1, a, [7]
$$\sum_{n=1}^{\infty}\underbrace{\frac{(-5)^n}{\sqrt{n}}}_{a_n}\cdot x^n \; ; \; \sqrt[n]{|a_n|}=\frac{5}{\sqrt[n]{\sqrt{n}}}\longrightarrow 5 \Rightarrow \underline{R=\frac{1}{5}} \quad 3$$

Végpontok: $x=\frac{1}{5}$; $\sum_{n=1}^{\infty}\frac{(-5)^n}{\sqrt{n}}\left(\frac{1}{5}\right)^n=\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ $\underline{\text{konvergens}}$, mert Leibniz 2

$x=\frac{-1}{5}$; $\sum_{n=1}^{\infty}\frac{(-5)^n}{\sqrt{n}}\left(\frac{-1}{5}\right)^n=\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}}$ $\underline{\text{divergens}}$ $(\alpha=\frac{1}{2}<1)$ 2

Tehát a konvergencia-tartomány: $\underline{\underline{KT=\left(-\frac{1}{5},\frac{1}{5}\right]}}$

[7] b,
$$\sum_{n=1}^{\infty}\underbrace{\frac{(-5)^n}{n\sqrt{n}}}_{a_n}(x+3)^n \; ; \; \sqrt[n]{|a_n|}=\frac{5}{\sqrt[n]{n^{3/2}}}\longrightarrow 5 \Rightarrow \underline{R=\frac{1}{5}} \quad 3$$

A végpontokban abszolút konvergens, tehát konvergens a sor:
$$\sum_{n=1}^{\infty}\left|\frac{(-5)^n}{n\sqrt{n}}\left(\pm\frac{1}{5}\right)^n\right|=\sum_{n=1}^{\infty}\frac{1}{n^{3/2}}<\infty$$ $\underline{\text{konvergens}}$, mert $\alpha=\frac{3}{2}>1$. 3

$K.T.=\left[-3-\frac{1}{5},-3+\frac{1}{5}\right]=\underline{\underline{\left[-\frac{16}{5},-\frac{14}{5}\right]}}$ 1

2, a, [6]
$$f(x)=\frac{1}{3+7x}=\frac{1}{3}\,\frac{1}{1-\left(\frac{-7}{3}\right)x}=\underline{\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{-7}{3}\right)^n x^n} \quad 4$$ ha $\left|\frac{-7}{3}x\right|<1$, azaz $\underline{|x|<\frac{3}{7}}$ 2

[6] b,
$$g(x)=f(x^2)=\underline{\frac{1}{3}\sum_{n=0}^{\infty}\left(\frac{-7}{3}\right)^n x^{2n}} \quad 4$$ ha $|x^2|<\frac{3}{7}$, azaz $\underline{|x|<\sqrt{\frac{3}{7}}}$ 2

[8] c,
$$h(x)=\frac{1}{(3+7x)^2}=\frac{-1}{7}\cdot f'(x)=\underline{\frac{-1}{21}\sum_{n=1}^{\infty}\left(\frac{-7}{3}\right)^n\cdot n\,x^{n-1}} \quad 5 \; ; \; R=\frac{3}{7} \quad 1$$

nem változik

$x=\pm\frac{3}{7}$: $\lim\left(\frac{-7}{3}\right)^n n\left(\pm\frac{3}{7}\right)^{n-1}\underset{n\to\infty}{\not\longrightarrow}0$, tehát a határokon a sor divergens. $\underline{K.T.=\left(-\frac{3}{7},+\frac{3}{7}\right)}$ 2

3, $\boxed{15}$ $f(x)=\sum_{n=0}^{\infty}(2n+1)x^{2n} \underset{x^2=y}{=} \sum_{n=0}^{\infty}\underbrace{(2n+1)}_{a_n}y^n;$

$$\left|\frac{a_{n+1}}{a_n}\right|=\left|\frac{2n+3}{2n+1}\right| \xrightarrow{n\to\infty} 1 \Longrightarrow R_y=1 \Longrightarrow \underline{\underline{R_x=\sqrt{1}=1}}$$ (3)

$$\int_{t=0}^{x} f(t)\,dt=\int_{t=0}^{x}\left(\sum_{n=0}^{\infty}(2n+1)t^{2n}\right)dt=\sum_{n=0}^{\infty}\int_{t=0}^{x}(2n+1)t^{2n}\,dt=$$

$$=\sum_{n=0}^{\infty}\left[t^{2n+1}\right]_0^x=\sum_{n=0}^{\infty}x^{2n+1}=x\cdot\sum_{n=0}^{\infty}(x^2)^n=\frac{x}{1-x^2},$$ (9) ha $|x^2|<1$, $|x|<1$

$$f(x)=\left(\frac{x}{1-x^2}\right)'=\frac{1-x^2-x(-2x)}{(1-x^2)^2}=\underline{\underline{\frac{1+x^2}{(1-x^2)^2}}}$$ (3)

