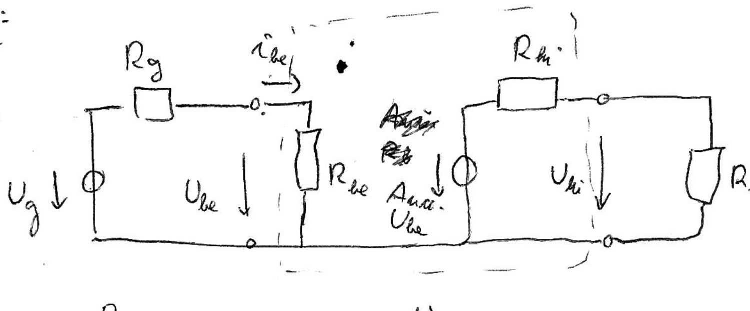
PULZOXITERM

$$\rightarrow \ln\left(\frac{I_{ki\,max}}{I_{ki\,min}}\right)\Bigg|_{\lambda_1}, \quad \ln\left(\frac{I_{ki\,max}}{I_{ki\,min}}\right)\Bigg|_{\lambda_2}, \quad R = \frac{\ln\left(\frac{I_{ki\,max}}{I_{ki\,min}}\right)\Big|_{\lambda_1}}{\ln\left(\frac{I_{ki\,max}}{I_{ki\,min}}\right)\Big|_{\lambda_2}}$$

$$R = \frac{a_{HbO_{2_1}}\cdot C_{HbO_{2_1}} + a_{Hb_1}\cdot C_{Hb_1}}{a_{HbO_{2_2}}\cdot C_{HbO_{2_2}} + a_{Hb_2}\cdot C_{Hb_2}}$$

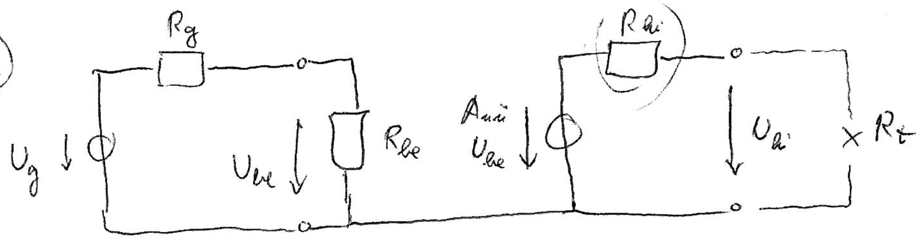
Feladatok:

1. Erősítő:



$$U_{be} = U_g\cdot\frac{R_{be}}{R_g + R_{be}}; \quad R_{be} = \frac{U_{be}}{i_{be}}; \quad U_{ki} = A_{uü}\cdot U_{be}\cdot\frac{R_t}{R_t + R_{ki}}; \quad R_{ki} = -\frac{U_{ki\,üresjárási}}{I_{ki\,zárlati}}\ !$$

(2.)



a.) $R_t \to \infty$ (nehadás) $\to U_{ki_1} = 1V$

$R_t = 10\,k\Omega \longrightarrow U_{ki_2} = 0{,}5V$

$$U_{ki_1} = A_{uü} \cdot U_{be} \cdot \frac{R_t}{R_{ki}+R_t} = A_{uü} U_{be} \cdot \underbrace{\left(\frac{\infty}{\infty}\right)}_{1} = 1V$$

$$U_{ki_2} = A_{uü} \cdot U_{be} \cdot \frac{R_t}{R_{ki}+R_t} = A_{uü} U_{be} \cdot \frac{10k\Omega}{R_{ki}+10k\Omega} = 0{,}5V$$

$\Rightarrow R_{ki} = ?$

$$\frac{U_{ki_2}}{U_{ki_1}} = \frac{10k\Omega}{R_{ki}+10k\Omega} \to U_{ki_2} = U_{ki_1} \cdot \frac{10k\Omega}{R_{ki}+10k\Omega} = 1V \cdot \frac{10k\Omega}{R_{ki}+10k\Omega}$$

$$\cancel{R_{ki} = \frac{U_{ki\,üresjárási}}{I_{ki\,rövidzár}} = \frac{1V}{\left(\frac{A_{uü}U_{be}}{R_{ki}}\right)}}$$

$R_{ki} = 10\,k\Omega$ ✓

b.) $R_g = 0 \to U_{ki} = 2V$

$R_g = 100\,k\Omega \to U_{ki} = 1V$

$R_{be} = ?$

$\Rightarrow R_g(U_{ki}, R_{be})$?

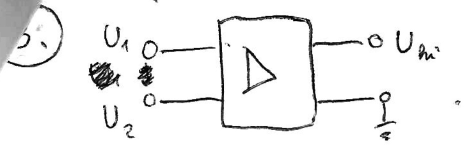
$$U_{be} = U_g \cdot \frac{R_{be}}{R_g+R_{be}}$$

$$U_{ki} = A_{uü} U_{be} \cdot \frac{R_t}{R_{be}+R_t} = A_{uü} \cdot U_g \cdot \frac{R_{be}}{R_g+R_{be}} \cdot \frac{R_t}{R_{be}+R_t} = konst. \cdot \frac{R_{be}}{R_{be}+R_g}$$

$$\frac{U_{ki_1}}{U_{ki_2}} = 2 = \frac{\frac{\cancel{R_{be}}}{R_{be}+R_{g_1}}}{\frac{\cancel{R_{be}}}{R_{be}+R_{g_2}}} = \frac{R_{be}+R_{g_2}}{R_{be}+R_{g_1}} \Rightarrow 2(R_{be}+R_{g_1}) = R_{be}+R_{g_2}$$

$$R_{be} = R_{g_2} - 2R_{g_1} = 100\,k\Omega$$ ✓

3.



a.) $U_1 = 9{,}95\,V$, $U_2 = 10{,}05\,V$, $U_{ki} = -0{,}9\,V$

b.) $U_1 = 5{,}1\,V$, $U_2 = 5{,}0\,V$, $U_{ki} = 1{,}0505\,V$

$\Big\}$ A_{us} ? A_{uk} ? E_{ku} ?

