$$W_{PID}(s) = A_P\left(1+\frac{1}{sT_I}+\frac{sT_D}{1+sT_C}\right)$$

$$W_{PID}(s) = \frac{A_P}{T_I}\,\frac{1+s(T_I+T_C)+s^2(T_D+T_C)T_I}{s(1+sT_C)} = \frac{A_P}{T_I}\,\frac{(1+s\tau_1)(1+s\tau_2)}{s(1+sT_C)} =$$

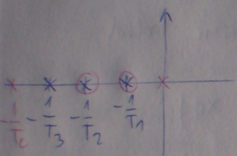
$$= \frac{A_P}{T_I}\,\frac{1+s(\tau_1+\tau_2)+s^2\tau_1\tau_2}{s(1+sT_C)}$$

$$\tau_1+\tau_2 = T_I + T_C$$

$$\tau_1\tau_2 = T_1(T_D+T_C)$$

Szakasz: $W(s) = A\,\frac{1}{(1+sT_1)(1+sT_2)(1+sT_3)}$

$$T_1 \geq T_2 \geq T_3$$



Ha: $\tau_1 = T_1$ és $\tau_2 = T_2$

Felnyitott kör:

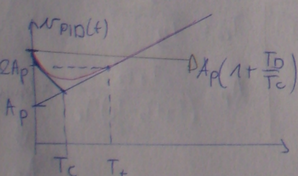
$$W_0(s) = \frac{A_P A}{T_I}\,\frac{1}{s(1+sT_3)(1+sT_C)}$$

$$\tau_1, \tau_2, T_C \longrightarrow T_I = \tau_1+\tau_2-T_C$$

$$T_D + T_C = \frac{\tau_1\tau_2}{T_I} \rightarrow T_D = \frac{\tau_1\tau_2}{T_I} - T_C$$

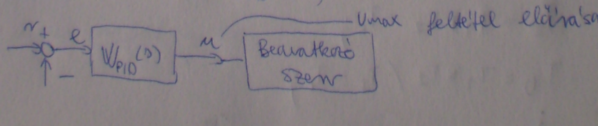
$$T_I = T_1+T_2-T_C$$

$$T_D = \frac{\tau_1\tau_2}{T_1+T_2-T_C} - T_C; \qquad T_D+T_C = \frac{T_1T_2}{T_1+T_2-T_C}$$



$$v_{PID}(0) = A_P\left(1+\frac{T_D}{T_C}\right) = A_P\,\frac{T_D+T_C}{T_C} =$$

$$= W_{PID}(\infty)$$



$A_p, T_c, w_c \longleftrightarrow \underbrace{\varphi_t, u_{max}}$

Ezek adottak, ebből meghatározható a másik három (A_p, T_c, w_c).

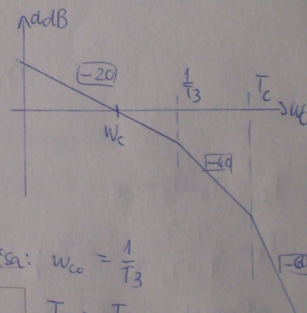
$$\left.\begin{aligned} &|W_0(jw_c)| - 1 = 0 \\ &\pi + \arg W_0(jw_c) - \varphi_t[\mathrm{rad}] = 0 \\ &A_p\left(1 + \frac{T_D}{T_c}\right) - U_{max} = 0 \iff A_p \frac{T_D + T_c}{T_c} - U_{max} = 0 \end{aligned}\right\} \bar{f}(\bar{x}) = \bar{0}$$

Nem lineáris egyenletrendszer

M.o.: Matlab: "fsolve" Optimalization Toolbox

$X = \begin{pmatrix} A_p \\ T_c \\ W_c \end{pmatrix}$ [X, fval] = fsolve (@ myfun, XØ)

```
% myfun fv. megírása:
% call fsolve
A = 5; T1 = 10 ; T2 = 1; T3 = 0.1;
wcØ = 1/T3;    Umax = 5; phim = pi/4;
TcØ = T3;
ApØ = umax·(T1+T2-Tc0)·Tc0 / (T1·T2);
```



