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FIZIKA 3

6 EA

$k = \frac{2E}{\hbar \omega}$ $\quad \xi = x\sqrt{\frac{m\omega}{\hbar}}$ $\quad \varphi_a = e^{-\frac{\xi^2}{2}}$

Teljes megoldás:

$$\varphi_{telj.} = e^{-\frac{\xi^2}{2}} \, v(\xi) \quad ; \quad v(\xi) = ?$$

$$\frac{d^2\varphi(\xi)}{d\xi^2} + (k - \xi^2)\,\varphi(\xi) = 0$$

$$\varphi' = \varphi_a' v + v' \varphi_a$$

$$\varphi'' = \varphi_a'' v + 2\,\varphi_a' v' + \varphi_a v''$$

$$\varphi_a'' = (\xi^2 - 1)\,\varphi_a$$

$$\varphi_a'' v + 2\varphi_a' v' + \varphi_a v'' + k\varphi_a v - \xi^2 \varphi_a v = 0$$

$$\left(-e^{-\frac{\xi^2}{2}} + \xi^2 e^{-\frac{\xi^2}{2}} + k e^{-\frac{\xi^2}{2}} - \xi^2 e^{-\frac{\xi^2}{2}}\right) v -$$

$$-2\xi e^{-\frac{\xi^2}{2}} \cdot v' + e^{-\frac{\xi^2}{2}} \cdot v'' = 0$$

$$\boxed{v'' - 2\xi v' + (k-1)\,v = 0}$$

$$v(\xi) = \sum_{r=0}^{n} c_r \, \xi^r$$

$$v' = \sum_{r=0}^{n} r \, c_r \, \xi^{(r-1)} \qquad v'' = \sum_{r=0}^{n} r(r-1) \, c_r \, \xi^{(r-2)}$$

trafó: $r \to r+2$

$$v'' = \sum (r+2)(r+1) \, c_{r+2} \, \xi^r$$

$$\sum_r \underbrace{\left\{ (r+2)(r+1) \, c_{r+2} - \left[2r - (k+1) \right] c_r \right\}}_{0 \text{ kell hogy legyen}} \xi^r = 0$$

$$\boxed{c_{r+2} = \frac{2r+1-k}{(r+2)(r+1)} \, c_r}$$

ha $c_0 \neq 0 \to c_1 = 0$

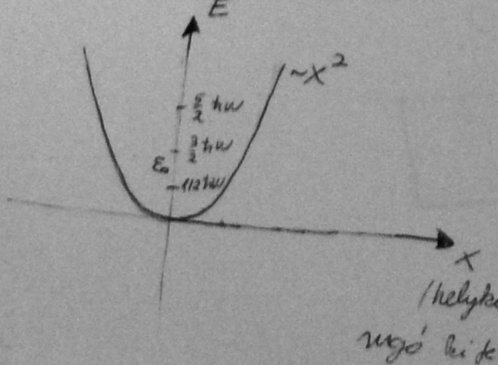
ha $c_1 \neq 0 \to c_0 = 0$

$$2r+1 = k = \frac{2E}{\hbar \omega}$$

$$\boxed{E_n = \hbar \omega \left(n + \frac{1}{2} \right)} \qquad n = 0, 1, 2 \ldots$$

energia nem folytonos

$$\boxed{E_0 = \frac{1}{2} \hbar \omega}$$ zérusponti energia



(helykoordináta)

rugó kitérése

Speciális fv. - Hermite-féle polinom

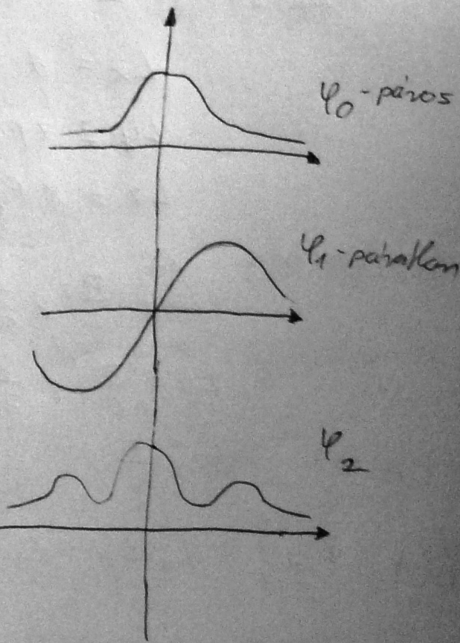
$$\boxed{\frac{d^2H}{d\xi^2} - 2\xi\frac{dH}{d\xi} + 2nH = 0}$$