~~a.) $A_{U_S} = U_1 - U_2 = -0{,}1\,V$~~

~~$A_{U_K} = \frac{U_1 + U_2}{2} = 10\,V$~~

$U_S = U_1 - U_2 = -0{,}1\,V$ | $U_S = 0{,}1\,V$

$U_K = \frac{U_1+U_2}{2} = 10\,V$ | $U_K = \frac{10{,}05\,V}{2} = 5{,}025$

~~$E_{KU} = \frac{A_{U_K}}{A_{U_S}} = -100 \to -40\,dB$~~

~~$E_{KU} = \frac{A_{U_S}}{A_{U_K}} = -0{,}01\,V$~~

~~$U_{ki} = A_{U_S} \cdot U_S + A_{U_K} \cdot U_K$~~

~~$-0{,}9\,V = -0{,}1\,V \cdot A_{U_S} + 10\,V \cdot A_{U_K}$~~

~~b.) $U_{ki} = 0{,}1\,V \cdot A_{U_S} + 10{,}05\,V \cdot A_{U_K}$~~

~~$+ \qquad = 1{,}0505\,V$~~

~~a.) + b.) : $0{,}1505 = 20{,}05\,A_{U_K} \Rightarrow A_{U_K} = \frac{20{,}05}{0{,}1505} \approx 133{,}22$~~

~~$\to A_{U_K} = 0{,}0075$~~

$$U_{ki} = A_{U_S} \cdot U_S + A_{U_K} \cdot U_K$$

$$-0{,}9 = A_{U_S} \cdot (-0{,}1) + A_{U_K} \cdot 10 \quad \cancel{\Rightarrow A_{U_S} =}$$

$$+ \quad 1{,}0505 = A_{U_S} \cdot 0{,}1 + A_{U_K} \cdot \overset{5{,}025}{\cancel{10{,}05}} \longrightarrow A_{U_S} = \frac{1{,}0505 - A_{U_K} \cdot \overset{5{,}025}{\cancel{10{,}05}}}{0{,}1} = 10{,}505 - \overset{50{,}25 A_{U_K}}{\cancel{100{,}5 A_{U_K}}}$$

$$0{,}1505 = A_{U_K} \cdot \underset{15{,}025}{\cancel{20{,}05}} \longrightarrow \boxed{\begin{array}{l} \cancel{A_{U_K} = 0{,}0075} \\ \cancel{A_{U_S} = 9{,}7506} \end{array}}$$

$A_{U_K} \cong 0{,}01$ ✓

$A_{U_S} \cong 10{,}002$ ✓

$E_{KU} = \frac{A_{U_S}}{A_{U_K}} \cong 10^3 = \overset{60\,dB}{\cancel{80\,dB}}$

4.

van köztük kapcsolat!

$$R = \rho \cdot \frac{l}{A} \to \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta l}{l} - \frac{\Delta A}{A}$$

$$V = l \cdot A \to \frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta A}{A}$$

$$\Bigg\} \quad \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + 2 \cdot \frac{\Delta l}{l} \Rightarrow \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta l}{l}\right)} = 2 \quad \text{(Gauge-faktor)}$$

5. Milyen ~~erősítő kell~~ szűrő kell?

- tiszta jelre → Mindent áteresztő
- kisfrekvenciás zajra → Felüláteresztő (különben alapjelvándorlás)
- nagyfrekvenciás zajra → Aluláteresztő (különben nullponteltolódás)

$A(s) = \frac{R_2}{R_1 + R_2}$ feszültségosztó

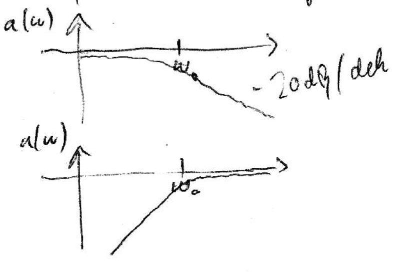
$\omega = 2\pi f$

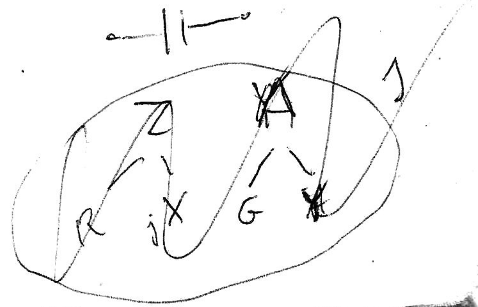
$Z_C = \frac{1}{j\omega C}$

FA: $A(s) = \frac{sRC}{1 + sRC}$

AA: $A(s) = \frac{1}{1 + sRC}$

$\Big\}$ BODE amplitúdó diagramm?





