2008. 02. 19.

$$\overline{\overline{T}}_{0,6} = \overline{\overline{T}}_{0,1}\,\overline{\overline{T}}_{1,2}\,\overline{\overline{T}}_{2,3}\,\overline{\overline{T}}_{3,4}\,\overline{\overline{T}}_{4,5}\,\overline{\overline{T}}_{5,6}$$

$$\overline{\overline{T}}_{0,6}(\bar{q}), \quad \bar{q} = \begin{pmatrix} q_1 \\ \vdots \\ q_6 \end{pmatrix}$$

$$K_B \xrightarrow{\overline{\overline{T}}_{B,0}} K_0 \xrightarrow{\overline{\overline{T}}_{0,6}(\bar{q})} K_6 \xrightarrow{\overline{\overline{T}}_{6,E}} K_E$$

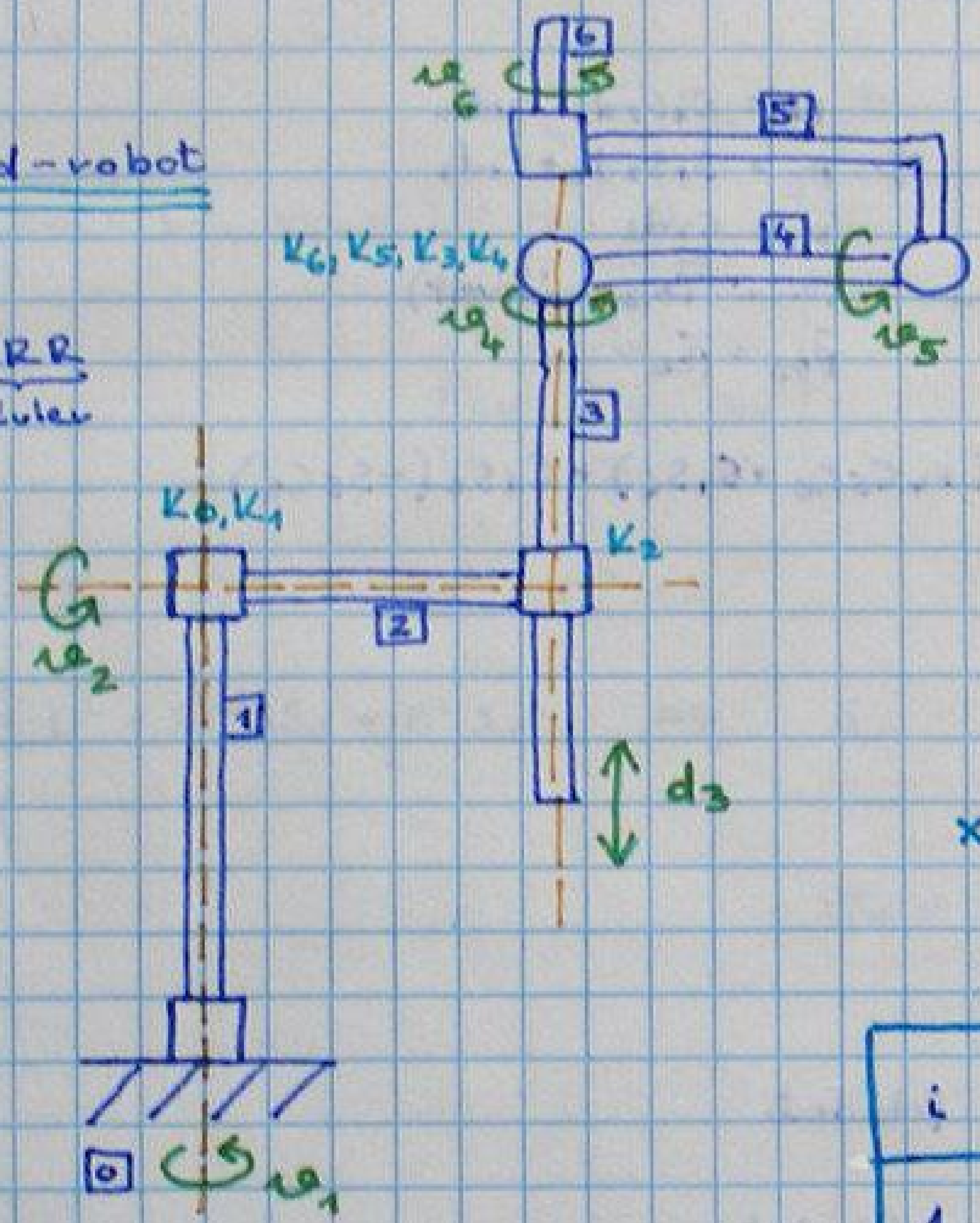
$\overline{\overline{T}}_{B,E}$ ROBOTGRÁF