XØ választása: $w_{c0} = \frac{1}{T_3}$

$T_{c0} = T_3$

$A_{p0} \frac{T_1 T_2}{(T_1 + T_2 - T_c) T_c} = U_{max}$

$\Downarrow$

A_{p0}

```
[X, fval] = fsolve (@ myfun, XØ);
XØ = [ApØ  TcØ  wcØ]';
Ap = X(1); Tc = X(2) ; wc = X(3);
TI = T1 + T2 - Tc;
TD = T1 * T2 / TI - Tc;
⋮
```

% myfun-t menteni kell : myfun.m-be. Myfun: X.12 P. 5.h.

```
function f = myfun (x)
  Ap = x(1); Tc = x(2); wc = x(3);
  A = 5; T1 = 10; T2 = 1; T3 = 0.1; umax = 5; plim = pi/4;
```

← kétszer kell leírni, erre majd lesz trükk.

$$f_1 = \frac{A_p \cdot A}{(T_1+T_2-T_c)} * \frac{1}{wc * sqrt(1+(wc\cdot T_3)\hat{}2) * sqrt(1+(wc\cdot T_c)\hat{}2)} - 1;$$

$$f_2 = pi - pi/2 - atan(wc \cdot T_3) - atan(wc \cdot T_c) - phim;$$

$$f_3 = A_p * \frac{T_1 * T_2}{(T_1+T_2-T_c)\cdot T_c} - Umax;$$

```
f = [f1 f2 f3]';
```

Lehetőségek (ne kelljen 2-szer írni az adatokat!)

```
% call fsolve
global A T1 T2 T3 umax plim     % globális változók
  ⋮
function f = myfun(x)
global A T1 T2 T3 umax plim
```

Másik lehetőség: Nested Function (beágyazás)

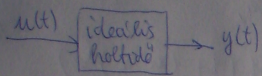
```
function [x, fval] = myfsolve (xΦ, A, T1, T2, T3, umax, plim)

      [x, fval] = fsolve (@myfun, xΦ)
    function f = myfun (x)
      ⋮
      f = [f1 f2 f3]';
    end
end
```

% A modul neve myfsolve.m

Holtidős rendszerek irányítása

(rendszeren történő áthaladás ideje: pl.: futószalagon meddig megy 1db cuccos.)

Ideális holtidő

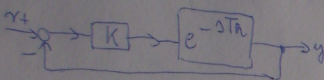


$y(t) = u(t - T_h)$

$Y(s) = e^{-sT_h} U(s)$

$W(s) = e^{-sT_h}$

P szabályozás



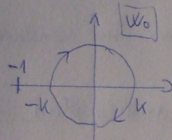
(Felnyitott Kör)

$FK: \; W_0(s) = Ke^{-sT_h}$

$W_0(j\omega) = Ke^{-j\omega T_h}$

$|W_0(j\omega)| = K$

$\varphi(\omega) = -\omega T_h \, [rad]$



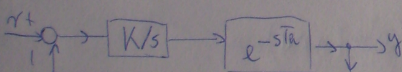
(Zárt Rendszer)

ZR: stabil: $\underline{K \leq 1}$

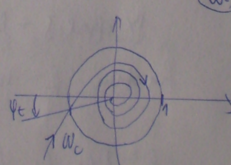
Zavarás csökkentés: $\dfrac{1}{1+K} \cong \dfrac{1}{2}$! kicsi

A P szabályzás itt használhatatlan, mert a zavarást nem tudja lecsökkenteni.

