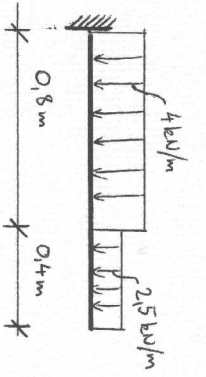
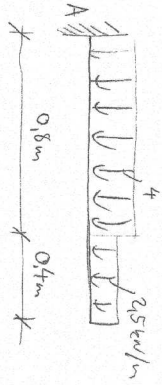


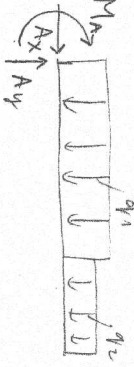
1



a) SZÁMITSUK KI A REAKCIÓKAT!
 b) RAJZOLJUK MEG A TRÉTO
 ILLYERVENÉTELI ÁBRÁKAT



a) ELÜLVÉNYÍTÉS



EGYENSÜVTI KIJELENÍTÉS

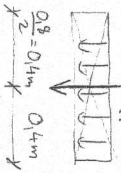
$((Q_1), (Q_2), A, M_A) \equiv 0$

$A_x = ? , A_y = ? , M_A = ?$

c) MEGOLDÁS

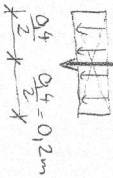
$(Q_1) \equiv Q_1$

$Q_1 = 4 \cdot 0.8 = 3.2 \text{ kN}$



$(Q_2) \equiv Q_2$

$Q_2 = 2.5 \cdot 0.4 = 1.0 \text{ kN}$

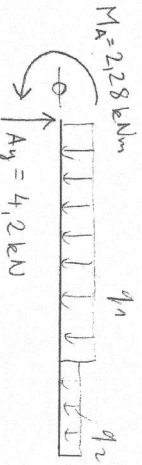


$\sum F_{ix} : A_x = 0$

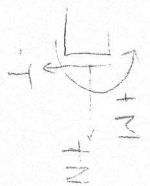
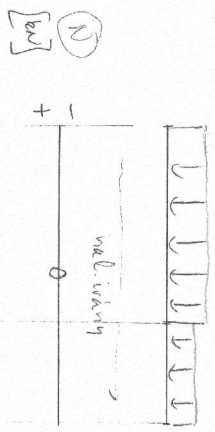
$\sum F_{iy} : +3.2 + 1.0 - A_y = 0 \Rightarrow A_y = 4.2 \text{ kN} (\uparrow)$

$\sum M_i^{(A)} : +3.2 \cdot 0.4 + 1.0 \cdot (0.8 + 0.2) + M_A = 0 \Rightarrow M_A = -2.28 \text{ kNm} (\curvearrowright)$

EREDMÉNYSZÁMLÁLAT



b) REAKCIÓERŐK (nem kell, mert jobbról tudunk mindent számszerűsíteni)



$+2.5 \cdot 0.4 = +1.0 \text{ kN}$

$+2.5 \cdot 0.4 + 4.0 \cdot 0.8 = 4.2 \text{ kN}$

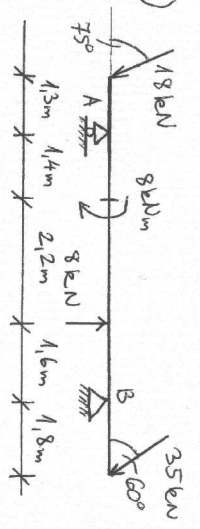
$-2.5 \cdot 0.4 \cdot \frac{0.4}{2} = -0.2$

$-2.5 \cdot 0.4 \cdot (0.8 + \frac{0.4}{2}) - 4.0 \cdot 0.8 \cdot \frac{0.8}{2} = -2.28$

$\frac{4 \cdot 0.8^2}{8} = 0.32$

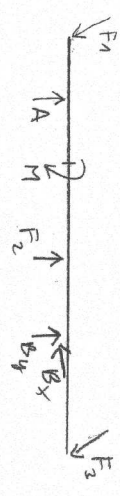
$\frac{4.5 \cdot 0.4^2}{8} = 0.05$

2



a) SZÁMÍTSD KI A REAKCIÓKAT!
b) RAJZOLDD UPK MEG AZ IJÉNYKÉPVEZETCI ÁBRÁKAT!

