BIBO stability cond.: DT $h[k]$ / CT $h(t)$ is absolute summable / integrable

DT $|\lambda_i| < 1$
CT $\mathrm{Re}\,\lambda_i < 0$

State variables: $\underline{x}[k+1] = \underline{\underline{A}}\,\underline{x}[k] + \underline{B}\cdot u[k]$ $\quad \dfrac{d\underline{x}(t)}{dt} = \underline{\underline{A}}\,\underline{x}(t) + \underline{B}\cdot u(t)$

$y[k] = \underline{C}^T\cdot \underline{x}[k] + D\cdot u[k]$ $\quad y(t) = \underline{C}^T\cdot\underline{x}(t) + D\cdot u(t)$

$\boxed{DT}$

Response calculation:

1, Step-by-step

if the input signal is causal $\underline{x}(0) = \underline{0}$

2, Decomposition to free and generated components $y[k] = y_f[k] + y_g[k]$

2.1, Determine the eigenvalues $\det(\underline{\underline{A}} - \lambda\underline{\underline{I}}) = 0 \Rightarrow \lambda_i$

$y_f[k] = M_1\cdot\lambda_1^k + M_2\cdot\lambda_2^k + \dots + M_N\cdot\lambda_N^k$

2.2, Generated component

$y_g[k] = \underline{C}^T\cdot\underline{x}_g[k] + D\,u[k]$ $\quad \underline{x}_g[k] = \begin{bmatrix} A_1 \\ A_1 \\ \vdots \\ A_N \end{bmatrix}\cdot u[k]$ generated comp has the same form that of the input

constants to be found

$\underline{x}_g[k+1] = \underline{\underline{A}}\,\underline{x}_g[k] + \underline{B}\,u[k]$

from this we can calculate $A_1 .. A_N$

$y_g[k]$ we get (it must be similar to the input)

2.3, Matching the values for $k=0$ and $k=1$ to get $M_1 .. M_N$

For $y[0]$ and $y[1]$ values we need the table

$x[k] = \sum_{p=1}^{N} C_p\,\underline{m}_p\,\lambda_p^k + \underline{x}_g[k]$ where $\underline{m}_i$ are eigenvectors of $\underline{\underline{A}}$

$\underline{\underline{A}}\cdot\underline{m}_i = \lambda_i\cdot\underline{m}_i$

3, Matrix powers (MIMO)

$\underline{x}[k] = \underline{\underline{A}}^k\cdot\underline{x}[0] + \sum_{p=0}^{k-1}\underline{\underline{A}}^p\cdot\underline{\underline{B}}\cdot\underline{u}[k-1-p]$ if $k \geq 1$

$y[k] = \begin{cases} \underline{\underline{C}}\cdot\underline{\underline{A}}^k\cdot\underline{x}[0] + \sum_{0}^{k-1}\underline{\underline{C}}\cdot\underline{\underline{A}}^p\cdot\underline{\underline{B}}\cdot\underline{u}[k-1-p] + \underline{\underline{D}}\,\underline{u}[k] & \text{if } k\geq 1 \\ \underline{\underline{C}}\,\underline{x}[0] + \underline{\underline{D}}\,\underline{u}[0] & \text{if } k=0 \end{cases}$

Impulse response calculation:

$h[k] = y[k]$ if $u[k] = \delta[k]$

1, Steps: 1.1, Determine the eigenvalues of $\underline{\underline{A}} \Rightarrow y_f[k] = M_1\cdot\lambda_1^k + \dots + M_N\cdot\lambda_N^k$

1.2, $y_g[k] = 0$ if $k\geq 1$ $\quad h[k] = M_1'\cdot\lambda_1^k + \dots + M_N'\cdot\lambda_N^k = M_1\cdot\lambda_1^{k-1} + \dots + M_2\cdot\lambda_N^{k-1}$

1.3, Matching for $k=1$ and $k=2$

We calculate the table and we get M_i values

$h[k] = D\cdot\delta[k] + \varepsilon[k-1]\left(M_1\lambda_1^{k-1} + \dots + M_N\cdot\lambda_N^{k-1}\right)$

2, Matrix powers (SISO system)

$h[k] = \begin{cases} D & \text{if } k=0 \quad (u[0]=1) \\ \underline{C}^T\cdot\underline{\underline{A}}^{k-1}\cdot\underline{B} & \text{if } k\geq 1 \end{cases}$

$h[k] = D\cdot\delta[k] + \varepsilon[k-1]\cdot\underline{C}^T\cdot\underline{\underline{A}}^{k-1}\cdot\underline{B}$

