Valószám pótzh

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1) A: a páciens tbc-s

B: -"- t " -nek gondolják

1.v. $P(A) = 0{,}0003$ (1)

$P(\bar{A}) = 0{,}9997$

$P(B|A) = 0{,}99$ (1)

$P(B|\bar{A}) = 0{,}002$ (1)

2.v. $P(A) = 0{,}0001$

$P(\bar{A}) = 0{,}9999$

$P(B|A) = 0{,}95$

$P(B|\bar{A}) = 0{,}001$

A keresett vg: $P(\bar{A}|B)$. (1)

$$P(\bar{A}|B) = \frac{P(\bar{A} \cdot B)}{P(B)} = \frac{P(B|\bar{A}) \cdot P(\bar{A})}{P(B)} \quad (2)$$

$$P(B) = P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A}) \quad (2)$$

teljes vg. tétele alapján

$P(B) = 0{,}99 \cdot 0{,}0003 + 0{,}002 \cdot 0{,}9997$

$= 0{,}0049694$ (1)

$P(\bar{A}|B) = 0{,}402$ (1)

$P(B) = 0{,}95 \cdot 0{,}0001 + 0{,}001 \cdot 0{,}9999$

$= 0{,}0010949$

$P(\bar{A}|B) = 0{,}913$

2) kell: $\int_{-\infty}^{+\infty} f(x)\,dx = 1$ (2)

1.v. $1 = \int_0^2 \alpha(2x - x^2)\,dx =$ (1)

$= \alpha\left[x^2 - \frac{x^3}{3}\right]_0^2 = \alpha \cdot \frac{4}{3}$

$\alpha = \frac{3}{4}$ (1)

2.v. $1 = \int_1^3 \alpha(2x + x^2)\,dx =$

$= \alpha\left[x^2 + \frac{x^3}{3}\right]_1^3 = \alpha \cdot \frac{50}{3}$

$\alpha = \frac{3}{50}$

2) folyt.

$$F(x) = \int_{-\infty}^{x} f(u)\,du \qquad 2$$

1.v $\quad F(x) = \int_0^x \frac{3}{4}(2u - u^2)\,du \qquad 1$

$$= \frac{3}{4}\left[u^2 - \frac{u^3}{3}\right]_0^x = \frac{3}{4}\left(x^2 - \frac{x^3}{3}\right) \qquad 1$$

ha $x \in (0,2)$

$F(x) = 0$ ha $x < 0$ $\qquad 1$

$F(x) = 1$ ha $x > 2$ $\qquad 1$

2.v $\quad F(x) = \int_1^x \frac{3}{50}(2u + u^2)\,du$

$$= \frac{3}{50}\left[u^2 + \frac{u^3}{3}\right]_1^x = \frac{3}{50}\left(x^2 + \frac{x^3}{3} - \frac{4}{3}\right)$$

ha $x \in [1,3)$

$F(x) = 0$ ha $x < 1$

$F(x) = 1$ ha $x > 3$

3) 1.v $\quad f_{Y|X}(y|x) = \dfrac{f_{X,Y}(x,y)}{f_X(x)} \qquad 2$

$$f_X(x) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dy \;\; 2 \;\; = \int_0^1 (x+y)\,dy = \left[\frac{y^2}{2} + yx\right]_0^1 = x + \frac{1}{2} \qquad 1$$

$$f_{Y|X}(y|x) = \frac{x+y}{x+\frac{1}{2}} \qquad 0 < x, y < 1 \qquad 1$$

Fgtlenek, ha $f_X(x)\, f_Y(y) = f_{X,Y}(x,y)$ $\qquad 2$

/vagy $f_{Y|X}(y|x) = f_Y(y)$ /

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx = \ldots = y + \frac{1}{2}$$

$\rightarrow$ nem fgtlenek. $\qquad 2$

2.v. Pontozás ugyanígy $\qquad f_{X|Y}(x|y) = \dfrac{f_{X,Y}(x,y)}{f_Y(y)}$

$$f_Y(y) = \int_{-\infty}^{+\infty} f_{X,Y}(x,y)\,dx = \int_0^1 \frac{12}{11}(x^2 + xy + y^2)\,dx = \frac{12}{11}\left(\frac{1}{3} + \frac{y}{2} + y^2\right)$$

$$f_{X|Y}(x|y) = \frac{x^2 + xy + y^2}{\frac{1}{3} + \frac{y}{2} + y^2} \qquad 0 < x, y < 1$$

$f_X(x) = \frac{1}{3} + \frac{x}{2} + x^2$ $\qquad \rightarrow$ nem fgtlenek.

4) 1. v.

$$Z = 3Y \cdot \frac{1}{1+2X}$$

2 fgtlenek, mert X és Y fgtlen

1 $\Rightarrow$ szorzat várható értéke = várható értékek szorzata

$$EZ = E(3Y)\cdot E\left(\frac{1}{1+2X}\right) \quad 2$$

$$E(3Y) = 3\cdot E(Y) = \frac{3}{2} \quad 1$$

$$E\left(\frac{1}{1+2X}\right) = \int \frac{1}{1+2x}\cdot f_X(x)\,dx \quad 2$$

$$= \int_0^1 \frac{1}{1+2x}\cdot 1\cdot dx \quad 1$$

$$= \left[\frac{1}{2}\cdot \ln(1+2x)\right]_0^1 = \frac{\ln 3}{2} \quad 1$$

$$EZ = \frac{3\ln 3}{4}$$

2. v.

$$Z = 2Y \cdot \frac{1}{1+3X}$$

$$EZ = E(2Y)\cdot E\left(\frac{1}{1+3X}\right)$$

$$E(2Y) = 2\cdot E(Y) = 1$$

$$E\left(\frac{1}{1+3X}\right) = \int \frac{1}{1+3x} f_X(x)\,dx$$

$$= \int_0^1 \frac{1}{1+3x}\cdot 1\cdot dx$$

$$= \left[\frac{1}{3}\ln(1+3x)\right]_0^1 = \frac{\ln 4}{3}$$

$$EZ = \frac{\ln 4}{3}$$

5) 1. v.

