2014.03.13. Int. an 2. I. ZH, α variáns $\boxed{-1-}$

1,

$\boxed{12}$

$$y' = \frac{\operatorname{arctg}(2x)}{\operatorname{ch}(3y)} \Rightarrow \underbrace{\int \operatorname{ch}(3y)\,dy}_{I_1} = \underbrace{\int \operatorname{arctg}(2x)\,dx}_{I_2} \quad ②$$

Szeparálható

$$I_1 = \frac{1}{3}\operatorname{sh}(3y) + C \quad ③$$

f'/f alak

$$I_2 = \int \underset{u'}{1} \cdot \underset{v}{\operatorname{arctg}(2x)}\,dx = x\operatorname{arctg}(2x) - \frac{1}{4}\int \frac{2x\cdot 4}{1+4x^2}\,dx =$$

$$u = x \quad v' = \frac{2}{1+4x^2} \quad \Bigg| = x\operatorname{arctg}(2x) - \frac{1}{4}\ln(1+4x^2) + \tilde{C} \quad ⑥$$

$$\boxed{\frac{1}{3}\operatorname{sh}(3y) = x\operatorname{arctg}(2x) - \frac{1}{4}\ln(1+4x^2) + C} \quad ①$$

2,

$\boxed{12}$

$$y' - \frac{2}{x}y = \frac{x^2}{1+x^2}$$ elsőrendű lineáris

$$(H): \quad y' = +\frac{2}{x}y \underset{(y\equiv 0 \text{ mo.})}{\Rightarrow} \int \frac{dy}{y} = +2\int\frac{dx}{x} \Rightarrow \ln|y| = +2\ln|x| + C$$

$$\underline{y_{H,\text{ált}}(x) = K\cdot x^{+2}; \quad K \in \mathbb{R}} \quad ⑤$$

$y_{I,p}(x) = K(x)\cdot x^{+2}$; Beírva:

$$\underbrace{K'\cdot x^{+2} + \cancel{2K\cdot x^{+1}}}_{y'} - \cancel{\frac{2}{x}\cdot \underbrace{K x^2}_{y}} = \frac{x^2}{1+x^2}$$

$$K' = \frac{1}{1+x^2} \Rightarrow K(x) = \operatorname{arctg} x;$$

$$\underline{y_{I,p}(x) = \operatorname{arctg} x \cdot x^2} \quad ⑤$$

$$y_{I,\text{ált}}(x) = y_{H,\text{ált}}(x) + y_{I,p}(x) = \underline{\underline{x^2(K + \operatorname{arctg} x)}}; \quad K \in \mathbb{R} \quad ②$$

α

3, $\boxed{16}$ $x^2 y' = y^2 - 4xy + 6x^2 \Rightarrow y' = \left(\frac{y}{x}\right)^2 - 4\frac{y}{x} + 6$

$u(x) = \frac{y}{x}$ helyettesítés; $y(x) = x \cdot u(x)$; $y' = u + xu'$

$u + xu' = u^2 - 4u + 6 \Rightarrow u' = \frac{u^2 - 5u + 6}{x} = \frac{(u-2)(u-3)}{x}$ (5)

$u \equiv 2$ ill. $u \equiv 3$, tehát $\underline{y = 2x,\ y = 3x \text{ megoldás.}}$ (1)

$$\int \frac{du}{(u-2)(u-3)} = \int \frac{dx}{x}$$

$$\frac{1}{(u-2)(u-3)} = \frac{A}{u-2} + \frac{B}{u-3} = \frac{u(A+B) - 3A - 2B}{(u-2)(u-3)}; \quad \left.\begin{aligned} A + B &= 0 \\ -3A - 2B &= 1 \end{aligned}\right\}$$

$A = -1;\ B = +1$

$$\int \frac{du}{(u-2)(u-3)} = \int \frac{-1}{u-2}\,du + \int \frac{1}{u-3}\,du = -\ln|u-2| + \ln|u-3| + C =$$

$$= +\ln\left|\frac{u-3}{u-2}\right| + C = \ln\left|\frac{y-3x}{y-2x}\right| + C \quad (5)\ (1)$$

$\int \frac{dx}{x} = \ln|x| + \tilde{C}$; (2) $\quad \ln\left|\frac{y-3x}{y-2x}\right| = \ln|x| + C$

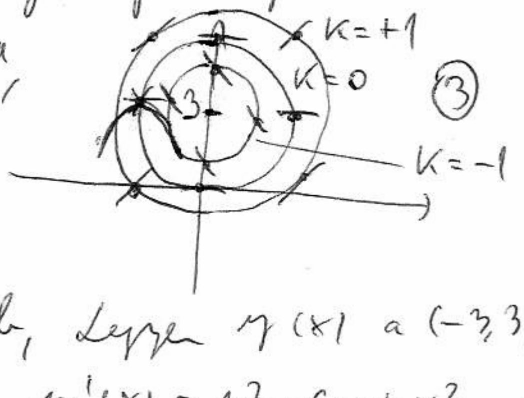
(2)

$$\frac{y-3x}{y-2x} = K \cdot x \Rightarrow y - 3x = Kxy - 2Kx^2 \Rightarrow \boxed{y = \frac{3x - 2Kx^2}{1 - Kx}}$$

— o —

4, $\boxed{10}$ $y' = y^2 - 6y + x^2$ izoklína: $y^2 - 6y + x^2 = K$

$(y-3)^2 + x^2 = K + 9$ kör (2)

a, 

(3)

b, Legyen $y(x)$ a $(-3,3)$-on átmenő megoldás.