I szabályozás



W_0 Nyquist



$FK: \; W_0(s) = \dfrac{K}{s} e^{-sT_h}$

$W_0(j\omega) = \dfrac{K}{j\omega} e^{-j\omega T_h}$

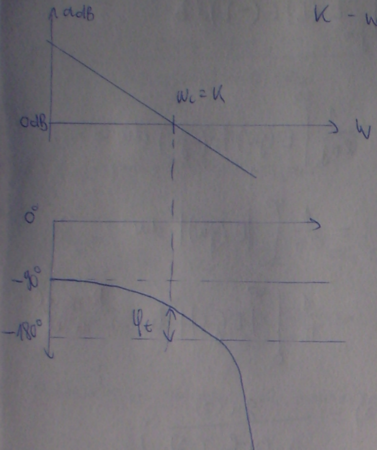
$|W_0(j\omega)| = \dfrac{K}{\omega}, \quad \varphi(\omega) = -\dfrac{\pi}{2} - \omega T_h \, [rad]$

Bode →

X. 12. p.
5. h.

Bode:

Ha $-20\,dB$-el jön a görbe, akkor
K-nál metszi a tengelyt!



$$\omega_c = K$$

$$\varphi_t = 180^\circ + \varphi(\omega_c)\ [\text{fok}]$$

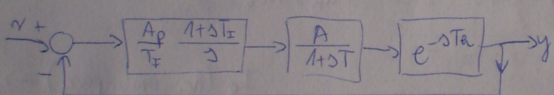
$$\Updownarrow$$

$$\varphi_t = \pi - \frac{\pi}{2} - \omega_c T_h$$

$$\boxed{\omega_c = K = \frac{\frac{\pi}{2} - \varphi_t\ [\text{rad}]}{T_h} \qquad \omega_c = K = \frac{90^\circ - \varphi_t\ [\text{fok}]}{T_h} \cdot \frac{\pi}{180^\circ}}$$

PI-szabályozás egytárolós holtidős rendszer esetén

$$W_{PI}(s) = A_p\left(1 + \frac{1}{sT_I}\right) = \frac{A_p}{T_I} \cdot \frac{1+sT_I}{s}$$



$$T_I = T$$

$$W_0(s) = \frac{K}{s} e^{-sT_h}, \quad \text{ahol } K = \frac{A_p A}{T_I} \quad ; \ \omega_c = K \checkmark \Longrightarrow A_p = \frac{K T_I}{A}$$

Hibanégyzet integrál minimalizálása

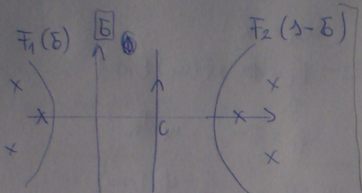
Konvolúciós tétel:

$$F(s) = F_1(s) F_2(s) \longleftrightarrow f(t) = \int_0^t f_1(\tau) f_2(t-\tau)\, d\tau$$

$$f(t) = f_1(t) \cdot f_2(t) \longrightarrow F(s) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F_1(\sigma) F_2(s-\sigma)\, d\sigma$$

$$I_2 = I_Q = \int_0^\infty [e(t)]^2 dt = \lim_{t\to\infty} \int_0^t [e(\tau)]^2 d\tau = \lim_{s\to 0} \cancel{s} \frac{1}{\cancel{s}} \mathcal{L}[e^2(t)] = \longrightarrow$$

$$\lim_{s\to 0}\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} E(\sigma)\,E(s-\sigma)\,d\sigma = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} E(\sigma)\,E(-\sigma)\,d\sigma$$



$$= \frac{1}{2\pi j}\int_{-\infty}^{\infty} E(j\omega)\,E(-j\omega)\,d\omega =$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|E(j\omega)\right|^2 d\omega$$

Hiba integrál: $I_Q = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|E(j\omega)\right|^2 d\omega = \frac{1}{\pi}\int_{0}^{\infty} \left|E(j\omega)\right|^2 d\omega$ Parseval-tétel