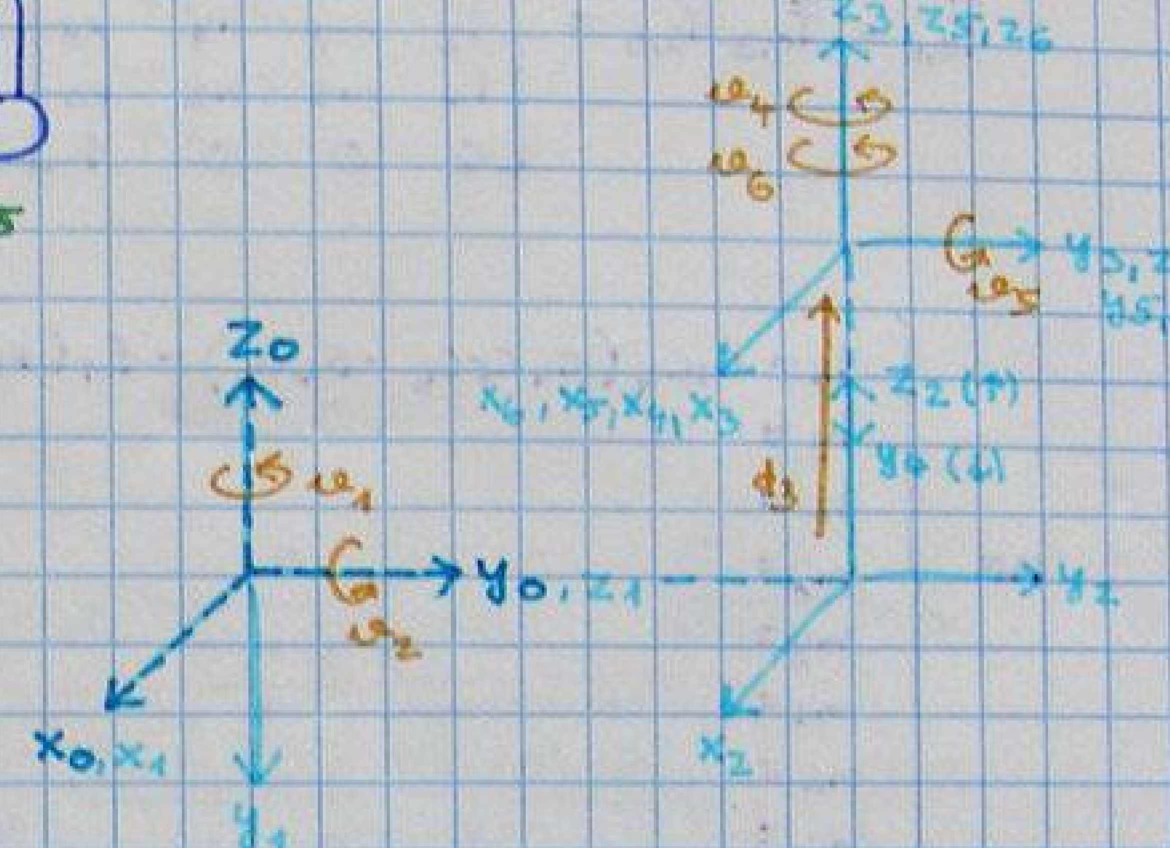
$$\overline{\overline{T}}_{i-1,i}$$

$$\overline{\overline{T}}_{B,E} = \overline{\overline{T}}_{B,0} \cdot \overline{\overline{T}}_{0,6}(\bar{q}) \cdot \overline{\overline{T}}_{6,E} \Rightarrow \boxed{\overline{\overline{T}}_{0,6}(\bar{q}) = \overline{\overline{T}}_{B,0}^{-1} \cdot \overline{\overline{T}}_{B,E} \cdot \overline{\overline{T}}_{6,E}^{-1}}$$

Stanford-robot

- RRT $\underbrace{RRR}_{\text{Euler}}$





i	q_i	ϑ_i	d_i	a_i	α_i	$S_{\alpha i}$	$C_{\alpha i}$
1	ϑ_1	ϑ_1	0	0	-90°	-1	0
2	ϑ_2	ϑ_2	d_2	0	90°	1	0
3	d_3	0°	d_3	0	0°	0	1
4	ϑ_4	ϑ_4	0	0	-90°	-1	0
5	ϑ_5	ϑ_5	0	0	0°	1	0
6	ϑ_6	ϑ_6	0	0	0°	0	1

$$\overline{\overline{T}}_{i-1,i} = \begin{bmatrix} C_{\vartheta_i} & -S_{\vartheta_i}C_{\alpha i} & S_{\vartheta_i}S_{\alpha i} & a_i C_{\vartheta_i} \\ S_{\vartheta_i} & C_{\vartheta_i}C_{\alpha i} & -C_{\vartheta_i}S_{\alpha i} & a_i S_{\vartheta_i} \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{1,2} = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \overline{\overline{T}}_{2,3} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \overline{\overline{T}}_{3,4} = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{4,5} = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \overline{\overline{T}}_{5,6} = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \overline{\overline{T}}_{0,1} = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