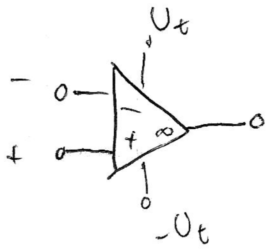
6. Műveleti erősítő:

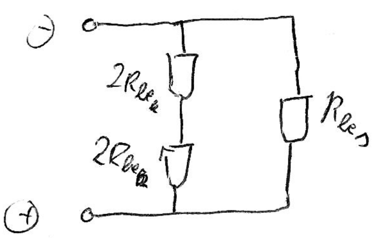
$$\left.\begin{array}{l} A_{U_S} \to \infty \\ A_{U_K} = 0 \end{array}\right\} E_{ku} = \frac{A_{U_S}}{A_{U_K}} \to \infty$$

$R_{be_S} \Rightarrow \infty$ $\quad$ ~~$i_{be} = 0$~~

$R_{be_K} \to \infty$ $\quad$ $i_{be}^+ = i_{be}^- = 0$

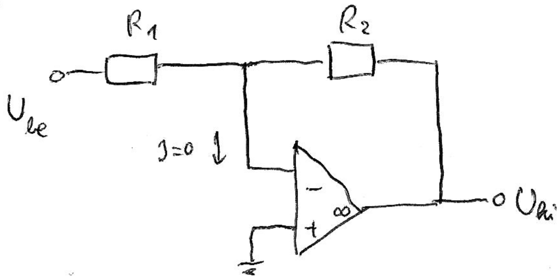
$R_{ki} \to 0$ $\quad$ ~~$U_{ki}^+ = 0$~~, $U_{be}^+ = U_{be}^-$





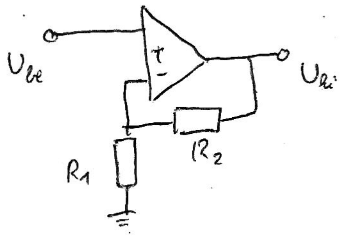
7. Erősítő alapkapcsolások:

- invertáló alapkapcsolás:



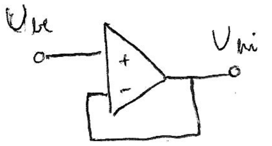
$$\frac{U_{ki}}{U_{be}} = -\frac{R_2}{R_1}$$

- nem invertáló alapkapcsolás:



$$\frac{U_{ki}}{U_{be}} = 1 + \frac{R_2}{R_1}$$

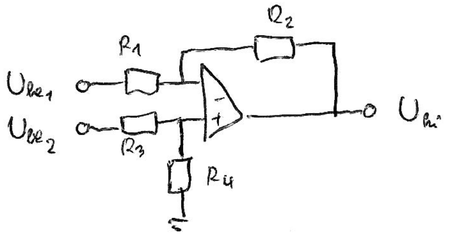
- követő erősítő:



nem terheli az áramkört

$R_{be} \to \infty$

- mérőerősítő:



SZUPERPOZÍCIÓVAL

$$U_{ki} = -\frac{R_2}{R_1} \cdot U_{be_1} + \frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} \cdot U_{be_2}$$

$\to R_1 := R_3, \; R_2 := R_4$

$$U_{ki} = -\frac{R_2}{R_1}\left(U_{be_1} - U_{be_2}\right)$$

Feladat: $\left.\begin{array}{l} R_2 = 10 R_1 \\ h_R = 0{,}01 \end{array}\right\} E_{ku}$?

$$A_{U_S} = \frac{U_{ki}}{U_{be_1} - U_{be_2}} = -\frac{R_2}{R_1}$$

U_S számolása: 

$$A_{U_K} = \frac{U_{ki}}{U_{be_K}} \; ; \quad U_{be_K} = \frac{U_1 + U_2}{2} \; ;$$

U_{be_K} számolása: $U_{be_1} = U_{be_2}$!!!

$$U_{ki} = -\frac{R_2}{R_1} \cdot U_{be_K} + \frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} \cdot U_{be_K}$$

$$U_{ki} = U_{be_K} \cdot \left(\frac{R_4}{R_3+R_4} \cdot \frac{R_1+R_2}{R_1} - \frac{R_2}{R_1} \right)$$

$$U_{ki} = U_{be_K} \cdot \frac{R_4R_1 + R_4R_2 - R_2R_3 - \cancel{R_2R_4}}{R_1 \cdot (R_3+R_4)} = U_{be_K} \cdot \frac{R_1R_4 - R_2R_3}{R_1(R_3+R_4)}$$

$$\frac{U_{ki}}{U_{be,n}} = \frac{R_1R_4 - R_2R_3}{R_1(R_3+R_4)}$$

$h_R = 0{,}01$

mikor lesz a legnagyobb hiba?

$$\left.\frac{d\,U_{ki}/U_{be}}{d\,R_1}\right|_{\substack{R_1=R_3\\R_2=R_4}} = \frac{R_1R_2}{(R_1+R_2)R_1^2} = \frac{R_2}{R_1(R_1+R_2)}$$

$$\left.\frac{d\,U_{ki}/U_{be}}{d\,R_3}\right|_{\substack{R_1=R_3\\R_2=R_4}} = \cancel{\frac{R_2}{(R_1+R_2)^2}} - \frac{R_2}{R_1(R_1+R_2)} =$$

$$\frac{-\cancel{R_1R_2} - R_2(R_1+R_2)}{R_1(R_1+R_2)^2}$$

$$= \frac{\cancel{+R_1R_2 - R_1R_2 - R_2^2}}{R_1(R_1+R_2)^2} = \frac{-2R_1R_2 - R_2^2}{R_1(R_1+R_2)^2}$$

$$= -\frac{R_2}{R_1}\cdot\frac{\cancel{2}R_1+R_2}{(R_1+R_2)^2} = -\frac{R_2}{R_1}\cdot\frac{1}{R_1+R_2} \checkmark$$