$\boxed{\begin{array}{l} e(t) \rightarrow E(s) \text{ stabil} \\ e(\infty) = 0 \end{array}}$

$$E(s)\,E(-s) = \frac{g_n(s)}{h_n(s)\,h_n(-s)};$$

$$g_n(s) = b_{n-1}s^{2(n-1)} + b_{n-2}s^{2(n-2)} + \dots + b_1 s^2 + b_0$$

$$h_n(s) = a_n s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0$$

$$I_Q = \frac{M_n}{2a_n\Delta_n}$$

I_Q (hr. szab. par.) $\rightarrow$ min

$$\Delta_n = \begin{vmatrix} a_{n-1} & a_{n-3} & \dots & 0 \\ a_n & a_{n-2} & \dots & 0 \\ 0 & a_{n-1} & \dots & 0 \\ \vdots & & & \\ 0 & \dots & \dots & a_n \end{vmatrix}$$

Mint Hurwitz sehnaknál

$$M_n = \begin{vmatrix} b_{n-1} & b_{n-2} & \dots & b_0 \\ a_n & a_{n-2} & \dots & 0 \\ 0 & a_{n-1} & \dots & 0 \\ \vdots & & & \\ 0 & \dots & & a_n \end{vmatrix} (-1)^{n-1}$$

$$I_2 = I_a = \frac{M_n}{2 a_n \Delta_n} \qquad \Delta_2 = \begin{vmatrix} a_1 & 0 \\ a_2 & a_0 \end{vmatrix} = a_1 a_0$$

$$E(s) = \frac{c_1 s + c_0}{a_2 s^2 + a_1 s + a_0}$$

$$g_2(s) = (c_1 s + c_0)(-c_1 s + c_0) = -c_1^2 s^2 + c_0^2$$

$$h_2(s) = a_2 s^2 + a_1 s + a_0$$

$$M_2 = \begin{vmatrix} -c_1^2 & c_0^2 \\ a_2 & a_0 \end{vmatrix} (-1)^{2-1} = c_1^2 a_0 + c_0^2 a_2$$

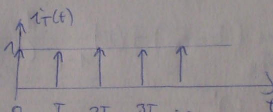
$$I_2 = I_a = \frac{c_1^2 a_0 + c_0^2 a_2}{2 a_2 a_1 a_0} \longrightarrow \min$$

Mintavételes rendszerek analízise

eltolt Dirac impulzus(ok)

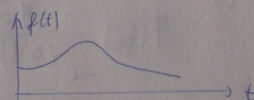
$$f(t) \xrightarrow{T} f^*(t) = \sum_{n=0}^{\infty} f(nT)\,\delta(t-nT) = f(t) \cdot i_T(t)$$

hordozó: $i_T(t) = \sum_{n=0}^{\infty} \delta(t-nT)$



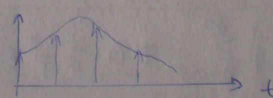
végtelen magas, 1 területű impulzusok

analóg jel: $f(t)$



modulált jel:

$$f^*(t) = f(t) \cdot i_T(t)$$



Hordozó jel Laplace-tr.-ja:

$$I_T(s) = \sum_{n=0}^{\infty} e^{-snT} = \sum_{n=0}^{\infty} \left(e^{-sT}\right)^n = \frac{1}{1-e^{-sT}}$$

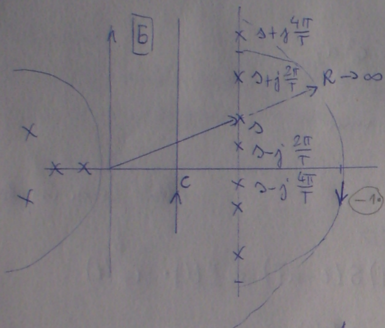
pólus: $1 - e^{-sT} = \emptyset \Rightarrow 1 = e^{-sT} = 1 \cdot e^{jk2\pi}$; $k = 0, \pm 1, \pm 2, \ldots$

$$-sT = jk2\pi \longrightarrow s_k = -jk\frac{2\pi}{T}, \quad k = 0, \pm 1, \pm 2, \ldots$$

$$F^*(s) = \sum_{n=0}^{\infty} f(nT)e^{-snT} = ?$$

$$F^*(s) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(\sigma)\, I_T(s-\sigma)\, d\sigma = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} F(\sigma)\frac{1}{1-e^{-(s-\sigma)T}}\, d\sigma$$

