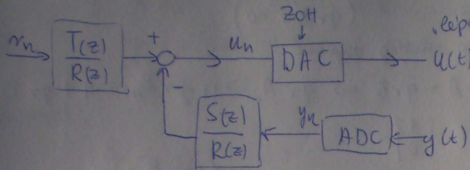
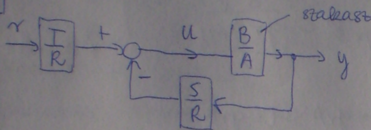
X. 26. p.
7. h

2-DOF szabályzó

Nullad rendű tartószerv



$\boxed{ZR}$



$$\frac{\frac{T}{R}\,\frac{B}{A}}{1+\frac{S}{R}\cdot\frac{B}{A}} = \frac{TB}{AR+BS} = \frac{B_m}{A_m}\cdot\frac{A_o}{A_o} = \frac{TB^+B^-}{A\cancel{B^+}(z-1)^l R_1' + \cancel{B^+}B^-S}$$

$B = B^+B^-$

$R = B^+(z-1)^l \cdot R_1'$ ← integrátorok száma

$B_m = B^- B_m'$

$T = B_m' \cdot A_o$

$A(z-1)^l R_1' + B^- S = A_m A_o$

$\boxed{AX + BY = C}$

diofantoszi egyenlet megoldhatósága

$gr\,X < gr\,B$

$gr\,Y < gr\,A$

pólusok ↓ zérusok ↓

$$\left.\begin{aligned} &gr\,A_m - gr\,B_m \geq gr\,A - gr\,B \\ &gr\,S = gr\,A + l - 1 \\ &gr\,A_o \geq 2\,gr\,A + l - 1 - gr\,B^+ - gr\,A_m \\ &gr\,R_1' = gr\,A_m + gr\,A_o - gr\,A - l \end{aligned}\right\}\ \text{Fokszám feltételek}$$

A_o ↑ megfigyelő polinom

Eddig ismétlés. Új anyag: →

Monik: A, B^+, R_1', A_m, A_σ

Nem monik: B^-, B_m', S $\quad (\mathrm{gr}\, B_m' = 0)$ $\quad$ gr = fokszám

$\mathrm{gr}\, A - \mathrm{gr}\, B = 1$ ← Ez mindig teljesül

(1) $\mathrm{gr}\, A_m - \mathrm{gr}\, B_m \geq \mathrm{gr}\, A - \mathrm{gr}\, B \Rightarrow \mathrm{gr}\, A_m \geq 1 + \mathrm{gr}\, B_m = 1 + \mathrm{gr}\, B^-$

$* \mathrm{gr}\, A_m = 1 + \mathrm{gr}\, B^- + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$

(2) $* \mathrm{gr}\, S = \mathrm{gr}\, A + l - 1$

(3) $\mathrm{gr}\, A_\sigma \geq \mathrm{gr}\, A + \mathrm{gr}\, A + l - 1 - \mathrm{gr}\, B^+ - \boxed{\mathrm{gr}\, A_m}$ $\quad 1 + \mathrm{gr}\, B^- + \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$

$* \mathrm{gr}\, A_\sigma = \mathrm{gr}\, A + l - 1 - \begin{Bmatrix} 1 \\ 0 \end{Bmatrix}$

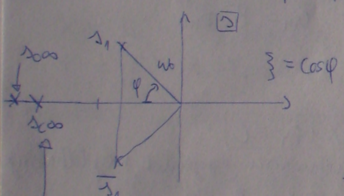
* Ezeket használjuk.