overline{\overline{T}}_{3,6} = \underbrace{\overline{\overline{T}}_{3,4}\,\overline{\overline{T}}_{4,5}\,\overline{\overline{T}}_{5,6}}_{HF} = \overline{\text{Euler}}(\underbrace{\varphi}_{q_4}, \underbrace{\vartheta}_{q_5}, \underbrace{\psi}_{q_6}) \qquad \overline{\overline{T}}_{0,3} = \underbrace{\overline{\overline{T}}_{0,1}\,\overline{\overline{T}}_{1,2}\,\overline{\overline{T}}_{2,3}}_{HF}$$

$$\bar{\bar{T}}_{3,6} = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & 0 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & 0 \\ -S_5C_6 & S_5S_6 & C_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{0,3} = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & C_1S_2d_3 - S_1d_2 \\ S_1C_2 & C_1 & S_1S_2 & S_1S_2d_3 + C_1d_2 \\ -S_2 & 0 & C_2 & C_2d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\bar{\bar{T}}_{0,6} = \bar{\bar{T}}_{0,3}\,\bar{\bar{T}}_{3,6} = \begin{bmatrix} l_x & m_x & n_x & p_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{array}{l} p_x = C_1S_2d_3 - S_1d_2 \\ p_y = S_1S_2d_3 + C_1d_2 \\ p_z = C_2d_3 \end{array}$$

(ami $\bar{\bar{T}}_{0,3}$-nál volt)

$\bar{p}_{03} = \bar{p}_{06}$

$$\text{HF} \left\{ \begin{array}{l} l_x = C_1C_2(C_4C_5C_6 - S_4S_6) - S_1(S_4C_5C_6 + C_4S_6) + C_1S_2(-S_5C_6) \\ l_y = \\ l_z = \\ m_x = \\ m_y = \\ m_z = \\ n_x = \\ n_y = \\ n_z = -S_2(C_4S_5) + C_2(C_5) \end{array} \right.$$