$$s-\sigma_k = -j\frac{2\pi}{T}k \Rightarrow \sigma_k = s+j\frac{2\pi}{T}k\ , \quad k=0,\pm 1,\pm 2,\dots$$



körüljárás miatt

$$F^*(s) = \overset{-1\cdot}{\cancel{2\pi j}}\ \mathrm{Res}_{\sigma_k} \frac{1}{\cancel{2\pi j}} F(\sigma)\frac{1}{1-e^{-(s-\sigma)T}}$$

$$F^*(s) = -\mathrm{Res}_{\sigma_k} F(\sigma)\frac{1}{1-e^{-(s-\sigma)T}}$$

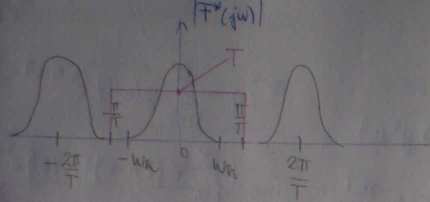
$$\mathrm{Res}_{\sigma_k} = \lim_{\sigma\to\sigma_k}(\sigma-\sigma_k)F(\sigma)\frac{1}{1-e^{-(s-\sigma)T}} = F(\sigma_k)\lim_{\sigma\to\sigma_k}\frac{\sigma-\sigma_k}{1-e^{-(s-\sigma)T}} =$$

Lopital : $\frac{(\,)'}{(\,)'}$

$$= F(\sigma_k)\left.\frac{1}{-e^{-(s-\sigma)T}T}\right|_{\sigma=\sigma_k} = -\frac{1}{T}$$

$$\boxed{F^*(s) = \frac{1}{T}\sum_{k=-\infty}^{\infty} F\left(s+j\frac{2\pi}{T}k\right)}$$

$$\boxed{F^*(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} F\left(j\left(\omega+\frac{2\pi}{T}k\right)\right)}$$



$f(t)$ sávkorlátozott, $\omega_h \leq \frac{\pi}{T}$

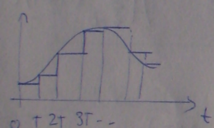
$2\pi f_h \leq \pi \frac{1}{T}$

$T \leq \frac{1}{2 f_h}$

Shanon tétel:
$\omega_h \leq \frac{\pi}{T} = \omega_N \to$ Nyquist

Probléma:
Az ideális aluláteresztő szűrő nem kauzális.
Helyette tartószerveket használunk.

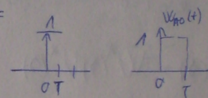
ZOH (Zero on hold)



Rontja a fázistöbbletet!

Nulladrendű tartószerv átviteli fv.-e:

súlyfv.: $W_{H0}(t) =$



$W_{H0}(t) = 1(t) - 1(t-T) \Rightarrow$

$W_{H0}(s) = \frac{1}{s} - \frac{1}{s}e^{-sT} = \frac{1-e^{-sT}}{s}$

$$W_{H0}(j\omega) = \frac{1-e^{-j\omega T}}{j\omega} = e^{-j\frac{\omega T}{2}} \frac{e^{j\frac{\omega T}{2}} - e^{-j\frac{\omega T}{2}}}{2j} \frac{1}{\frac{\omega T}{2}} T =$$

$$= T e^{-j\frac{\omega T}{2}} \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

$\boxed{\omega_c T \approx 0.2}$

$\frac{\omega_c T}{2} \in [\pm 5°] \frac{\pi}{180°}$; $\omega_c T \approx 0.17$

$$|W_{H0}(j\omega)| = T \left| \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}} \right| \quad ; \quad \varphi_{H0}(\omega) \overset{\text{rad}}{=} -\frac{\omega T}{2} + \arg \frac{\sin\frac{\omega T}{2}}{\frac{\omega T}{2}}$$

Hol lesz a váltás?

$\frac{\omega T}{2} = \pi \Rightarrow \omega = \frac{2\pi}{T}$