(4) $* \mathrm{gr}\, R_1' = \mathrm{gr}\, A_m + \mathrm{gr}\, A_0 - \mathrm{gr}\, A - l = \mathrm{gr}\, B^-$

$$1 = \frac{B^m(1)}{A_m(1)} = \frac{B^-(1)\, B_m'}{A_m(1)} \Rightarrow B_m' = \frac{A_m(1)}{B^-(1)}$$

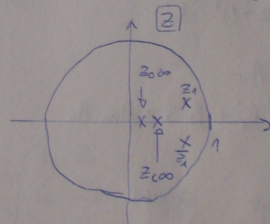
ZR (zárt rendszer) specifikáció: s-ben specifikálunk

σ = observer



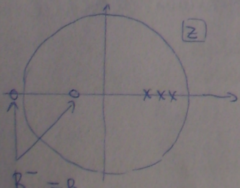
Ha kell még pólus, akkor innen választjuk

$\Rightarrow$ $z_i = e^{s_i T}$



Példa:

Szakasz: $W(s) \xrightarrow[\text{'zoh'}]{\text{c2d}} D(z) = \dfrac{B(z)}{A(z)}$



$\boxed{l = 1}$

$B^- = B$

$B^+ = 1$

→

Fokszámok:

$l=1$; $gr\, A_m = 1+2=3$; itt korrigálni nem kell.

$gr\, S = 3+1-1=3$; $gr\, A_o = 3+1-1=3$

$gr\, R_1' = 2$

Polinomok:

$$R_1' = z^2 + r_1 z + r_2$$

$$S = s_0 z^3 + s_1 z^2 + s_2 z + s_3$$

$$A_m = (z - z_1)(z - \bar{z}_1)(z - z_{c\infty})$$

$$A_o = (z - z_{o\infty})^3$$

Diophantoszi egyenlet polinomjai :

$$AX + BY = C$$

$$C := A_m \cdot A_o = z^6 + c_1 z^5 + c_2 z^4 + c_3 z^3 + c_4 z^2 + c_5 z + c_6$$

$$A := A(z-1) = z^4 + \tilde{a}_1 z^3 + \tilde{a}_2 z^2 + \tilde{a}_3 z + \tilde{a}_4$$

$$B := B^- = b_0 z^2 + b_1 z + b_2$$

$$X := R_1' = z^2 + r_1 z + r_2$$

$$Y := S = s_0 z^3 + s_1 z^2 + s_2 z + s_3$$

Megoldás: 2 Töplic blokkból áll:

Ez 6 kell, hogy legyen az integrálás miatt.

$$\left[\begin{array}{cc|cccc} 1 & 0 & b_0 & 0 & 0 & 0 \\ \tilde{a}_1 & 1 & b_1 & b_0 & 0 & 0 \\ \tilde{a}_2 & \tilde{a}_1 & b_2 & b_1 & b_0 & 0 \\ \tilde{a}_3 & \tilde{a}_2 & 0 & b_2 & b_1 & b_0 \\ \tilde{a}_4 & \tilde{a}_3 & 0 & 0 & b_2 & b_1 \\ 0 & \tilde{a}_4 & 0 & 0 & 0 & b_2 \end{array}\right] \begin{pmatrix} r_1 \\ r_2 \\ s_0 \\ s_1 \\ s_2 \\ s_3 \end{pmatrix} = \begin{pmatrix} c_1 - \tilde{a}_1 \\ c_2 - \tilde{a}_2 \\ c_3 - \tilde{a}_3 \\ c_4 - \tilde{a}_4 \\ c_5 \\ c_6 \end{pmatrix}$$

1. töplic blokk

2.

Ezért a 6. elem a nulla.

$$\boxed{B_m' = \frac{A_m(1)}{B^-(1)}}$$

$$S(z) = s_0 z^3 + s_1 z^2 + s_2 z + s_3$$

$$T(z) = B_m' A_o = B_m' (z - z_{o\infty})^3$$

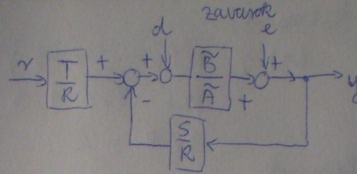
$$R(z) = B^+(z-1) R_1' = (z-1)(z^2 + r_1 z + r_2)$$