$$\left.\frac{d\,U_{ki}/U_{be}}{d\,R_2}\right|_{\substack{R_1=R_3\\R_2=R_4}} = -\frac{R_1}{R_1(R_3+R_2)} = -\frac{1}{R_1+R_2}$$

$$\left.\frac{d\,U_{ki}/U_{be}}{d\,R_4}\right|_{\substack{R_1=R_3\\R_2=R_4}} = \frac{R_1^{\cancel{2}} + \cancel{R_1}R_2}{\cancel{R_1}(R_1+R_2)^2}$$

$$= \frac{R_1+R_2}{(R_1+R_2)^2} = \frac{1}{R_1+R_2}$$

$R_2 = 10R_1 \Rightarrow$

$$\frac{dA}{dR_1} = \frac{10R_1}{R_1\cdot 11R_1} = \left(\frac{10}{11}\right)\cdot\frac{1}{R_1}$$

$$\frac{dA}{dR_2} = -\frac{1}{R_1+10R_1} = \left(-\frac{1}{11}\right)\frac{1}{R_1}$$

$$\frac{dA}{dR_3} = -\frac{10\cancel{R_1}}{\cancel{R_1}}\cdot\frac{1}{11R_1} = \left(-\frac{10}{11}\right)\frac{1}{R_1}$$

$$\frac{dA}{dR_4} = \frac{1}{11R_1} = \left(\frac{1}{11}\right)\cdot\frac{1}{R_1}$$

$$\frac{d\,\frac{U_{ki}}{U_{be}}}{d\,R_1} = ? \qquad \frac{U_{ki}}{U_{be,n}} = \frac{R_4}{R_3+R_4} - \frac{R_2R_3}{R_1(R_3+R_4)}$$

$$\hookrightarrow \left(\frac{R_2R_3}{R_3+R_4}\right)\cdot\frac{1}{R_1^2}$$

konstans

$$\left[\left(\frac{f(x)}{g(x)}\right)' = \frac{f'g - fg'}{g^2}\right]$$

$$\frac{d\,\frac{U_{ki}}{U_{be}}}{d\,R_2} = -\frac{R_3}{R_1(R_3+R_4)}$$

$$~~\frac{d\,U_{ki}/U_{be}}{d\,R_3} = \cancel{-\frac{R_4}{(R_3+R_4)^2}} = -\frac{R_4}{(R_3+R_4)^2}\cdot 1 - \frac{R_2}{R_1(R_3+R_4)}~~$$

valami rossz

$$\frac{d\,U_{ki}/U_{be}}{d\,R_4} = \frac{R_1^2(R_3+R_4) - (R_1R_4 - R_2R_3)\cdot R_1}{[R_1(R_3+R_4)]^2} =$$

$$= \frac{\cancel{R_1^2}(R_3+R_4)}{\cancel{R_1^2}(R_3+R_4)^2} - \frac{\cancel{R_1}(R_1R_4 - R_2R_3)}{R_1^{\cancel{2}}(R_3+R_4)^2} = \frac{1}{R_3+R_4} - \frac{R_1R_4 - R_2R_3}{R_1(R_3+R_4)^2}$$

$$= \cancel{\left(\frac{R_1 - R_1R_4 + R_2R_3}{R_1(R_3+R_4)}\right)} = \frac{R_1(R_3+R_4) - R_1R_4 + R_2R_3}{R_1(R_3+R_4)^2}$$

$$= \frac{R_1R_3 + \cancel{R_1R_4} - \cancel{R_1R_4} + R_2R_3}{R_1(R_3+R_4)^2} = \frac{R_1R_3 + R_2R_3}{R_1(R_3+R_4)^2}$$

$d\,A/dR_3$:

$$\frac{-R_2(R_1(R_3+R_4)) - R_1(R_1R_4 - R_2R_3)}{R_1^2(R_3+R_4)^2} = \frac{-\cancel{R_1}R_2R_3 - \cancel{R_1}R_2R_4 - R_1^{\cancel{2}}R_4 + \cancel{R_1}R_2R_3}{R_1^{\cancel{2}}(R_3+R_4)^2}$$

$$= \frac{\cancel{-R_2R_3} - R_2R_4 - R_1R_4 + \cancel{R_2R_3}}{R_1(R_3+R_4)^2} \quad \underline{\underline{\substack{R_1=R_3\\R_2=R_4}}} \quad \frac{-R_2^2 - R_1R_2}{R_1(R_1+R_2)^2}$$

$$= \frac{-R_2(R_1+R_2)}{R_1(R_1+R_2)^{\cancel{2}}} = -\frac{R_2}{R_1}\cdot\frac{1}{R_1+R_2} \checkmark$$

max hiba :

$R_1 \to R_1(1+h_R)$

$R_2 \to R_2(1-h_R)$

$R_3 \to R_3(1-h_R)$

$R_4 \to R_4(1+h_R)$

$\checkmark$

$R_1=$
$R_2=$

$$A_{Uh} = \frac{R_1R_4 - R_2R_3}{R_1R_3 + R_1R_4} = \frac{R_1(1+h_R)\cdot R_4(1+h_R) - R_2(1-h_R)\cdot R_3(1-h_R)}{R_1(1+h_R)\cdot R_3\cdot(1-h_R) + R_1(1+h_R)\cdot R_4(1+h_R)} \Longrightarrow$$

$$A_{Uh} = \frac{R_1(1+h)R_2(1+h) - R_2(1-h)\cdot R_1(1-h)}{R_1(1+h)R_1(1-h) + R_1(1+h)\cdot R_2(1+h)} = \frac{R_1R_2(1+h)^2 - R_1R_2(1-h)^2}{R_1^2(1-h^2) + R_1R_2(1+h)^2} \xrightarrow{R_2=10R_1}$$