1) 1: ELKÜLÖNÍTÉS



2: BŐRÖNSŐÖRTI MÉRLEKLETES:

$$(F_1, F_2, F_3, M, B_x, A) \equiv 0$$

$$A_x = ? \quad B_y = ? \quad B_x = ?$$

$$F_{1y} = 18 \cdot \sin 75^\circ = 17,39 \text{ kN}$$

$$F_{1x} = 18 \cdot \cos 75^\circ = 4,659 \text{ kN}$$

$$F_{2y} = 35 \cdot \sin 60^\circ = 30,31 \text{ kN}$$

$$F_{3x} = 35 \cdot \cos 60^\circ = 17,5 \text{ kN}$$

3: HSGÖRÖK

$$\sum N_i^{(A)}: -17,39 \cdot 6,5 + 8 \cdot 4,6 + 30,31 \cdot 1,8 + 8 + A \cdot 5,2 = 0$$

$$A = +7,246 \text{ kN (↑)}$$

$$\sum M_i^{(A)}: -17,39 \cdot 1,3 - 8 \cdot 3,6 + 30,31 \cdot 7,0 + 8 - B_y \cdot 5,2 = 0$$

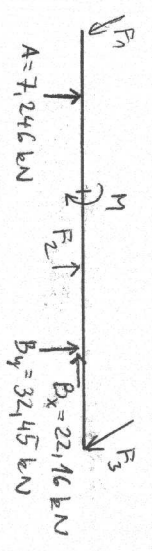
$$B_y = +32,45 \text{ kN (↑)}$$

$$\sum F_{x}: +4,659 + 17,5 - B_x = 0$$

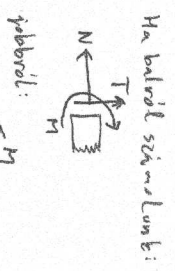
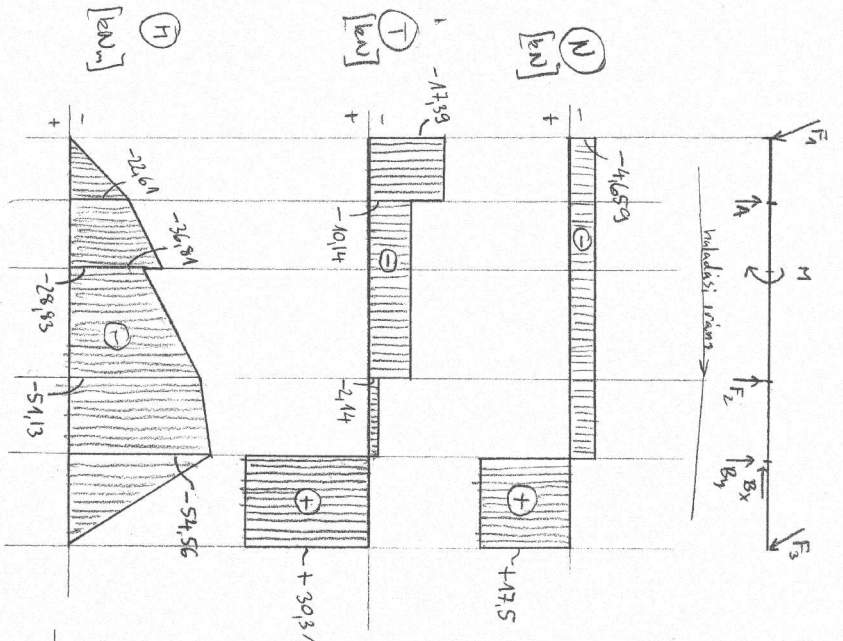
$$B_x = +22,16 \text{ kN (←)}$$

BOL: $\sum F_{iy}: +17,39 - 8 + 30,31 - 32,45 - 7,246 \stackrel{?}{=} 0$
0,004 ≈ 0 ✓

