(1) $\sum_{n=1}^{\infty}{}' \frac{(-4)}{\sqrt[3]{n}} x^n$ KT=? Hol abs. konv! | 2015.04.16. Inf. an2. ZH2 α

$a_n = \frac{(-4)^n}{\sqrt[3]{n}}$ így $\frac{1}{R} = \lim_{n\to\infty} \frac{\sqrt[n]{|4^n|}}{\sqrt[n]{\sqrt[3]{n}}} = \frac{4}{\lim_{n\to\infty}\sqrt[3]{\sqrt[n]{n}}} = 4 \Rightarrow R = \frac{1}{4}$ $x_0 = 0$

Végpontok: $x = \frac{1}{4}$ $\sum_{n=1}^{\infty}{}' \frac{(-4)^n}{\sqrt[3]{n}}\frac{1}{4^n} = \sum_{n=1}^{\infty}{}' \frac{(-1)^n}{\sqrt[3]{n}}$ konv, mi Leibniz, de nem abs konv, mi $\frac{1}{3}<1$

$x = -\frac{1}{4}$ $\sum_{n=1}^{\infty}{}' \frac{(-4)^n}{\sqrt[3]{n}}\left(-\frac{1}{4}\right)^n = \sum_{n=1}^{\infty}{}' \frac{1}{\sqrt[3]{n}}$ div, így

KT: $\left(-\frac{1}{4}, \frac{1}{4}\right]$, abs. konv: $\left(-\frac{1}{4}, \frac{1}{4}\right)$.

(2) a, $f(x) = x^3 e^{-x^2}$ $T(x) = x^3 \sum_{k=0}^{\infty}{}' \frac{(-x^2)^k}{k!} = \sum_{k=0}^{\infty}{}' \frac{(-1)^k}{k!} x^{2k+3}$ KT = ℝ

b, $\int_0^{0.1} f(x)\,dx \approx \int_0^{0.1} x^3 - x^5\,dx = \left[\frac{x^4}{4} - \frac{x^6}{6}\right]_0^{0.1} = \frac{1}{4}10^{-4} - \frac{1}{6}10^{-6}$

felcserélhető a 'T' konv. miatt

c, $\int_0^{0.1} f(x)\,dx = \int_0^{0.1} T(x)\,dx = \int_0^{0.1} \sum_{k=0}^{\infty}{}' \frac{(-1)^k}{k!} x^{2k+3}\,dx \overset{\oplus}{=} \sum_{k=0}^{\infty}{}' \int_0^{0.1} \frac{(-1)^k}{k!} x^{2k+3}\,dx =$

$\sum_{k=0}^{\infty}{}' \frac{(-1)^k}{k!}\left[\frac{x^{2k+4}}{2k+4}\right]_0^{0.1} = \sum_{k=0}^{\infty}{}' \frac{(-1)^k}{k!\,(2k+4)} \cdot \frac{1}{10^{2k+4}}$ ez Leibniz típusú sor, így a

$|\text{Hiba}| \le |a_{N+1}| = \frac{1}{2}\,\frac{1}{4+4}\cdot\frac{1}{10^8}$

(3) a, $f(x) = (8-x^2)^{-\frac{1}{3}} = \frac{1}{2}\left(1 - \frac{x^2}{8}\right)^{-\frac{1}{3}} = \frac{1}{2}\sum_{k=0}^{\infty}{}' \binom{-\frac{1}{3}}{k}\left(-\frac{x^2}{8}\right)^k = \sum_{k=0}^{\infty}{}' \frac{1}{2}\binom{-\frac{1}{3}}{k}\left(-\frac{1}{8}\right)^k x^{2k}$

b, $\left|-\frac{x^2}{8}\right| < 1 \Rightarrow |x| < \sqrt{8}$ $R = \sqrt{8}$

c, $f^{(6)}(0) = 6!\cdot a_6 = 6!\cdot\frac{1}{2}\binom{-\frac{1}{3}}{3}\left(-\frac{1}{8}\right)^3 = 6\cdot5\cdot4\cdot3\cdot2\cdot1\cdot\frac{1}{2}\cdot\frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3\cdot2\cdot1}\cdot\left(-\frac{1}{8\cdot8\cdot8}\right)$

(4) a, $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^2+2y^2} = \lim_{r\to\infty} \frac{2r^2\cos^2\varphi\; r\sin\varphi}{r^2\cos^2\varphi + 2r^2\sin^2\varphi} = \lim_{r\to 0} 2r\,\frac{\overbrace{\cos^2\varphi\sin\varphi}^{\le 1}}{\underbrace{1+\sin^2\varphi}_{\ge 1}} = 0$

b, $\lim_{(x,y)\to(0,0)} \frac{3(x+1)y}{x^2+y^2}$ = Ha ∃ is a lim $\lim_{\substack{x\to0\\ y=mx}} \frac{3(x+1)mx}{m^2x^2+x^2} = \lim_{x\to0} \frac{\overbrace{3(x+1)\cdot m}^{\to 3m}}{\underbrace{m^2x + x}_{\to 0}}$ ∄

(5) $f(x,y) = \begin{cases} \frac{(y+1)x^2}{x^2+y^2} & (x,y)\ne(0,0) \\ 0 & (x,y)=(0,0)\end{cases}$ $P(1,2)$ $e = \begin{bmatrix}-2\\3\end{bmatrix}\frac{1}{\sqrt{13}}$

a, $f'_x = \frac{2x(y+1)(x^2+y^2) - (y+1)x^2\cdot 2x}{(x^2+y^2)^2}$ ha $(x,y)\ne(0,0)$ | $f'_x(0,0) = \lim_{h\to0}\frac{\frac{1\cdot(\cancel{0}+h)^2}{h^2+0} - 0}{h}$ ∄

$f'_y = \frac{x^2(x^2+y^2) - x^2(y+1)2y}{(x^2+y^2)^2}$ ha $(x,y)\ne(0,0)$ | $f'_y(0,0) = \lim_{h\to0}\frac{\frac{(h+1)\cdot 0}{0+h^2} - 0}{h} = 0$

b, nem, mi ∄ f'_x ami szüks. felt

c, $\left.\frac{df}{de}\right|_P = \langle \operatorname{grad} f(1,2) \mid e \rangle = \left\langle \left(\frac{30-6}{25}, \frac{5-12}{25}\right), \left(\frac{-2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)\right\rangle = \frac{-48}{25\sqrt{13}} - \frac{21}{25\sqrt{13}}$