$$A_{Uh} = \frac{10R_1^2\left[(1+h)^2-(1-h)^2\right]}{R_1^2(1-h)^2 + 10R_1^2(1+h^2)} = \frac{\cancel{10(1+2h}\ 10(1+2h+h^2-1+2h-h^2)}{1-2h+h^2+10+10h^2} = \frac{20h}{11-2h+11h^2} \xrightarrow{h=0{,}01}$$

$$A_{Uh} = \frac{0{,}2}{11-0{,}02+0{,}0011} = \frac{0{,}2}{10{,}9811} \cong 0{,}0182$$

$$A_{Us} = -\frac{R_2}{R_1} = -\frac{10R_1}{R_1} = -10$$

$$E_{hu} = \frac{A_{Us}}{A_{Uh}} = -\frac{10}{0{,}0182} = -549{,}45$$

(55dB)

$$\left(1/E_{hu} = 0{,}00182\right)$$

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egyszerűbben :

$$A_{Uh} = \frac{R_1R_4 - R_2R_3}{R_1(R_3+R_4)} \longrightarrow A_{Uh\,max} = \frac{R_1(1+h)\cdot R_4(1+h) - R_2(1-h) - R_3(1-h)}{R_1(R_3+R_4)}$$

← nevezővel nem foglalkozunk „hiba hatása"

$$A_{Uh\,max}\Big|_{\substack{R_1=R_3\\R_2=R_4}} = \frac{R_1R_2(1+h)^2 - R_1R_2(1-h)^2}{R_1(R_1+R_2)} = \frac{R_2\left[(1+2h+h^2)-(1-2h+h^2)\right]}{R_1+R_2} = \frac{R_2}{R_1+R_2}\cdot 4h \quad ✓$$

$$A_{Us} = -\frac{R_2}{R_1}$$

$$\left|A_{Us\,min}\right| = \frac{R_2(1-h)}{R_1(1+h)} \xrightarrow{h=0{,}01} \frac{R_2}{R_1}\cdot\frac{0{,}99}{1{,}01}$$

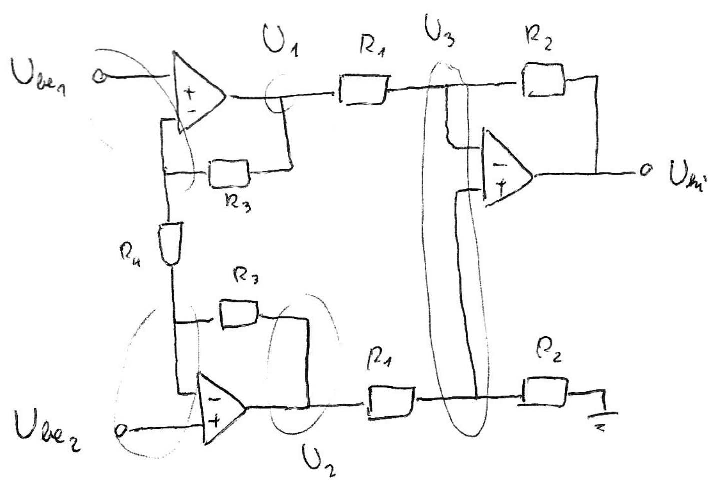
$$E_{hu} = \frac{A_{Us}}{A_{Uh}} = \frac{(-)\frac{R_2}{R_1}\cdot\frac{0{,}99}{1{,}01}}{+\frac{R_2}{R_1+R_2}\cdot 4h} = \frac{R_1+R_2}{R_1}\cdot\frac{0{,}99}{1{,}01}\cdot\frac{1}{4h}$$

$\Rightarrow$

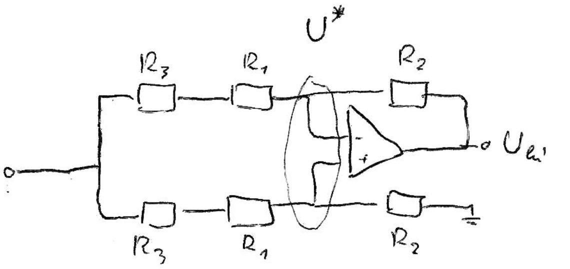
$$E_{hu}\Big|_{R_2=10R_1} = \frac{11R_1}{R_1}\cdot\frac{0{,}99}{1{,}01}\cdot\frac{1}{0{,}04} = 269{,}55 \approx 270 \longrightarrow \cancel{E_{hu}}\ \frac{1}{E_{hu}} = 0{,}0037 \approx 0{,}004 \quad ✓$$

$\Rightarrow$ 48dB

8.



A_{U_k} ? $U_{be_1} = U_{be_2}$ → R_4-en nem folyik áram → U_{be_k}



$$\frac{U_{be_k}-U^*}{R_1+R_3} = -\frac{U_{ki}}{R_2}$$

$$U^* = U_{be_k}\cdot\frac{R_2}{R_1+R_3+R_2}$$

$$\frac{U_{be_k}\left(1-\frac{R_2}{R_1+R_3+R_2}\right)}{R_1+R_3}\cdot(-R_2) = U_{ki}$$

$$\frac{U_{ki}}{U_{be_k}} = A_{U_k} = \frac{-R_2}{R_1+R_3}\cdot\frac{R_1+R_3-R_2}{R_2+R_1+R_3} = \frac{-R_1R_2-R_2R_3+R_2^2}{(R_1+R_3)^2} = -\frac{R_2}{(R_1+R_3)^2}\cdot(R_1+R_3-R_2))$$

$$\cancel{\frac{U_{be}}{U_{ki}}} = \frac{U_{ki}}{U_{be_k}} = A_{U_k} = \frac{\left(\frac{R_1+R_2+R_3-R_2}{R_1+R_2+R_3}\right)}{R_1+R_3}\cdot(-R_2) = \frac{(R_1+R_3)\cdot(-R_2)}{(R_1+R_2+R_3)(R_1+R_3)} = \frac{-R_2}{R_1+R_2+R_3}$$