4: ELŐJELTUDVÁZLAT



b) A: REAKCIÓERŐK (MISD AZ ELŐZŐ OLDALT)



$$-30,31 \cdot 1,8 = -54,56$$

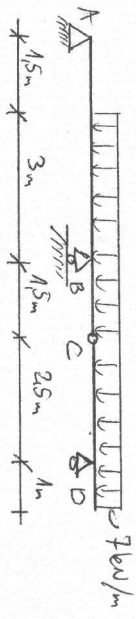
$$-30,31 \cdot 3,4 + 32,45 \cdot 1,6 = -5,413$$

$$-30,31 \cdot 5,6 + 32,45 \cdot 3,8 + 8 \cdot 2,2 = -28,83$$

$$-17,39 \cdot 1,3 = -22,61$$

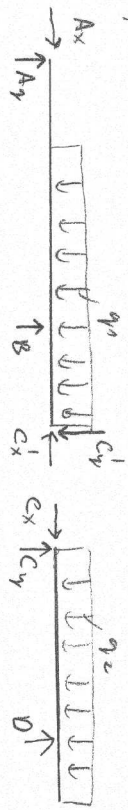
$$-17,39 \cdot 2,7 + 7,246 \cdot 1,4 = -36,81$$

3



FAKDAT: REAKUCIJSOM SVAETIPIKSA, IBEHTIPICTEBU ABEBU DAPZOLAKA

q, EYUVEHTIBS



h, EYTOBECYUTI KI TPEBETIBS:

I: $((q_1), C, B, A) = 0$

II: $((q_2), C, D) = 0$

Σ: $((q_1), (q_2), B, A, D) = 0$

$A_x = ?$, $A_y = ?$, $B = ?$, $C_x = ?$, $C_y = ?$, $D = ?$

q, MEYUVAK:

II: $Σ F_{2x} : C_x = 0 \text{ kN}$

Σ M^(B): $-24.5 \cdot 0.75 + C_y \cdot 2.5 = 0 \Rightarrow C_y = 7.35 \text{ kN} (\uparrow)$

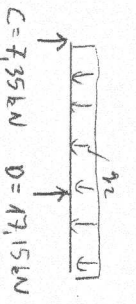
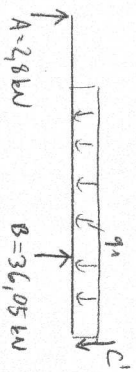
Σ M^(C): $+24.5 \cdot 1.75 - D \cdot 2.5 = 0 \Rightarrow D = 17.15 \text{ kN} (\uparrow)$

I: $Σ H^{(B)}$: $+3 \cdot 1.5 \cdot 3.75 + 7.35 \cdot 6 - B \cdot 4.5 = 0 \Rightarrow B = 36.05 \text{ kN} (\uparrow)$

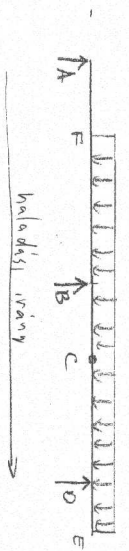
Σ M^(A): $-3 \cdot 1.5 \cdot 0.75 + 7.35 \cdot 1.5 + A_y \cdot 4.5 = 0 \Rightarrow A_y = 2.8 \text{ kN} (\uparrow)$