— o —

4, $\boxed{8}$ $f(x)=(x-2)^2+\sin(2x)=2-4x+x^2+2x-\underbrace{\frac{1}{3!}(2x)^3}_{\frac{4}{3}x^3}+\frac{1}{4!}f^{(4)}(\xi)=$

$$=\underline{\underline{2-2x+x^2-\frac{4}{3}x^3+\underbrace{\frac{16}{24}}_{2/3}\sin(2\xi)}}$$ (5) (3)

$f^{(4)}(x)=2^4\sin(2x)=16\sin(2x)$ $\qquad |\xi|<|x|$

— o —

5,

$$f(x,y)=\begin{cases} y\cos\left(\frac{1}{x^2+y^2}\right), & \text{ha } (x,y)\neq(0,0) \\ 0 & \text{ha } (x,y)=(0,0)\end{cases}$$

a, $\boxed{4}$ $\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{(x,y)\to(0,0)} \underset{\downarrow\,0}{y}\,\underbrace{\cos\left(\frac{1}{x^2+y^2}\right)}_{\text{korlátos}}=0=f(0,0)$

<u>folytonos az origóban!</u>

b, $(x,y) \neq (0,0)$

$\boxed{6}$ $f'_x(x,y) = -y \sin\left(\frac{1}{x^2+y^2}\right) \cdot \frac{-2x}{(x^2+y^2)^2}$ ③

$f'_y(x,y) = \cos\left(\frac{1}{x^2+y^2}\right) - y \sin\left(\frac{1}{x^2+y^2}\right) \cdot \frac{-2y}{(x^2+y^2)^2}$ ③

c, $f'_x(0,0) = \lim_{x\to 0} \frac{f(x,0)-f(0,0)}{x} = \lim_{x\to 0} \frac{0-0}{x} = 0$ ③

$\boxed{10}$

$f'_y(0,0) = \lim_{y\to 0} \frac{f(0,y)-f(0,0)}{y} = \lim_{y\to 0} \cos\frac{1}{y^2} = \nexists$, mert

ha $\frac{1}{y_n^2} = 2n\pi$; $y_n = \frac{1}{\sqrt{2n\pi}} \xrightarrow{n\to\infty} 0$, akkor $\cos\frac{1}{y_n^2} = 1$

$\nparallel$

ha $\frac{1}{z_n^2} = 2n\pi + \frac{\pi}{2}$; $z_n = \frac{1}{\sqrt{2n\pi+\frac{\pi}{2}}} \xrightarrow{n\to\infty} 0$, akkor $\cos\frac{1}{z_n^2} = 0$ ⑦

Ellentmond az átviteli elvnek!

— o —

6, $f(x,y) = g(3xy+y^2)$

$\boxed{11}$ $f'_x(x,y) = g'(3xy+y^2)\cdot 3y$ ②; $f'_y(x,y) = g'(3xy+y^2)\cdot(3x+2y)$ ②

$f''_{xx}(x,y) = (3y)^2 \cdot g''(3xy+y^2)$ ②

$f''_{xy}(x,y) \underset{\uparrow}{=} f''_{yx}(x,y) = 3\cdot g'(3xy+y^2) + 3y\cdot g''(3xy+y^2)(3x+2y)$ ②

Young-tétel ①

$f''_{yy}(x,y) = 2g'(3xy+y^2) + (3x+2y)^2\cdot g''(3xy+y^2)$ ②

7,
$\boxed{12}$

$$f(x) = \sqrt[3]{27+x^2} = 3\left(1+\frac{x^2}{27}\right)^{1/3} = 3\sum_{n=0}^{\infty}\binom{1/3}{n}\left(\frac{x^2}{27}\right)^n$$ (6)

$$T_7(x) = 3\binom{1/3}{0} + 3\binom{1/3}{1}\frac{x^2}{27} + 3\binom{1/3}{2}\left(\frac{x^2}{27}\right)^2 + 3\binom{1/3}{3}\left(\frac{x^2}{27}\right)^3 =$$

$$= 3 + \frac{x^2}{27} + 3\cdot\frac{(1/3)\cdot(-2/3)}{2}\cdot\frac{x^4}{27^2} + 3\,\frac{(1/3)(-2/3)(-5/3)}{3\cdot 2}\cdot\frac{x^6}{27^3}$$

(6)