A_{U_s} ? $\frac{U_{be_1}}{R_4} = \frac{U_1}{R_3+R_4} \to U_1 = \frac{R_3+R_4}{R_4}\cdot U_{be_1}$

$U_2 = -\frac{U_{be_1}}{R_4}\cdot R_3$ $\frac{U_2-U_3}{R_1} = \frac{U_3}{R_2} \to U_3\left(\frac{1}{R_2}+\frac{1}{R_1}\right) = U_2/R_1$

$$U_3 = U_2\cdot\frac{R_1R_2}{(R_1+R_2)R_1} = \frac{R_2}{R_1+R_2}U_2$$

⇓

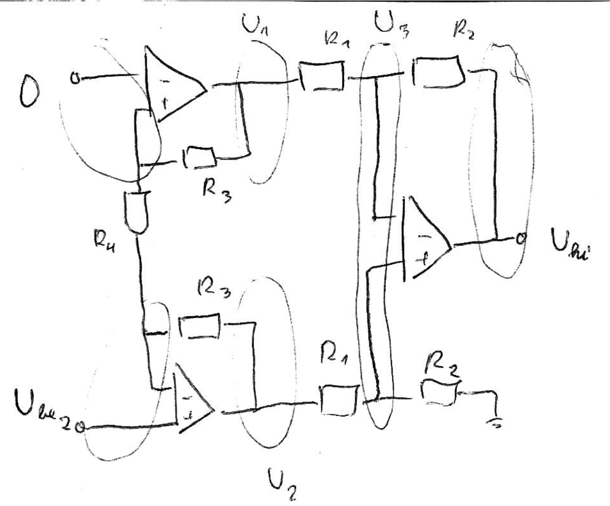
$$U_3 = -\frac{R_3}{R_4}\cdot\frac{R_2}{R_1+R_2}\cdot U_{be_1} \Longleftarrow$$

$$\frac{U_1-U_3}{R_1} = -\frac{U_{ki}}{R_2} \Rightarrow \frac{U_1+\frac{R_3}{R_4}\cdot\frac{R_2}{R_1+R_2}\cdot U_{be_1}}{R_1}\cdot(-R_2) = U_{ki}$$

$$\left.\frac{U_{ki}}{U_{be_1}}\right|_{U_{be_2}=0} = \frac{\frac{R_3+R_4}{R_4}+\frac{R_3}{R_4}\cdot\frac{R_2}{R_1+R_2}}{R_1}\cdot(-R_2) = -\frac{R_2}{R_1}\cdot\frac{(R_3+R_4)(R_1+R_2)+R_3\cdot R_2}{R_4(R_1+R_2)}$$

$$= -\frac{R_2}{R_1}\cdot\frac{R_3R_1+R_3R_2+R_4R_1+R_4R_2+R_2R_3}{R_4(R_1+R_2)} = -\frac{R_2}{R_1}\cdot\frac{2R_2R_3+R_1R_3+R_1R_4+R_2R_4}{R_4(R_1+R_2)}$$

$\cancel{U_{be_2}}\ \left.\frac{U_{ki}}{U_{be_2}}\right|_{U_{be_1}=0} = ?$



$$\frac{U_{be_2}}{R_4} = -\frac{U_1}{R_3} \rightarrow U_1 = U_{be_2}\cdot\left(-\frac{R_3}{R_4}\right) \qquad \frac{R_3+R_4}{R_3R_4}$$

$$\cancel{\frac{U_{be_2}-U_2}{R_3} = \frac{U_2-U_{be_2}}{R_3}\ \frac{U_{be_2}}{R_4}} \qquad \frac{U_2-U_{be_2}}{R_3} = \frac{U_{be_2}}{R_4} \rightarrow \frac{U_2}{R_3} = U_{be_2}\left(\frac{1}{R_4}+\frac{1}{R_3}\right)$$

$$U_2 = U_{be_2}\cdot\frac{R_3+R_4}{R_4}$$

$$\frac{U_2-U_3}{R_1} = \frac{U_3}{R_2} \rightarrow U_3\left(\underbrace{\frac{1}{R_1}+\frac{1}{R_2}}_{\frac{R_1+R_2}{\cancel{R_1}R_2}}\right) = \frac{U_2}{\cancel{R_1}} \rightarrow U_3 = \frac{R_2}{R_1+R_2}\cdot U_2$$

$$U_3 = \frac{R_2}{R_1+R_2}\cdot\frac{R_3+R_4}{R_4}\cdot U_{be_2}$$

$$\frac{U_1-U_3}{R_1} = -\frac{U_{ki}}{R_2} \rightarrow U_{ki} = -\frac{R_2}{R_1}(U_1-U_3)$$

$$U_{ki} = -\frac{R_2}{R_1}\cdot\left[\left(-\frac{R_3}{R_4}U_{be_2}\right)-\left(\frac{R_2}{R_1+R_2}\cdot\frac{R_3+R_4}{R_4}\right)U_{be_2}\right] = -\frac{R_2}{R_1}U_{be_2}\cdot\left(-\frac{R_3}{R_4}-\frac{R_2}{R_1+R_2}\cdot\frac{R_3+R_4}{R_4}\right)$$