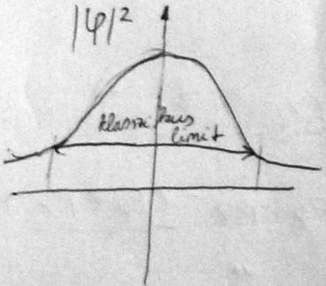
$$\varphi_n = e^{-\frac{\xi^2}{2}} H_n(\xi)$$

1. $\varphi_0 = \left(\frac{\alpha}{\pi}\right)^{1/4} e^{-\frac{\alpha}{2}x^2}$

2. $\varphi_1 = 2\left(\frac{\alpha^3}{4\pi}\right)^{1/4} x\, e^{-\frac{\alpha x^2}{2}}$

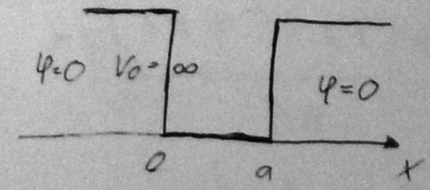
3. $\varphi_2 = \left(\frac{\alpha}{4\pi}\right)^{1/4}(1-2\alpha x^2)\, e^{-\frac{\alpha x^2}{2}}$





Példa 3 potenciálgödör

potenciálugrás végtelen nagy



Schröd.

$$\frac{d^2\varphi(x)}{dx^2} + \frac{2m}{\hbar^2}E\,\varphi(x) = 0 \qquad 0 \le x \le a$$

$$\varphi(0) = \varphi(a) = 0$$

$\varphi(x) = A\sin(kx)$ $\qquad k^2 = \frac{2m}{\hbar^2}E \qquad k_n = n\frac{\pi}{a}$

$$\boxed{E_n = \frac{\hbar^2}{2m}\frac{\pi^2 n^2}{a^2}}$$

n - kvantumszám

$n = 1, 2, 3 \ldots$

~~[illegible]~~

3D-re áttérünk

impulzusmomentum

helyvektor, impulzusvektor

klasszikusan: $\bar{L} = \bar{r} \times \bar{p}$

$$L_x = y\,p_z - z\,p_y$$

$$L_y = z\,p_x - x\,p_z$$

$$L_z = x\,p_y - y\,p_x$$

kvantumosan:

$$\hat{L}_x = \frac{\hbar}{i}\left(y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y}\right)$$

$$\hat{L}_y = \frac{\hbar}{i}\left(z\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) \qquad \text{operátorok}$$

$$\hat{L}_z = \frac{\hbar}{i}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

$$[L_x ; L_y] = L_x L_y - L_y L_x = (y p_z - z p_y)(z p_x - x p_z) -$$

$$-(z p_x - x p_z)(y p_z - z p_y) = y p_z z p_x - y p_z x p_z - z p_y z p_x +$$

$$+ z p_y x p_z - z p_x y p_z + z p_x z p_y + x p_z y p_z - x p_z z p_y =$$

⁚ - esik; nem esik ki

$$= y p_x \underbrace{(p_z z - z p_z)}_{\hbar/i} - x p_y \underbrace{(p_z z - z p_z)}_{\hbar/i} =$$

$$= -\frac{\hbar}{i}(x p_y - y p_x) = -\frac{\hbar}{i} L_z = i\hbar\, L_z$$

$$[p_x\, x] = \frac{\hbar}{i} \qquad [p_x ; p_y] = 0$$

$$[p_x , y] = 0$$

összefoglalva:

$$[L_x; L_y] = i\hbar L_z$$

$$[L_y; L_z] = i\hbar L_x$$

$$[L_z, L_x] = i\hbar L_y$$

$$\hat{L} = ? \qquad \hat{L}^2 = \hat{L}_x^2 + \hat{L}_y^2 + \hat{L}_z^2$$

$$[L^2; L_z] = L^2 L_z - L_z L^2 = (+\ +) L_z - L_z (++) =$$

$$= L_x^2 L_z - L_z L_x^2 + L_y^2 L_z - L_z L_y^2 \pm L_x L_z L_x \pm L_y L_z L_y =$$

$$= L_x (L_x L_z - L_z L_x) + (L_x L_z - L_z L_x) L_x + L_y (L_y L_z - L_z L_y) +$$

$$+ (L_y L_z - L_z L_y) L_y = 0$$