Σ F_{ix}: $A_x = 0$

EKIBOTIPIVATZLAT



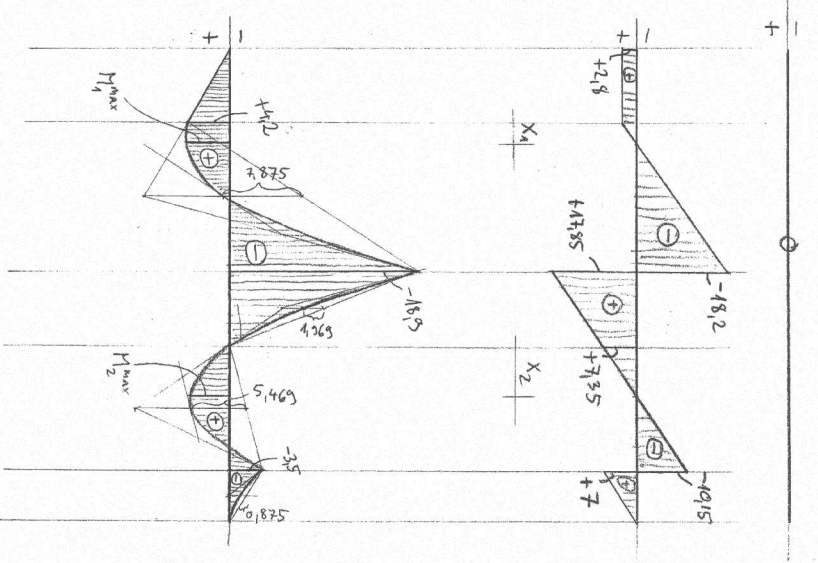
IGENITIB VETBU ABEBU



(N) [kN]

(T) [kN]

(M) [kNm]



$M_1^{max} = +2.8 \cdot (1.5 + 0.4) - 7 \cdot 0.4 \cdot \frac{0.4}{2} = +4.76 \text{ kNm}$

$M_2^{max} = +7.35 \cdot 1.05 - 7 \cdot 1.05 \cdot \frac{1.05}{2} = +3.859 \text{ kNm}$

$+3 \cdot 1.5 = +4.5$

$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

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$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

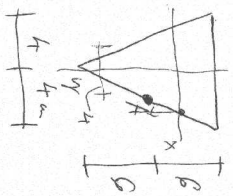
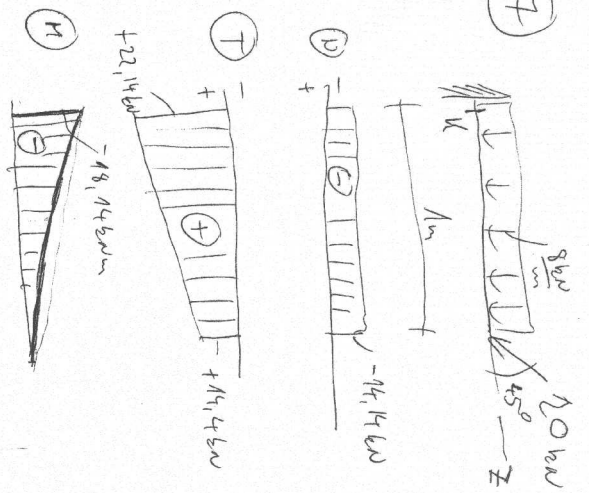
$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

$+7.35 + 7.35 = +14.7$

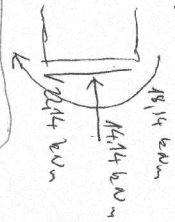
$+7.35 + 7.35 = +14.7$

7



FASZULTEVEN AZ A
PONTOK

K - Wrt:



$$\sigma_A = -\frac{M_x}{I_x} - \frac{M_y}{I_y} = -\frac{2214}{384} \cdot 4 = -2336 \frac{\text{N}}{\text{cm}^2}$$

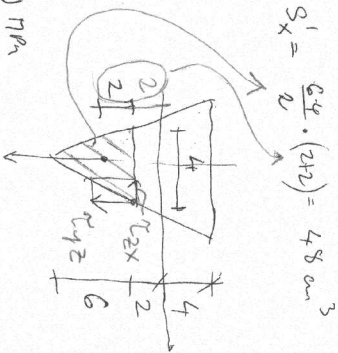
$$\sigma_{2y} = \frac{48 \cdot 18.14}{4 \cdot 384} = 0.5669 \frac{\text{N}}{\text{cm}^2} = 5.669 \text{ MPa}$$