$$\left.\frac{U_{ki}}{U_{be_2}}\right|_{U_{be_1}=0} = +\frac{R_2}{R_1}\cdot\frac{R_3(R_1+R_2)+R_2(R_3+R_4)}{R_4(R_1+R_2)} = \frac{R_2}{R_1}\cdot\left(\frac{R_3}{R_4}+\frac{R_2}{R_4}\cdot\frac{R_3+R_4}{(R_1+R_2)}\right)$$

$$U_{ki} = U_{be_1}\cdot\left[\overbrace{-\frac{R_2}{R_1}\cdot\frac{(R_1+R_2)(R_3+R_4)+R_2\cdot R_3}{R_4(R_1+R_2)}}^{K}\right] + U_{be_2}\cdot\left[\overbrace{\frac{R_2}{R_1}\cdot\frac{R_3(R_1+R_2)+R_2(R_3+R_4)}{R_4(R_1+R_2)}}^{K}\right]$$

$U_{ki} = K(U_{be_2}-U_{be_1})$

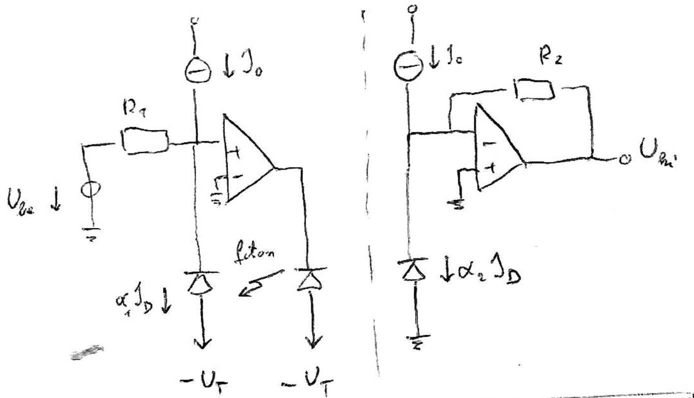
$R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4 + R_2R_3$ $\qquad$ $R_1R_3 + R_2R_3 + R_2R_3 + R_2R_4$

$A_{U_K} = -K$

$R_1R_3 + R_1R_4 + R_2R_4 + 2R_2R_3$ $\qquad$ $R_1R_3 + 0 + R_2R_4 + 2R_2R_3$

ist bisschen noch abweichend! :)

9. Galvanikusan leválasztott erősítő:



$$\frac{U_{be}}{R_1} + I_0 = \alpha_1 I_D$$

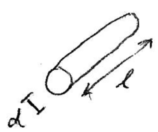
$$\alpha_2 I_D = I_0 + \frac{U_{ki}}{R_2}$$

$$\left(\frac{U_{be}}{R_1} + I_0\right) - \left(\frac{U_{ki}}{R_2} + I_0\right) = (\alpha_1 I_D) - (\alpha_2 I_D)$$

$$\frac{U_{be}}{R_1} - \frac{U_{ki}}{R_2} = I_D(\alpha_1 - \alpha_2)$$

ha $\alpha_1 = \alpha_2 \Rightarrow \frac{U_{be}}{R_1} = \frac{U_{ki}}{R_2}$; $\boxed{U_{ki} = \frac{R_2}{R_1} U_{be}}$

10. Nyúlásmérő:



$l = 200\,mm$
$d = 60\,\mu m$
$\rho_{20°C} = 5 \cdot 10^{-7}\,\Omega m$
$\alpha = 3 \cdot 10^{-5}\,\frac{1}{°C}$

$$R_{20°C} = \rho \cdot \frac{l}{A} = 5 \cdot 10^{-7}\,\Omega m \cdot \frac{200\,mm}{(30\,\mu m)^2 \pi} = 5 \cdot 10^{-7} \cdot \frac{0,2}{900 \cdot 10^{-12} \pi} = \cancel{35,3}\,\Omega \quad 35,37\,\Omega$$

l megnyújtása 1%-kal → változik A is
$A = \frac{V}{l}$ ↑ eredetileg 0,1% volt!!!

$$R'_{20°C} = \rho \cdot \frac{l'^2}{V} = \left(\frac{5 \cdot 10^{-7}}{(900 \cdot 10^{-12} \pi) \cdot 0,2}\right) \cdot 0,202^2 = 36,08\,\Omega$$

$$R'_{20°C} = \rho \cdot \frac{(l')^2}{A \cdot l} = R_{20°C} \cdot \frac{(l')^2}{l^2} = R_{20°C} \cdot \frac{l^2 \cdot 1,01^2}{l^2} = R_{20°C} \cdot 1,01^2 = 36,08\,\Omega$$

$\rho' \cdot \frac{l}{A}$

$R_{20°C} = 35,36776513\,\Omega$
$R_{25°C} = 35,3730703\,\Omega$
$R_{15°C} = 35,36245997\,\Omega$

$\cancel{R'_{20°C} - R'}$ $\quad R'_{20°C} - R_{20°C} = 0,71\,\Omega$ ✓

$$GF = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta l}{l}\right)} = \frac{0,71 / 35,37}{0,02 / \cancel{0,2}\ 0,2} = 0,2$$ ✓

$$\Delta T = 5°C \rightarrow R_{25°C} = R_{20°C} \cdot (1 + \Delta T \cdot \alpha) = R_{20°C} \cdot (1 + 5 \cdot 3 \cdot 10^{-5}) = 35,3753\,\Omega$$

