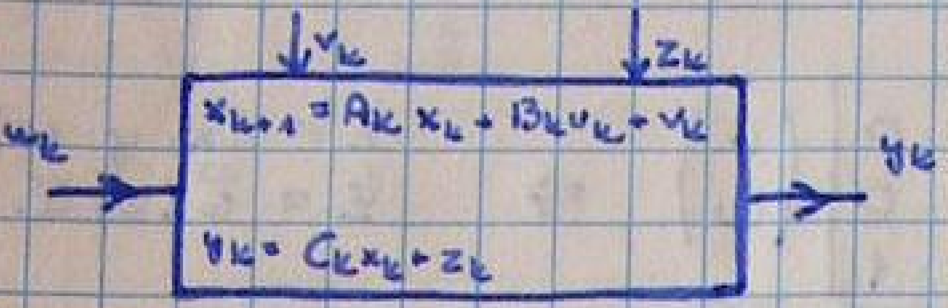
2008.04.08.

Kalman-szűrő



$x(0) \rightarrow E x(0) = x_0$,

$\Sigma_0 = E[(x(0) - x0)(x(0) - x0)^T]$

$E v_k = 0$

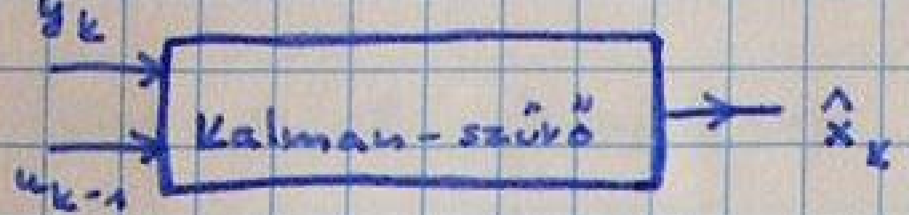
$E[v_k v_l^T] = R_{v,k} \delta_{k,l}$

$E z_k = 0$

$E[z_k z_l^T] = R_{z,k} \delta_{k,l}$

$E v_l z_l^T = 0$

$x(0)$ nem korrelál z_k, v_k



$\hat{x}_k = F_k \hat{x}_{k-1} + G_k y_k + H_{k-1} u_{k-1}$

$\hat{x}_0 = E x(0) = x0$

(1) $E(x_k - \hat{x}_k) = 0, \ \forall k$

(2) $\Sigma_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] \rightarrow \inf$

$x_{k+1} = A_k x_k + B_k u_k + v_k$

$\hat{x}_{k+1} = F_k \hat{x}_k + G_{k+1}[(A_k x_k + B_k u_k + v_k) + z_{k+1}] + H_k u_k$

$\boxed{H_k = B_k - G_{k+1} C_{k+1} B_k}$

$\boxed{F_k = (I - G_{k+1} C_{k+1}) A_k}$

$$x_{k+1} - \hat{x}_{k+1} = (I - G_{k+1} C_{k+1}) A_k x_k + (B_k - G_{k+1} C_{k+1} B_k - H_k) u_k + \\ + (I - G_{k+1} C_{k+1}) v_k - G_{k+1} z_{k+1} - F_k \hat{x}_k$$

$$x_{k+1} - \hat{x}_{k+1} = (I - G_{k+1} C_{k+1})[A_k x_k + B_k u_k + v_k - A_k \hat{x}_k - B_k u_k] - G_{k+1} z_{k+1}$$

$\boxed{\bar{x}_{k+1} = A_k \hat{x}_k + B_k u_k}$

$x_{k+1} - \hat{x}_{k+1} = (I - G_{k+1} C_{k+1})(x_{k+1} - \bar{x}_{k+1}) - G_{k+1} z_{k+1}$

$E[(x_{k+1} - \bar{x}_{k+1})(x_{k+1} - \bar{x}_{k+1})^T] = \Pi_{k+1}$

$\Sigma_{k+1} = (I - G_{k+1} C_{k+1}) \Pi_{k+1} (I - G_{k+1} C_{k+1})^T + G_{k+1} R_{z,k+1} G_{k+1}^T \rightarrow \inf$

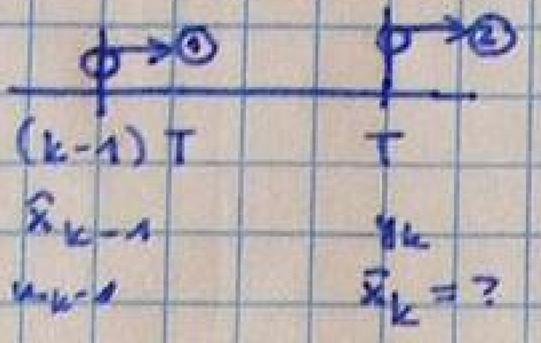
$-(I - G_{k+1} C_{k+1}) \Pi_{k+1} C_{k+1}^T + G_{k+1} R_{z,k+1} = 0$