$$\sigma_{2x} = \frac{1}{2} \cdot 5.669 = 1.890 \text{ MPa}$$

$$\sigma_z = 5.926 \text{ MPa}$$

$$\frac{I_x \cdot I_y}{b \cdot l \cdot x}$$

$$\frac{I_{xy}}{I_x} = \frac{4}{12}$$



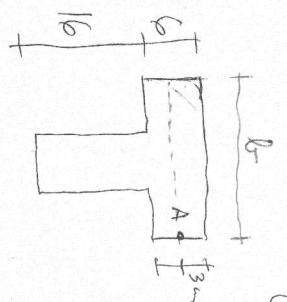
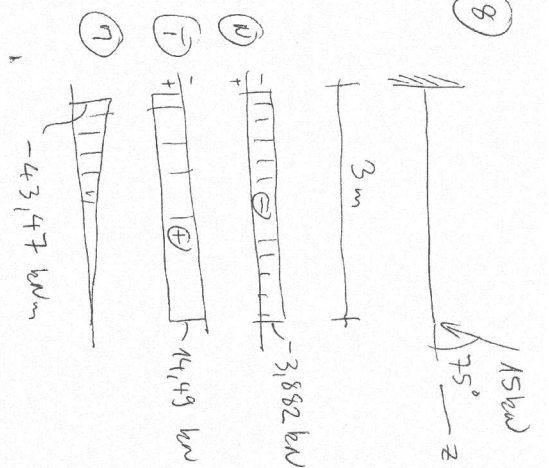
$$A = \frac{12 \cdot 8}{2} = 48 \text{ cm}^2$$

$$I_x = \frac{8 \cdot 12^3}{36} = 384 \text{ cm}^4$$

$$S_x' = \frac{6 \cdot 4}{2} \cdot (2+2) = 48 \text{ cm}^3$$

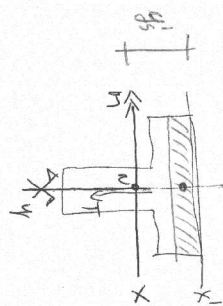
2a

8



$$r_A = \frac{z}{2}$$

$$\sigma_{max} = 2$$



$$\sigma_A = \frac{S_x' \cdot T_y}{b \cdot l \cdot x} = \frac{43962 \cdot 14.49}{20 \cdot 10585}$$

$$= 0.0295 \frac{\text{N}}{\text{cm}^2} = 0.295 \text{ MPa}$$

$$S_x' = 3 \cdot 20 \cdot (8.677 - 15) = 43962 \text{ cm}^3$$

$$I_x = 10585 \text{ cm}^4$$

$$I_x = \frac{20 \cdot 6^3}{12} + 20 \cdot 5.677^2 + \frac{8 \cdot 16^3}{12} + 128 \cdot 5.323^2$$

$$y_s' = 8.677 \text{ cm}$$

$$S_x = 120 \cdot 3 + 128 \cdot 14 = 2152 \text{ cm}^3$$

$$A = 6 \cdot 20 + 16 \cdot 8 = 120 + 128 = 248 \text{ cm}^2$$

2b

FELÜL:

$$\sigma_A = -\frac{3.882}{248} + \frac{4347}{10585} \cdot 8.677 = 3.548 \frac{\text{N}}{\text{cm}^2}$$

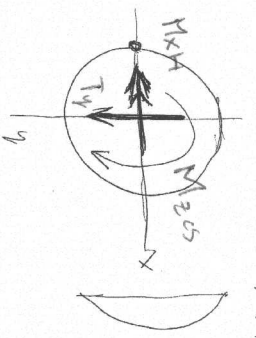
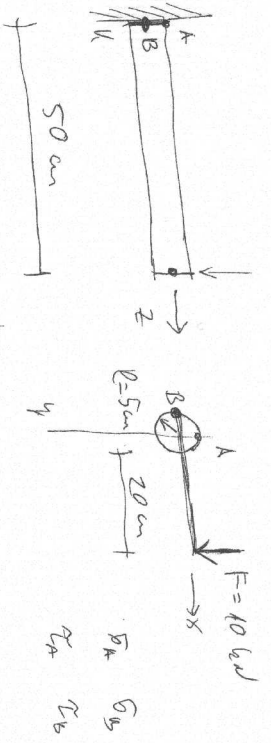
ALUL:

$$\sigma_a = -\frac{3.882}{248} - \frac{4347}{10585} \cdot 13.323 = -5.1487 \frac{\text{N}}{\text{cm}^2}$$

$$\frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x$$

$$= 0, \text{ MGEET } M_y = 0$$

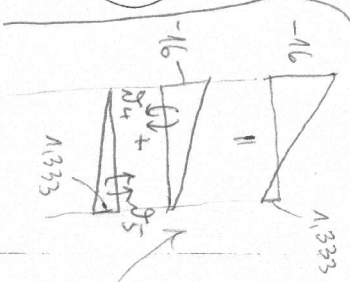
9



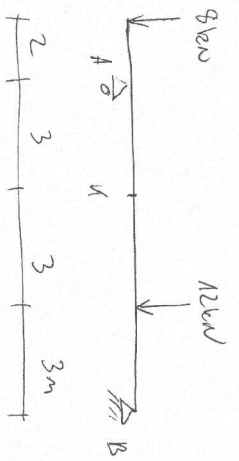
$I_x = 4 \text{ cm}^4$
 $I_y = 4 \text{ cm}^4$
 $I_z = 0$
 $M_x = 0$
 $M_y = 10 \cdot 50 = 500 \text{ kNm}$
 $M_z = 10 \cdot 25 = 250 \text{ kNm}$

$I = 785 \text{ cm}^2$
 $I_x = 490,6 \text{ cm}^4$
 $I_y = 981,3 \text{ cm}^4$
 $S_x = 0$
 $S_y = 78,5 \text{ cm}^3$
 $S_x = 0$
 $S_y = 83,33 \text{ cm}^3$

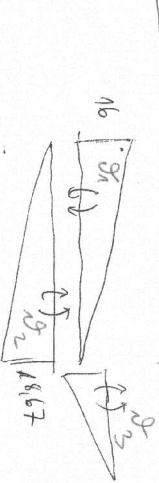
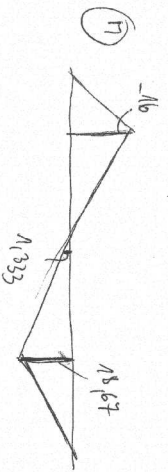
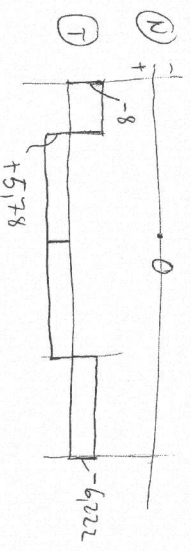
$G_B = 0$
 $G_A = + \frac{500}{490,6} \cdot 5 = 5,096 \text{ cm}^2$
 $G_A = \frac{250}{981,3} \cdot 5 = 1,274 \text{ cm}^2$
 $G_B = - \frac{250}{981,3} \cdot 5 + \frac{8333 \cdot 10}{10 \cdot 490,6} = -1,104 \text{ cm}^2$



10



9. REAKTIONEN
 $R_x = 0$
 $R_y = 6,222 \text{ kN}$
 $R_z = 13,778 \text{ kN}$

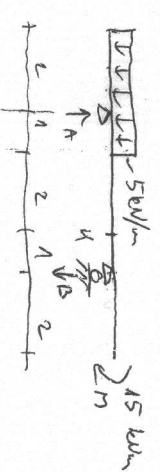


$E_B = 0 + 16 \cdot 9 + \frac{16 \cdot 6}{2} \cdot 7 - \frac{18,67 \cdot 6}{2} \cdot 5 - \frac{18,67 \cdot 3}{2} \cdot 2 = 0$
 $E_A = 0 + 6,2 \cdot 10^{-3} \cdot 3 + \frac{18,67}{2} \cdot 2 - \frac{13,33 \cdot 3}{2} \cdot 1 = 46,02 / EI$

