Info an. Szig. 2017.03.10. $\boxed{1-1}$

1, a, Ha $0 \le a_n \le b_n$ és $\sum_n a_n = \infty \Rightarrow \sum_n b_n = \infty$ (2)

b, $S_n^{(a)} \le S_n^{(b)}$, és $S_n^{(a)} \longrightarrow \infty \Rightarrow S_n^{(b)} \longrightarrow \infty$ (5)

c, $b_n = \frac{1}{\sqrt{n(n+3)}} \ge \frac{1}{\sqrt{(n+3)(n+3)}} = \frac{1}{n+3} =: a_n$, és $\sum_n a_n = \infty \Rightarrow \sum_n b_n = \infty$ (4)

2, a, $\left(\frac{n+4}{n+3}\right)^{2n} = \left(\frac{\left(1+\frac{4}{n}\right)^n}{\left(1+\frac{3}{n}\right)^n}\right)^2 \longrightarrow \left(\frac{e^4}{e^3}\right)^2 = \underline{\underline{e^2}}$ (5)

b, $\lim_{x \to 0+0} \frac{\ln(4x)}{(1/x)} \overset{L'H}{=} \lim_{x \to 0+0} \frac{(1/x)}{(-1/x^2)} = \lim_{x \to 0+0} (-x) = \underline{\underline{0}}$ (5)

3, a, $\boxed{8}$ $\int_2^3 \frac{x^2}{x^2-1}\,dx = \int_2^3 \left(1 + \frac{-1/2}{x+1} + \frac{+1/2}{x-1}\right) dx = 1 + \left[\frac{-1}{2}\ln(x+1) + \frac{1}{2}\ln(x-1)\right]_2^3$ (5) (2)

$= \underline{\underline{1 - \frac{1}{2}\ln\frac{4}{3} + \frac{1}{2}\ln\frac{2}{1}}}$ (1) $= 1 + \frac{1}{2}\ln\frac{3}{2}$

b, $\int \sin x \cos^3 x\,dx = \underline{\underline{-\frac{\cos^4 x}{4} + C}}$ (3)

$f' \cdot f^3$ alak

$\boxed{5}$ c, $\int \underset{u}{(x+3)}\, \underset{v'}{e^{2x}}\,dx = (x+3)\cdot\frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx = \underline{\underline{\frac{x+3}{2}e^{2x} - \frac{1}{4}e^{2x} + C}}$ (3) (2)

$u' = 1 \quad v = \frac{1}{2}e^{2x}$

4, (H) lin. diff. egy. $y \equiv 0$ mo.

$y' = \frac{x}{\sqrt{1+x^2}}\, y \overset{\downarrow}{\Rightarrow} \int \frac{dy}{y} = \int \frac{x\,dx}{\sqrt{1+x^2}} \Rightarrow \ln|y| = \sqrt{1+x^2} + C$

$\Rightarrow \underline{y_{H,\text{ált}}(x) = K \cdot e^{\sqrt{1+x^2}}}$; $K \in \mathbb{R}$ (5)

(I) $y_{i,p}(x) = K(x)\, e^{\sqrt{1+x^2}}$; (1) Beírva:

$\sqrt{1+x^2} \cdot K' \cancel{e^{\sqrt{1+x^2}}} + \cancel{\sqrt{1+x^2} \cdot K \cdot e^{\sqrt{1+x^2}} \cdot \frac{x}{\sqrt{1+x^2}} - xK e^{\sqrt{1+x^2}}} = \cancel{e^{\sqrt{1+x^2}}}$

$$K'(x)=\frac{1}{\sqrt{1+x^2}} \Longrightarrow K(x)=\int \frac{1}{\sqrt{1+x^2}}\,dx = \operatorname{arsh} x$$ (4)

$$y_{i,\acute a}(x)=y_{i,p}+y_{H,\acute alt}=\underline{\underline{(\operatorname{arsh} x+K)\,e^{\sqrt{1+x^2}}}}\ ;\ K\in\mathbb{R}.$$ (2)

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$\boxed{\frac{5}{10}}$ 5, Legyen a főegyüttható pozitív. Ekkor $\lim_{x\to\pm\infty} p(x)=\pm\infty$, így $\exists\,\Omega>0:\ p(-\Omega)<0,\ p(\Omega)>0$. Alkalmazható a folyt. fgv.-ekre érvényes Bolzano-tételt a $[-\Omega, +\Omega]$ intervallumra. (5) (5)

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6, a, $f(x)=\dfrac{1}{1+x}=\underline{\underline{\sum_{n=0}^{\infty}(-1)^n x^n}}\ ;\ |x|<1\ ;\ \underline{\underline{R=1}}$ (mértani sor) (4)

b, $g(x)=\ln(1+x)=0+\int_0^x f(x)\,dx=\sum_{n=0}^{\infty}(-1)^n\dfrac{x^{n+1}}{n+1}\ ;\ \underline{R=1}$ (nem változik) (6)

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7, $\boxed{13}$
$$\left.\begin{aligned} g'_x &= 4\cdot(2x-y)+12x^2=0 \quad (2)\\ g'_y &= \underbrace{-2(2x-y)-6}_{-4x+2y-6}=0 \quad (2)\end{aligned}\right\} \Longrightarrow y=3+2x$$
$$2x-3-2x+3x^2=0 \Longrightarrow x=\pm 1$$ (5)

$\underline{(x_1,y_1)=(+1,+5)}\ ;\ \underline{(x_2,y_2)=(-1,+1)}$

$$|\underline{H}|=\begin{vmatrix} 8+24x & -4\\ -4 & +2\end{vmatrix}=16+48x-16=48x$$ (2)

$8+24x>0$, $48x>0$: lok. min (1)

$48x<0$: Nyeregpont (1)

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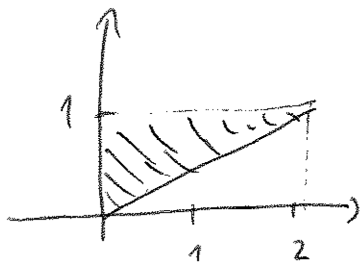
8, $\boxed{10}$
$$\int_{x=0}^{2}\left(\int_{y=x/2}^{1} e^{y^2}\,dy\right)dx=\int_{y=0}^{1}\left(\int_{x=0}^{2y} e^{y^2}\,dx\right)dy=$$ (5)



$$=\int_{y=0}^{1} e^{y^2}\cdot 2y\,dy=\left[e^{y^2}\right]_0^1=\underline{\underline{e-1}}$$ (3) (2)

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9, $\boxed{2+6=8}$
$$\mathcal{F}[g'](\omega)=\int_{x=-\infty}^{+\infty} e^{-i\omega x} g'(x)\,dx=\underbrace{\left[e^{-i\omega x} g(x)\right]_{-\infty}^{\infty}}_{=0}-\int_{-\infty}^{\infty}(-i\omega)\,e^{-i\omega x} g(x)\,dx$$
$$=\underline{\underline{i\omega\,\mathcal{F}[g](\omega)}}$$