$A^k = \sum_{p=1}^{N}\underline{\underline{L}}_p\cdot\lambda_p^k$

↳ Lagrange matrix $\quad \underline{\underline{L}}_p = \prod_{\substack{i=1 \\ i\neq p}}^{N}\dfrac{1}{\lambda_p - \lambda_i}(\underline{\underline{A}} - \lambda_i\underline{\underline{I}}) \quad \left(\sum_{p=1}^{N}\underline{\underline{L}}_p = \underline{\underline{I}}\right)$

Steps: 2.1, Eigenvalues

2.2, Lagrange matrices

2.3, $\underline{\underline{A}}^{k-1} = \underline{\underline{L}}_1\cdot\lambda_1^{k-1} + \underline{\underline{L}}_2\cdot\lambda_2^{k-1}$ (calculate $\underline{C}^T\cdot\underline{\underline{L}}_1\underline{B}$, $\underline{C}^T\cdot\underline{\underline{L}}_2\cdot\underline{B}$)

2.4, $h[k] = D\cdot\delta[k] + \varepsilon[k-1]\cdot\underline{C}^T\cdot\underline{\underline{A}}^{k-1}\cdot\underline{B}$

$\boxed{CT}$

Response calculation:

1, Numerical calculation with Euler-method

$x(\Delta t) \cong x(+0) + \underbrace{x'(\Delta t)}_{\underline{\underline{A}}\,\underline{x}(+0) + \underline{B}\cdot u(+0)} \cdot \Delta t$

$y(\Delta t) = \underline{C}^T \cdot \underline{x}(\Delta t) + D \cdot u(\Delta t)$

Δt is given, so the values can be found step-by-step

$\underline{x}(-0) = \underline{0}$ $\quad \underline{x}(+0) = \underline{x}(-0)$ if $u(t)$ is causal

$y(+0) = \underline{C}^T \cdot \underline{x}(+0) + D \cdot u(+0)$

2, Decomposition to free and generated components

$\underline{x}(t) = \sum_{p=1}^{N} C_p \cdot \underline{m}_p \cdot e^{\lambda_p t} + \underline{x}_g(t)$ $\qquad \underline{m}_p, \lambda_p$: eigenvs of $\underline{\underline{A}}$

$\hookrightarrow \underline{\underline{A}}\,\underline{x}_g(t) + \underline{B}\,u(t)$ $\qquad \underline{\underline{A}} \cdot \underline{m}_p = \lambda_p \cdot \underline{m}_p$

Steps: 2.1, Find λ_p, $\underline{m}_p$ eigenvs

$\underline{m} = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}$ from $\underline{\underline{A}}\,\underline{m} = \lambda\,\underline{m}$ we find $m_2 = (n+\lambda)\cdot m_1$ therefore m_1 can be chosen $m_1 = 1$

$\underline{m}_1 = \begin{bmatrix} m_1 \\ (n+\lambda_1) m_1 \end{bmatrix}$

$\underline{x}_f(t) = C_1 \cdot \underline{m}_1 \cdot e^{\lambda_1 t} + C_2\,\underline{m}_2 \cdot e^{\lambda_2 t}$

$\underline{m}_2 = \begin{bmatrix} m_1 \\ (n+\lambda_2) m_1 \end{bmatrix}$

2.2, Find the generated comp. $\underline{x}_g = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$ from the given equations

2.3, Find C_1 and C_2 $\qquad C_1 \cdot \underline{m}_1 + C_2 \cdot \underline{m}_2 + \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = \underline{x}(+0)$ (can be $\underline{0}$ or $\underline{B}$)

2.4, $y(t) = \underline{C}^T \underline{x}(t) + D \cdot u(t)$

Impulse response calculation:

1, Matrix functions

$h(t) = \varepsilon(t) \cdot \underline{C}^T \cdot e^{\underline{\underline{A}}t}\,\underline{B} + D\delta(t)$

$\hookrightarrow e^{\underline{\underline{A}}t} = \sum_{p=1}^{N} \underline{\underline{L}}_p \cdot e^{\lambda_p t}$

Asymptotic stability:

DT: $|\lambda_i| < 1$

CT: $\mathrm{Re}\,\lambda_i < 0$ $\qquad$ characteristic polynom: $\lambda^N + a_1 \cdot \lambda^{N-1} + a_2 \cdot \lambda^{N-2} + \ldots + a_N$

Jury criterion: 1, $P(\lambda=1) > 0$ $\qquad$ Hurwitz criterion: $a_1 \ldots a_N > 0$

2, $P(\lambda=-1) > 0$

3, $|a_N| < 1$