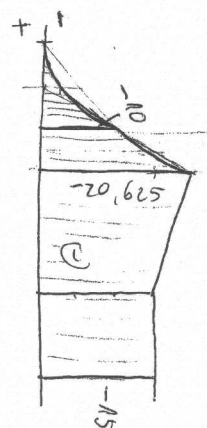
- K PUNT ABAN
 A PUNT ABAN
 - K PUNT ABAN

$A = \frac{66}{2} = 18 \text{ cm}^2$
 $I_x = \frac{66}{36} = 1 \text{ cm}^4$
 $S_x = \frac{3 \cdot 3}{2} \cdot (4m) = 9 \text{ cm}^3$
 $I_A = - \frac{133,3}{1} \cdot 1 = -133,3 \text{ cm}^4$
 $I_{2x} = \frac{9 \cdot 5,78}{8 \cdot 1} = 17,34 \text{ cm}^2$
 $I_{2y} = 18,28 \text{ cm}^2$

aA)



$A = 16,875 \text{ kN} (\uparrow)$
 $B = 18,75 \text{ kN} (\uparrow)$

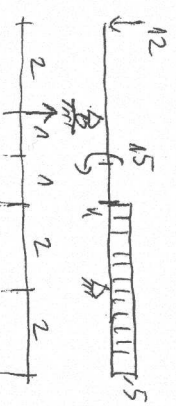


$-5 \cdot 2 \cdot \frac{2}{2} = -10$
 $\frac{5 \cdot 2^2}{8} = 2,5$
 $-10 - 1,875 \cdot 3 = -20,625$
 $\frac{5 \cdot 1^2}{8} = 0,625$

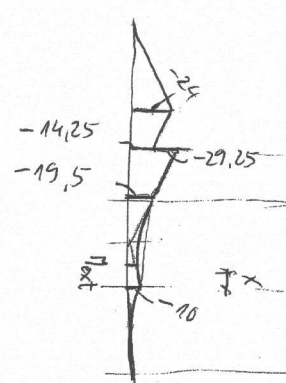
$M_A = -15 \cdot 1 \cdot 1,875 = -16,875 \text{ kNm}$
 $M_{\max} = -20,625 \text{ kNm}$



cA)



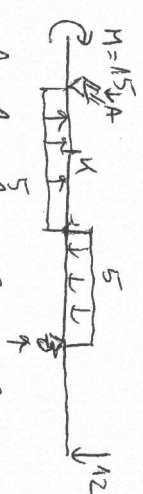
$A = 29,75 \text{ kN} (\uparrow)$
 $B = 10,25 \text{ kN} (\uparrow)$



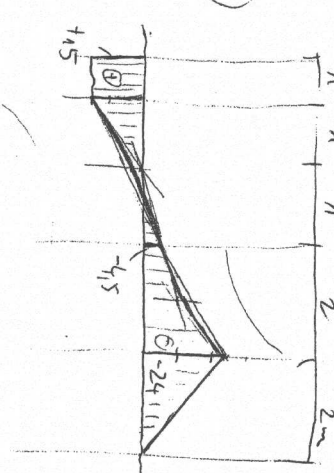
$M_A = -12 \cdot 4 + 29,75 \cdot 2 - 15 = -19,5 \text{ kNm}$
 $T(x) = -5 \cdot 2 + 10,25 - 5 \cdot x = 0$
 $x = 0,05 \text{ m}$
 $M_{\text{ext}} = -9,994 \text{ kNm}$
 $M_{\max} = -29,25 \text{ kNm}$



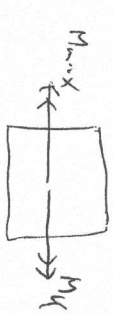
dA)



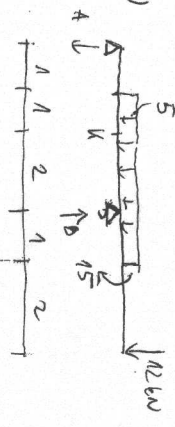
$A = 14,75 \text{ kN} (\downarrow)$
 $B = 26,75 \text{ kN} (\uparrow)$