Realizálás:

$$r,\ y \rightarrow \boxed{R(z)u = T(z)r - S(z)y} \rightarrow u$$

$$D(z) = \frac{B(z)}{A(z)} \longrightarrow T(z),\ S(z),\ R(z)$$

$$\tilde{D}(z) = \frac{\tilde{B}(z)}{\hat{A}(z)}$$ megváltozott szakasz



$$y = \frac{\frac{T}{R}\cdot\frac{\tilde{B}}{\hat{A}}}{1+\frac{S}{R}\frac{\tilde{B}}{\hat{A}}}\cdot r + \frac{\frac{\tilde{B}}{\hat{A}}}{1+\frac{S}{R}\cdot\frac{\tilde{B}}{\hat{A}}}d + \frac{1}{1+\frac{S}{R}\frac{\tilde{B}}{\hat{A}}}e = y$$

$$y = \frac{T\tilde{B}}{\hat{A}R+\tilde{B}S}r + \frac{\tilde{B}R}{\hat{A}R+\tilde{B}S}d + \frac{\hat{A}R}{\hat{A}R+\tilde{B}S}e$$

Feltételezzük, hogy van integrátor:

$l=1 \longrightarrow R = B^{+}(z-1)R_1' \longrightarrow R(1)=0$

$r = 1(t) \longrightarrow \frac{T(1)}{S(1)} = 1 \longrightarrow y_\infty = r$

$d = 1(t) \longrightarrow 0 \longrightarrow y_\infty = 0$

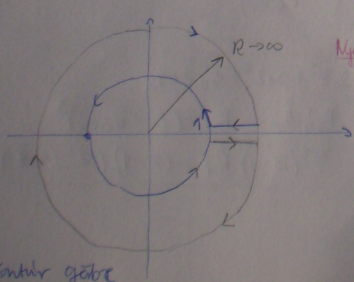
$e = 1(t) \longrightarrow 0 \longrightarrow y_\infty = 0$

Stabil lesz a rendszer:

$\hat{A}R+\tilde{B}S = 0$ gyökei: $|z_i| < 1$

STABIL egy jelentős tartományban

$D_0(z)$ felnyitott kör



Kontúr görbe

$z = e^{j\omega T}$

• $\pi = \omega T \Rightarrow \omega = \frac{\pi}{T} = \omega_N \leftarrow$ Nyquist

Nyquist-krit.:

$$\text{EKVSZ}\left(D_0(e^{j\omega T}, -1)\right) = P$$

Előjeles körül-vételek száma

P = felnyitott kör instabil pólusainak száma

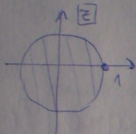
→

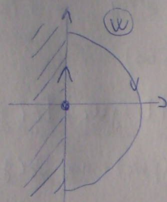
Bode-krit.:

ω_c, φ_t, ha $P=0$ $\qquad D_0(z=e^{j\omega T})=\dfrac{B_0(e^{j\omega T})}{A_0(e^{j\omega T})}$

Bilineáris transzformáció

$$w=\frac{2}{T}=\frac{z-1}{z+1}$$





Itt a kontúr görbe a komplex tengely.

$D_0(z)\longrightarrow D_0(w)$

$\dfrac{B_0(z)}{A_0(z)}$ $\qquad \dfrac{B_0(w)}{A_0(w)}$

$$w=\frac{2}{T}\,\frac{z-1}{z+1}\longrightarrow \frac{T}{2}w=\frac{z-1}{z+1}$$

$$\frac{T}{2}w(z+1)=z-1$$

$$\boxed{z=\frac{1+w\frac{T}{2}}{1-w\frac{T}{2}}}$$

$$D(z)=A_z\frac{\prod_{i=1}^{m}(z-z_{0i})}{(z-1)^{l}\prod_{i=1}^{n}(z-z_i)}$$

$$z-z_i=\frac{1+w\frac{T}{2}}{1-w\frac{T}{2}}-z_i=(1-z_i)\frac{1+w\frac{T}{2}\,\frac{1+z_i}{1-z_i}}{1-w\frac{T}{2}}$$