$\bar{q} \longmapsto \bar{\bar{T}}_{06}(\bar{q})$ direkt geometriai feladat

$\bar{\bar{T}}_{06}$ num $\longrightarrow \bar{q}$ inverz geometriai feladat

$$\bar{\bar{A}}_{06} = \bar{\bar{A}}_{03}\,\bar{\bar{A}}_{36} \Rightarrow \bar{\bar{A}}_{36} = \bar{\bar{A}}_{03}^{-1}\,\bar{\bar{A}}_{06} = \underset{\substack{\uparrow \\ q_1,\ q_2,\ q_3}}{\bar{\bar{A}}_{03}^{T}}\,\bar{\bar{A}}_{06}$$

$$\bar{\bar{A}}_{36}\text{ num} \xrightarrow{\text{inv. Euler}} q_4, q_5, q_6$$

1 Inverz pozícionáló feladat

$$\left.\begin{aligned} p_x &= C_1 S_2 d_3 - S_1 d_2 \\ p_y &= S_1 S_2 d_3 + C_1 d_2 \end{aligned}\right\} \overset{(1)}{\ } -S_1 p_x + C_1 p_y = d_2 \Rightarrow A C_\alpha + B S_\alpha = D \quad (2x)$$

$$p_z = C_2 d_3$$

$$\underbrace{(C_1 p_x + S_1 p_y)}_{S_2 d_3} C_2 - \underbrace{p_z}_{C_2 d_3} S_2 = 0 \quad \Rightarrow \overset{(2)}{\ } T_2 = \frac{C_1 p_x + S_1 p_y}{p_z}$$

(3)

$$\underbrace{(C_1 p_x + S_1 p_y)}_{S_2 d_3} S_2 + \underbrace{p_z}_{C_2 d_3} C_2 = d_3 = q_3$$

$\downarrow$ q_1, q_2, q_3

$$\bar{\bar{A}}_{36} = \overline{\overline{\text{Euler}}}(q_4, q_5, q_6) = \underbrace{\bar{\bar{A}}_{03}^{T} \, \bar{\bar{A}}_{06}}_{\begin{bmatrix} L_x & M_x & N_x \\ L_y & M_y & N_y \\ L_z & M_z & N_z \end{bmatrix}} \xrightarrow{\text{inv. Euler}} \begin{aligned} \varphi &= q_4 \\ \vartheta &= q_5 \\ \psi &= q_6 \end{aligned}$$

$$A C_\alpha + B S_\alpha = D \qquad S_\alpha^2 + C_\alpha^2 = 1$$

$$A^2 \underbrace{C_\alpha^2}_{1 - S_\alpha^2} = D^2 - 2DBS_\alpha + B^2 S_\alpha^2 \quad \Rightarrow \quad S_\alpha^2 - 2\frac{DB}{A^2 + B^2} \cdot S_\alpha + \frac{D^2 - A^2}{A^2 + B^2} = 0$$

$$\boxed{S_\alpha = \frac{DB + \delta_\alpha A \sqrt{A^2 + B^2 - D^2}}{A^2 + B^2}}$$

$$C_\alpha^2 - 2\frac{DA}{A^2 + B^2} \cdot C_\alpha + \frac{D^2 - B^2}{A^2 + B^2} = 0$$

$$\boxed{C_\alpha = \frac{DA + \gamma_\alpha B \sqrt{A^2 + B^2 - D^2}}{A^2 + B^2}}$$

ha nincs megoldás: $D < 0$ (csak valós lehet)
elhagytuk a munkateret

ha van megoldás: 2 megoldás van

$$\boxed{(\delta_\alpha, \gamma_\alpha) = \begin{cases} (+1, -1) \\ (-1, +1) \end{cases}}$$