$\hookrightarrow \boxed{G_{k+1} = \Pi_{k+1} C_{k+1}^T (C_{k+1} \Pi_{k+1} C_{k+1}^T + R_{z,k+1})^{-1}}$

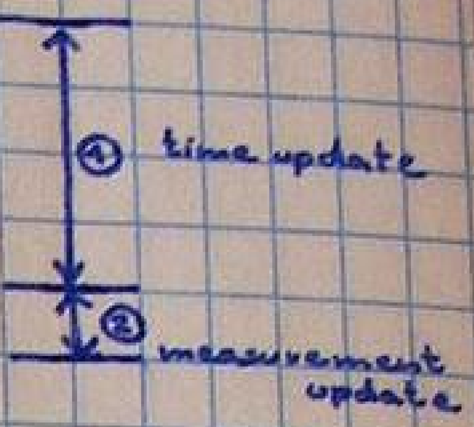
Aktuális Kalman-szűrő algoritmus

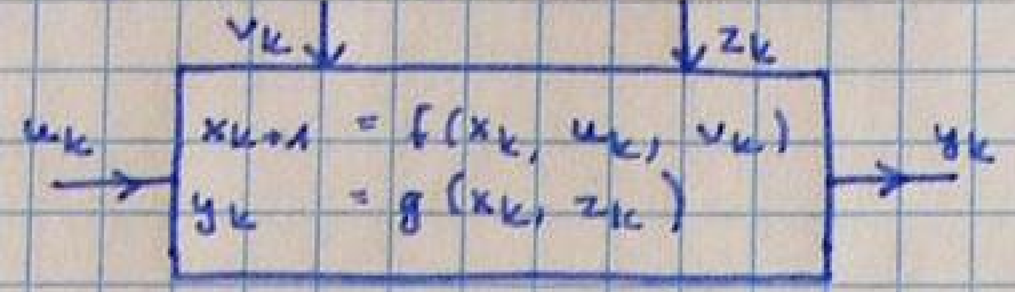
① Inic.: $\hat{x}_0 = Ex(0) = x_0, \Sigma_0$

② Elöretö rekurzió



$$\bar{x}_k = A_{k-1}\hat{x}_{k-1} + B_{k-1}u_{k-1}$$
$$\Pi_k = A_{k-1}\Sigma_{k-1}A_{k-1}^T + R_{v,k-1}$$
$$\Sigma_k = \Pi_k - \Pi_k C_k^T (C_k\Pi_k C_k^T + R_{z,k}) C_k\Pi_k$$
$$G_k = \Pi_k C_k^T (C_k\Pi_k C_k^T + R_{z,k})^{-1}$$
$$\hat{x}_k = \bar{x}_k + G_k(y_k - C_k\bar{x}_k)$$





$$\hat{x}_{k-1}, v_{k-1} = 0 \longrightarrow f(\hat{x}_{k-1}, u_k, 0) + \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial x}(x_k - \hat{x}_{k-1}) + \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial v} v_k,$$

$$A_{k-1} = \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, u)}{\partial x}, \quad B_{v,k-1} = \frac{\partial f(\hat{x}_{k-1}, u_{k-1}, 0)}{\partial v}$$

$$\bar{x}_k, z_k = 0 \longrightarrow g(\bar{x}_k, 0) + \frac{\partial g(\bar{x}_k, 0)}{\partial x}(x_k - \bar{x}_k) + \frac{\partial g(\bar{x}_k, 0)}{\partial z} z_k,$$

$$C_k = \frac{\partial g(\bar{x}_k, 0)}{\partial x}, \quad C_{z,k} = \frac{\partial g(\bar{x}_k, 0)}{\partial z}$$

$$E[B_{v,k-1}v_{k-1}v_{k-1}^T B_{v,k-1}^T] = B_{v,k-1}R_{v,k-1}B_{v,k-1}^T =: R_{v,k-1}$$

$$E[C_{z,k}\, z_k z_k^T C_{z,k}^T] = C_{z,k}R_{zk}, C_{z,k}^T =: R_{z,k}$$

EKF-szűrő:

Inic.: $\hat{x}_0 = Ex(0) = x_0, \Sigma_0$

Elöretartó rekurzió:

$\bar{x}_k = f(\hat{x}_{k-1}, u_{k-1}, 0)$
Nemlineáris rendszer linearizálása: A_{k-1}; B_{k-1}; C_k; $C_{z,k}$; kovr. $R_{v,k-1}$; $R_{z,k}$

② $\hat{x}_k = \bar{x}_k + G_k(y_k - g(\bar{x}_k, 0))$