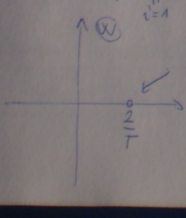
$$z-1=\frac{1+w\frac{T}{2}}{1-w\frac{T}{2}}-1=\frac{wT}{1-w\frac{T}{2}}$$

$$D(w)=A_w\frac{\left(1-w\frac{T}{2}\right)^{l+n-m}\prod_{i=1}^{m}\left(1+w\frac{T}{2}\,\frac{1+z_{0i}}{1-z_{0i}}\right)}{w^{l}\prod_{i=1}^{n}\left(1+w\frac{T}{2}\,\frac{1+z_i}{1-z_i}\right)}$$

$$A_w=A_z\frac{\prod_{i=1}^{m}(1-z_{0i})}{T^{l}\prod_{i=1}^{n}(1-z_i)}$$

$1-w\frac{2}{T}$

$\arctan\left(-\frac{2}{T}w\right)<0$



nem minimumfázisú zérus hely, ezért a fázisszöge negatív

Rontja a fázistöbbletet!

$D(w)$ szakasz $\qquad T_1 > T_2$ domináns időállandó

$$D_c(w) = A_{cw} \frac{(1+w\tau_1)(1+w\tau_2)}{w(1+wT_c)}$$

$$\tau_1 = T_1, \quad \tau_2 = T_2$$

$$U_{max} \longrightarrow D_c\left(w = \frac{2}{T}\right)$$

$$\{f_n\} \to f_0 + f_1 z^{-1} + f_2 z^{-2} + \dots = F(z)$$

$$f_0 = F(z=\infty)$$

Vágási frekvencia: $\quad |D(jw_c)\, D_c(jw_c)| - 1 = 0$

Fázistöblet: $\quad \pi + \arg D(jw_c) + \arg D_c(jw_c) - \varphi_t\,[rad] = 0$

$$D_c\left(w = \frac{2}{T}\right) - U_{max} = 0$$

$\bar{f}(\bar{x}) = \bar{0}$ fsolve

$$W(s) \xrightarrow[\text{'zoh'}]{c2d} D(z) \xrightarrow[\text{'tustin'}]{d2c} D(w) \xrightarrow{fsolve} D_c(w) \xrightarrow[\text{'tustin'}]{c2d} D_c(z)$$

realizálás

$\bar{x} = (A_{cw}, \tau_c, w_c)$

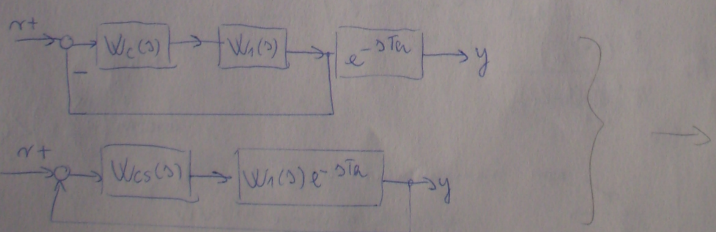
$\bar{f}(\bar{x}) = \bar{0}$ fsolve ('myfun', x0)

$\bar{x}0$

<u>Holtidős rendszer szabályozása Smith-prediktorral</u>

Szakasz: $W_1(s) \cdot e^{-sT_h}$

$W_1(s) \longrightarrow W_c(s)$ szabályozó



$$\frac{W_c \cancel{W_1 e^{-sT_h}}}{1+W_1W_c} = \frac{W_{cs}\cancel{W_1 e^{-sT_h}}}{1+W_{cs}W_1e^{-sT_h}}$$

$$W_c(1+W_{cs}W_1e^{-sT_h}) = W_{cs}(1+W_1W_c)$$

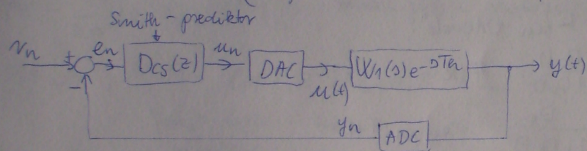
$$W_c + W_{cs}W_cW_1e^{-sT_h} = W_{cs}+W_{cs}W_1W_c$$

