1, $\boxed{14}$ a, $\boxed{6}$ $u(x,y) = g(x^2y)$

$u'_x(x,y) = 2xy\,g'(x^2y)$ (1) $\qquad u'_y(x,y) = x^2 g'(x^2y)$ (1)

$u''_{xx}(x,y) = 2y\,g'(x^2y) + 4x^2y^2 g''(x^2y)$ (1) ; $u''_{yy}(x,y) = x^4 g''(x^2y)$ (1)

$u''_{xy}(x,y) \overset{(1)}{=} u''_{yx}(x,y) = 2x\,g'(x^2y) + 2x^3y\,g''(x^2y)$ (1)

$\boxed{8}$ b, $u(x,y) = e^{x^2y}$; $u(1,2) = e^2$ (2)

$u'_x(x,y) = 2xy\,e^{x^2y}$; $u'_x(1,2) = 4e^2$ (2)

$u'_y(x,y) = x^2 e^{x^2y}$; $u'_y(1,2) = e^2$ (2)

Érintő sík: $z = u(1,2) + u'_x(1,2)(x-1) + u'_y(1,2)(y-2) =$

$\underline{\underline{z = e^2 + 4e^2(x-1) + e^2(y-2)}}$ (2)

2, $\boxed{6}$ $\dfrac{df(0,0)}{d\underline{v}} = \lim\limits_{t\to 0+} \dfrac{1}{t}\left(f(\underline{v}) - f(\underline{0})\right) \overset{(2)}{=} \lim\limits_{t\to 0+} \dfrac{1}{t}\sqrt{\left(\frac{3}{5}t\right)^2 + 3\cdot\left(\frac{4}{5}t\right)^4} \overset{(2)}{=}$

$= \lim\limits_{t\to 0+} \sqrt{\left(\frac{3}{5}\right)^2 + 3\cdot\left(\frac{4}{5}\right)^4 t^2} = \underline{\underline{\frac{3}{5}}}$ (2)

3, $\boxed{10}$ $f(x,y) = x^3 + y^3 - 3xy$

$f'_x(x,y) = 3x^2 - 3y \overset{(1)}{=} 0 \implies y = x^2$

$f'_y(x,y) = 3y^2 - 3x \overset{(1)}{=} 0$

$\implies 3x^4 - 3x = 0$

$x(x^3-1) = x(x-1)(\underbrace{x^2+x+1}_{>0}) = 0$

Vizsgálandó pontok:

(4) $A(0,0)$; $H(0,0) = -9 < 0 \implies$ A nyeregpont (1)

$B(1,1)$; $H(1,1) = 36 - 9 = 27 > 0$; $f''_{xx}(1,1) = 6 > 0 \implies$ B-ben lokális minimum van (1)

$H = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = 36xy - 9$ (2)

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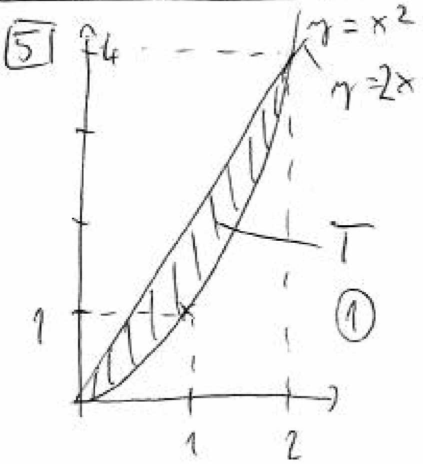
4, $\boxed{10}$ a, $\boxed{5}$



$$I = \int_{x=0}^{2} \left(\int_{y=x^2}^{2x} f(x,y)\, dy \right) dx = \quad ②$$

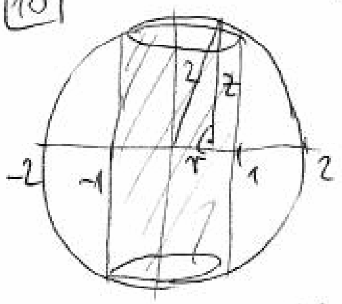
$$= \int_{y=0}^{4} \left(\int_{x=y/2}^{\sqrt{y}} f(x,y)\, dx \right) dy \quad ②$$

b, $\boxed{5}$

$$I = \int_{x=0}^{2} \left(\int_{y=x^2}^{2x} (x^2+y)\, dy \right) dx \overset{①}{=} \int_{x=0}^{2} \left[x^2 y + \frac{y^2}{2} \right]_{y=x^2}^{2x} dx =$$

$$= \int_{x=0}^{2} \left(2x^3 + 2x^2 - x^4 - \frac{x^4}{2} \right) dx \overset{②}{=} \left[\frac{2x^4}{4} + \frac{2x^3}{3} - \frac{3}{2}\,\frac{x^5}{5} \right]_{x=0}^{2} = \underline{\underline{8 + \frac{16}{3} - \frac{48}{5}}} \quad ②$$

5, $\boxed{10}$



Henger koordinátákkal dolgozunk.

A kivágott rész:

$$\left. \begin{array}{c} 0 \le r \le 1 \\ 0 \le \varphi \le 2\pi \\ -\sqrt{2^2 - r^2} \le z \le +\sqrt{2^2 - r^2} \end{array} \right\} ③$$

$$V = \int_{r=0}^{1} \int_{\varphi=0}^{2\pi} \int_{z=-\sqrt{4-r^2}}^{\sqrt{4-r^2}} 1 \cdot r \; dz\, d\varphi\, dr = \quad ②$$

$$= \underset{\int_{\varphi=0}^{2\pi} d\varphi}{\overset{①}{2\pi}} \cdot \int_{r=0}^{1} r \cdot 2\sqrt{4-r^2}\, dr \overset{①}{=} 2\pi \left[-\frac{(4-r^2)^{3/2}}{3/2} \right]_{r=0}^{1} \overset{②}{=}$$

$g' \cdot g^{1/2}$ alak

$$= 2\pi \cdot \frac{2}{3} \left(-3^{3/2} + 4^{3/2} \right) = \underline{\underline{\frac{4\pi}{3} \left(8 - 3^{3/2} \right)}} \quad ①$$