$y'(x) = y^2 - 6y + x^2 \Rightarrow y'(-3) = 9 - 18 + 9 = 0$ (1)

$y''(x) = 2yy' - 6y' + 2x \Rightarrow y''(-3) = 0 - 0 + 2\cdot(-3) = -6 < 0$ (2)

Tehát $y(x)$-nek $(-3,3)$-ban lok. maximuma van. (2)

IX

5,
[14]

$$y''' - 4y' = 3e^{2x}$$

(H) $\lambda^3 - 4\lambda = \lambda(\lambda+2)(\lambda-2) = 0 \Rightarrow y_{H,\acute{a}ll}(x) = C_1 + C_2 e^{2x} + C_3 e^{-2x}$

$C_1, C_2, C_3 \in \mathbb{R}$ (5)

Külső rezonancia!

$$y_{I,p}(x) = Axe^{2x} \quad (3) \qquad /0$$

$$y_{I,p}'(x) = Ae^{2x} + 2Axe^{2x} \qquad /\cdot(-4)$$

$$y_{I,p}''(x) = 4Ae^{2x} + 4Axe^{2x} \qquad /0$$

$$\oplus \quad y_{I,p}'''(x) = 12Ae^{2x} + 8Axe^{2x} \qquad /\cdot(1)$$

$$3e^{2x} = Ae^{2x}(12-4) + Axe^{2x}\cancel{\underbrace{(8+2\cdot(-4))}_{0}}$$

$$3 = A\cdot 8 \Rightarrow A = \frac{3}{8}; \quad y_{I,p}(x) = \frac{3}{8}xe^{2x} \quad (4)$$

$$y_{I,\acute{a}ll}(x) = y_{H,\acute{a}ll}(x) + y_{I,p}(x) = C_1 + C_2e^{2x} + C_3e^{-2x} + \frac{3}{8}xe^{2x} \quad (2)$$

6,
[12]

$$(2x+1)\,\mathrm{sh}(3x) = \frac{1}{2}\cdot(2x+1)e^{3x} - \frac{1}{2}(2x+1)e^{-3x}$$

$\lambda_{1,2} = 3$; $\lambda_{3,4} = -3$; Kar. polinom: $(\lambda-3)^2(\lambda+3)^2 =$
$= (\lambda^2-9)^2 = \lambda^4 - 18\lambda^2 + 81$

$$\Rightarrow y^{(4)} - 18y'' + 81y = 0 \quad (7)$$

$$y_{\acute{a}lt}(x) = C_1e^{3x} + C_2xe^{3x} + C_3e^{-3x} + C_4xe^{-3x} \quad (5)$$

$C_1, C_2, C_3, C_4 \in \mathbb{R}$.

K

7, $\boxed{10}$

$$f(n+2) = -\frac{5}{2} f(n+1) + \frac{3}{2} f(n)$$

$$q^2 = -\frac{5}{2} q + \frac{3}{2} \implies 2q^2 + 5q - 3 = 0 \quad (2)$$

$$q_{1,2} = \frac{-5 \pm \sqrt{25+24}}{4} = \frac{-5 \pm 7}{4} = \begin{cases} \frac{1}{2} \\ -3 \end{cases} \quad (2)$$

$$f(n) = A \cdot \left(\frac{1}{2}\right)^n + B(-3)^n \quad (2)$$

$$\left.\begin{aligned} f(0) = -3: &\quad A + B = -3 \\ f(1) = 16: &\quad \tfrac{1}{2}A - 3B = 16 \end{aligned}\right\} \;/\cdot(+3)$$

$\oplus$

$$\frac{7}{2}A = 7 \implies A = 2 \;;\; B = -5 \quad (2)$$

$$\underline{\underline{f(n) = 2 \cdot \left(\frac{1}{2}\right)^n - 5 \cdot (-3)^n}} \quad (2)$$

8, a, $\boxed{14}$

$$\sum_{n=0}^{\infty} \underbrace{\left(\frac{2n+1}{2n+3}\right)^{n^2+3n}}_{a_n > 0} \; ; \text{ Gyökkritériummal}$$

$$\sqrt[n]{a_n} = \left(\frac{2n+1}{2n+3}\right)^{n+3} = \underbrace{\left(\frac{2n+1}{2n+3}\right)^3}_{\downarrow \, 1} \cdot \frac{\left(1 + \frac{1/2}{n}\right)^n}{\left(1 + \frac{3/2}{n}\right)^n} \xrightarrow{n\to\infty} 1^3 \cdot \frac{e^{1/2}}{e^{3/2}} = e^{-1}$$

$$e^{-1} < 1$$

$e^{-1} < 1$, tehát $\sum_n a_n$ konvergens. (8p)

b, $$\sum_{n=0}^{\infty} \underbrace{\frac{(2n)!\, 5^{n-1}}{(3n)!}}_{a_n} \quad \text{Hányados kritériummal:}$$

$$\frac{a_{n+1}}{a_n} = \frac{(2n+2)!\, 5^n}{(3n+3)!} \cdot \frac{(3n)!}{(2n)!\, 5^{n-1}} = \frac{(2n+1)(2n+2) \cdot 5}{(3n+1)(3n+2)(3n+3)} \xrightarrow{n\to\infty} 0 < 1$$

Tehát $\sum_n a_n$ konvergens! (6p)