$$W_c = [1+W_1W_c(1-e^{-sT_h})]W_{cs} \Rightarrow \boxed{W_{cs} = \frac{W_c}{1+W_1W_c(1-e^{-sT_h})}}$$

e^{-sT_h} transztcendens => folytonos időben nem használható

Feltevés: $T_h = d\cdot T$; d egész szám

$$e^{-sT_h} \longrightarrow (e^{sT})^{-d} \longrightarrow z^{-d} \text{ shift-operátor}$$



$D_1(z)z^{-d}$ szakasz

$D_1(z) \longrightarrow D_c(z)$

$D_1 = \frac{B}{A}$

$D_c = \frac{S}{R}$

$$D_{cs}(z) = \frac{D_c(z)}{1+(1-z^{-d})D_c(z)D_1(z)} = \frac{u(z)}{e(z)}$$

$$= \frac{S/R}{1+(1-z^{-d})\frac{S}{R}\cdot\frac{B}{A}} = \frac{SA}{RA+(1-z^{-d})SB} = \frac{AS}{AR+BS-z^{-d}BS} = \frac{u}{e}$$

$$(AR+BS)u - z^{-d}BSu = ASe \qquad /\cdot z^{-(n_A+n_R)}$$

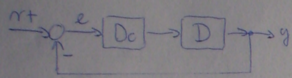
Könnyen realizálható:

$$\boxed{u_R = -Fu_F + Hu_H + Ge_G}$$

Véges beállási idejű stabilizozás (dead beat)

A Hibajel véges sok lépés után szűnjön meg!

$e_0 + e_1 z^{-1} + \dots = P_e(z^{-1})$ véges fokszámú polinom



$$e = \frac{1}{1+D_c D}\,\frac{1}{1-z^{-1}} = P_e(z)$$

$$r = \{1,1,1,\dots\} \to R(z) = \frac{1}{1-z_1}$$

$$e = r - y \Rightarrow P_e(z) = \frac{1}{1-z^{-1}} - \frac{D_c D}{1+D_c D}\,\frac{1}{1-z^{-1}} \quad ;$$

$$\frac{D_c D}{1+D_c D} = 1 - P_e(z)(1-z^{-1})$$ FIR zárt-rendszer

∀ polinom z^{-1}-ben értendő:

$$K(z^{-1}) = \frac{D_c D}{1+D_c D}$$ FIR

$$M(z^{-1}) = \frac{D_c}{1+D_c D}$$ legyen FIR

$$K = \frac{D_c}{1+D_c D}\cdot D = M\cdot D \Rightarrow K = M\frac{B}{A}$$

Szakasz $D(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})}$; $\frac{K}{M} = \frac{B}{A}\frac{L}{L}$

Korrekciós polinom: $L(z^{-1}) \Rightarrow K = BL$

$M = AL$

$$\frac{D_c}{1+D_c D} = M \Rightarrow D_c = M + D_c D M$$

$$D_c(1-MD) = M$$

korrekciós polinom

$$D_c = \frac{M}{1-MD} = \frac{AL}{1-AL\cdot\frac{B}{A}} \Rightarrow \boxed{D_c = \frac{AL}{1-BL}}$$

ZR átv. fv.

↓

$$K(z^{-1}) = k_0 + k_1 z^{-1} + \dots + k_{n_B+n_L}\cdot z^{-(n_B+n_L)} = \frac{k_0 z^{n_B+n_L} + k_1 z^{n_B+n_L-1} + \dots + k_{n_B+n_L}}{z^{n_B+n_L}}$$

$K(z^{-1})$ FIR $\Longleftrightarrow$ ∀ pólusa ZR-nek $z=0$-ban van! ✓