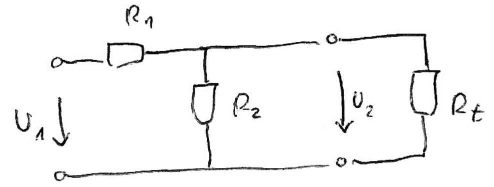
$$R_{15°C} = R_{20°C} \cdot (1 - 5 \cdot 3 \cdot 10^{-5}) = 35,3647\,\Omega$$

$$\Delta R_{+5°C} = 5,3 \cdot 10^{-3}\,\Omega$$

$$(\Delta R_{-5°C} = -5,3 \cdot 10^{-3}\,\Omega)$$

$$\frac{\Delta R_{5°C}}{\Delta R_{l'}} \cong 0,747 \approx 0,75 = 75\%$$ ✓

11. Terhelt feszültségosztó:



$R_1 = 9\,k\Omega$

$R_2 = 1\,k\Omega$

$U_1 = 1\,V$

$U_2 = ?$ ha $R_t = 100\,k\Omega$, $R_t = 5\,k\Omega$

$$U_1 \cdot \frac{R_2 \times R_t}{R_1 + R_2 \times R_t} = U_2 = \frac{\frac{R_2 \cdot R_t}{R_2 + R_t}}{R_1 + \frac{R_2 \cdot R_t}{R_2 + R_t}} = \frac{R_2 R_t}{R_1(R_2+R_t) + R_2 R_t} = \frac{1 \cdot R_t}{9(1+R_t) + 1 R_t} = \frac{R_t}{9(1+R_t)+R_t}$$

$$\frac{U_2}{U_1} = \frac{R_t}{9 + 10 R_t} \rightarrow \left.\frac{U_2}{U_1}\right|_{R_t = 100\,k\Omega} = \frac{100}{9+1000} = \frac{100}{1009} = 0{,}0991 \checkmark$$

$$\left.\frac{U_2}{U_1}\right|_{R_t = 5\,k\Omega} = \frac{5}{9+50} = \frac{5}{59} = 0{,}0847 \checkmark$$

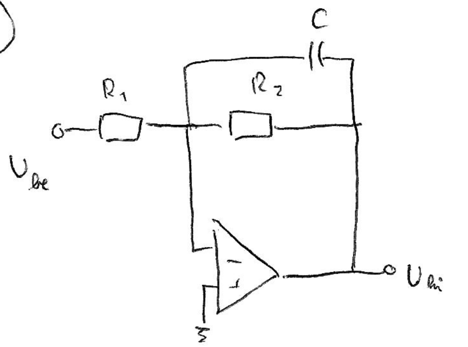
$U_2 = 0{,}08$

$R_t = ?$

$$U_2 = U_1 \cdot \frac{R_t}{9 + 10 R_t} = 1V \cdot \frac{R_t}{9 + 10 R_t} = 0{,}08$$

$$R_t = 0{,}72 + 0{,}8 R_t \rightarrow 0{,}2 R_t = 0{,}72 \rightarrow R_t = 3{,}6\,k\Omega \checkmark$$

12.



$R_1 = 10\,k\Omega$

$R_2 = 100\,k\Omega$

$U_{be} = 0{,}1\,V$

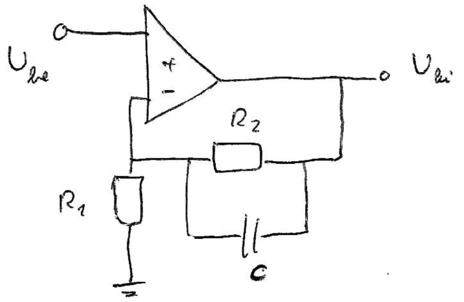
$U_{ki} = ?$

$$\left.U_{ki}\right|_{C=0} = -\frac{R_2}{R_1} \cdot U_{be} = -10 \cdot 0{,}1V = -1\,V$$

$$\left.\begin{array}{l} C = 10\,nF \\ \omega = 10^3 \frac{rad}{s} \end{array}\right\} Z_{R_2C} = \frac{R_2 \cdot \frac{1}{j\omega 10^{-8}}}{R_2 + \frac{1}{j\omega 10^{-8}}} = \frac{R_2}{1 + j\omega 10^{-8} R_2} = \frac{100\,000}{1 + j\,10^3 \cdot 10^{-8} \cdot 10^5} = \frac{10^5}{1 + j1}$$

$$U_{ki} = U_{be} \cdot \frac{-|Z_{R_2C}|}{R_1} = -\frac{10^5/\sqrt{2}}{10^4} = \cancel{10\sqrt{2}} = \cancel{-14{,}14} \quad \frac{-10/10}{\sqrt{2}} = -0{,}7071\,V \checkmark$$

3.



$R_1 = 10\,k\Omega$

$R_2 = 100\,k\Omega$

$U_{be} = 0{,}1\,V$

$C = 10\,nF$

$\omega = 10^3\,\frac{rad}{s}$

$$Z_{R_2C} = \frac{R_2 Z_c}{R_2 + Z_c} = \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{\cancel{R_2}\ 1 + j\omega R_2 C} = \frac{100000}{1 + j10^3 \cdot 10^5 \cdot 10^{-8}} = \frac{10^5}{1 + j \cdot 1}$$

$$\cancel{\frac{U_{ki}}{Z_{R_2C}}}) \quad \frac{U_{ki}}{Z_{R_2}C} = \frac{U_{be}}{R_1} \Rightarrow \frac{U_{ki}}{U_{be}} = \frac{Z_{R_2C}}{R_1} = \frac{10^5}{10^4(1+j)} = \frac{10}{1+j}$$