Y \ X	0	1	2	3	
0	$\left(\frac{3}{6}\right)^3$	0	0	0	$\frac{27}{216}$
1	$\frac{2}{6}\cdot\left(\frac{3}{6}\right)^2\cdot 3$	$\frac{1}{6}\left(\frac{3}{6}\right)^2\cdot 3$	0	0	$\frac{81}{216}$
2	$\left(\frac{2}{6}\right)^2\cdot\frac{3}{6}\cdot 3$	$\frac{1}{6}\cdot\frac{2}{6}\cdot\frac{3}{6}\cdot 6$	$\left(\frac{1}{6}\right)^2\cdot\frac{3}{6}\cdot 3$	0	$\frac{81}{216}$
3	$\left(\frac{2}{6}\right)^3$	$\frac{1}{6}\left(\frac{2}{6}\right)^2\cdot 3$	$\left(\frac{1}{6}\right)^2\cdot\frac{2}{6}\cdot 3$	$\left(\frac{1}{6}\right)^3$	$\frac{27}{216}$
	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$	

3

2. v.

Y \ X	0	1	2	3	
0	$\left(\frac{2}{6}\right)^3$	$\frac{1}{6}\left(\frac{2}{6}\right)^2\cdot 3$	$\left(\frac{1}{6}\right)^2\cdot\frac{2}{6}\cdot 3$	$\left(\frac{1}{6}\right)^3$	$\frac{27}{216}$
1	$\frac{3}{6}\left(\frac{2}{6}\right)^2\cdot 3$	$\frac{3}{6}\cdot\frac{2}{6}\cdot\frac{1}{6}\cdot 6$	$\left(\frac{1}{6}\right)^2\cdot\frac{3}{6}\cdot 3$	0	$\frac{81}{216}$
2	$\left(\frac{3}{6}\right)^2\cdot\frac{2}{6}\cdot 3$	$\left(\frac{3}{6}\right)^2\cdot\frac{1}{6}\cdot 3$	0	0	$\frac{81}{216}$
3	$\left(\frac{3}{6}\right)^3$	0	0	0	$\frac{27}{216}$
	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$	

Nem fgtlenek, mert pl. 2

$$0 = P(X=3, Y=1) \neq P(X=3)\cdot P(Y=1)$$

$$\mathrm{cov}(X,Y) = E(XY) - EX\cdot EY \quad 2$$

$$EX = \sum_{i=0}^{3} i\,P(X=i) = 0\cdot\frac{125}{216} + 1\cdot\frac{75}{216} + 2\cdot\frac{15}{216} + 3\cdot\frac{1}{216} = \frac{108}{216} = \frac{1}{2}$$

1 0,5

$$EY = 0\cdot\frac{27}{216} + 1\cdot\frac{81}{216} + 2\cdot\frac{81}{216} + 3\cdot\frac{27}{216} = \frac{189}{216}$$

0,5

5. folyt

1. v.

$E(XY) = 1\cdot1\cdot\frac{27}{216} + 1\cdot2\cdot\frac{36}{216} + 2\cdot2\cdot\frac{9}{216} +$

$+ 1\cdot3\cdot\frac{12}{216} + 2\cdot3\cdot\frac{6}{216} + 3\cdot3\cdot\frac{1}{216}$

$= 1$

0,5

$cov(X,Y) = 1 - \frac{1}{2}\cdot\frac{189}{216} = \frac{243}{432}$ 0,5

2. v.

$E(X\cdot Y) = 1\cdot1\cdot\frac{36}{216} + 1\cdot2\cdot\frac{27}{216} + 2\cdot1\cdot\frac{9}{216}$

$= \frac{108}{216} = \frac{1}{2}$

$cov(X\cdot Y) = \frac{1}{2} - \frac{1}{2}\cdot\frac{189}{216} = \frac{27}{216}$

6) 1. v.

$R(X-2Y, X+Y) = \frac{cov(X-2Y, X+Y)}{\sigma(X-2Y)\cdot\sigma(X+Y)}$ 2

2

$cov(X-2Y, X+Y) = \underbrace{cov(X,X)}_{\sigma^2 X} - 2\underbrace{cov(Y,X)}_{0} + \underbrace{cov(X,Y)}_{0} - 2\underbrace{cov(Y,Y)}_{\sigma^2 Y}$

1 1

mert X és Y fgtlen

$= 1 - 2\cdot2 = -3$

mert $\sigma^2 X = \lambda$ ha $X \sim Po(\lambda)$ 1

$\sigma^2(X+Y) = \sigma^2 X + \sigma^2 Y = 3$ 1

↑ mert fgtlenek

$\sigma^2(X-2Y) = \sigma^2 X + \sigma^2(2Y) = \sigma^2 X + 4\cdot\sigma^2 Y = 9$

2

$R(X-2Y, X+Y) = \frac{-3}{\sqrt{3}\cdot 3} = -\frac{1}{\sqrt{3}}$

2. v. Pontozás ugyanígy

$cov(X-Y, 2X+Y) = 2cov(X,X) - 2cov(Y,X) + cov(X,Y) - cov(Y,Y)$

$= 2\cdot3 - 2 = 4$

$\sigma^2(X-Y) = 3+2 = 5$

$\sigma^2(2X+Y) = 4\cdot3+2 = 14$

$R(X-Y, 2X+Y) = \frac{4}{\sqrt{5}\cdot\sqrt{14}}$