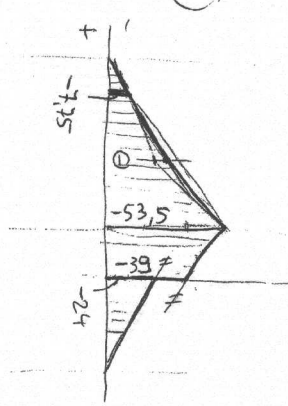
$-12 \cdot 4 + 26,75 \cdot 2 - 5 \cdot 2 \cdot 1 = -4,5$
 $\frac{5 \cdot 2^2}{8} = 2,5$
 $M_A = +15 - 14,75 \cdot 1 + 5 \cdot 1 \cdot 0,5 = +2,75 \text{ kNm}$
 $M_{\min} = -24 \text{ kNm}$



bA)



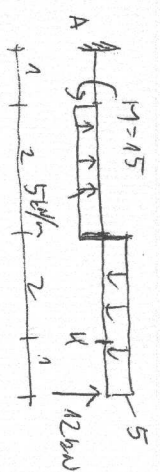
$A = 7,75 \text{ kN} (\downarrow)$
 $B = 39,75 \text{ kN} (\uparrow)$



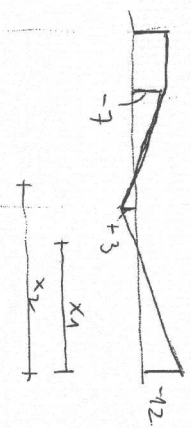
$-12 \cdot 3 - 15 - 5 \cdot 1 \cdot 0,5 = -53,5$
 $\frac{5 \cdot 1^2}{8} = 0,625$
 $\frac{5 \cdot 3^2}{8} = 5,625$
 $-7,75 \cdot 2 - 5 \cdot 1 \cdot 0,5 = -18 \text{ kNm} = M_A$
 $M_{\min} = -53,5 \text{ kNm}$



eA)



(T)



$$x_1 = \frac{12}{5} = 2.4 \text{ m}$$

$$x_2 = 3 + \frac{15 \cdot 12}{5} = 3.6 \text{ m}$$

$$M_1 = +12 \cdot 2.4 - 12 \cdot 1.2 = +14.4 \text{ kNm}$$

$$M_2 = +12 \cdot 6 - 15 \cdot 4.5 + 10 \cdot 2 + 15 = 31.5$$

$$\frac{5 \cdot x^2}{8} = 5.625 \quad \frac{5 \cdot x^2}{8} = 2.5$$

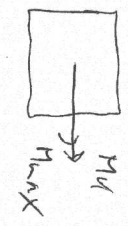
$$M_2 = 12 \cdot 3.6 - 15 \cdot 2.1 + 10 \cdot 0.5 + 9.3 = 12.6$$

$$+12 \cdot 3 - 15 \cdot 1.5 = 13.5$$

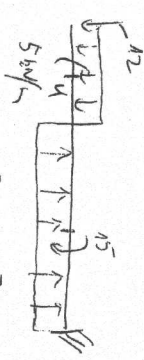
$$+12 \cdot 5 - 15 \cdot 3.5 + 10 \cdot 1 = 17.5$$

$$M_{\max} = 39.5 \text{ kNm}$$

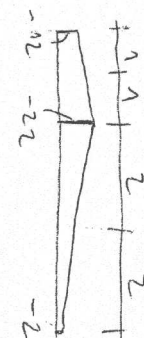
$$M_1 = 12 \cdot 1 - 5 \cdot 1 \cdot \frac{1}{2} = 9.5 \text{ kNm}$$



fA)



(T)



$$\frac{5 \cdot x^2}{8} = 2.5$$

$$-12 \cdot 2 - 5 \cdot 2 \cdot 1 = -34$$

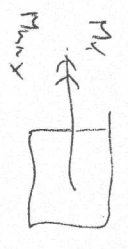
$$-12 \cdot 4 - 5 \cdot 2 \cdot 3 + 5 \cdot 2 \cdot 1 = -68$$

$$-68 + 15 = -53$$

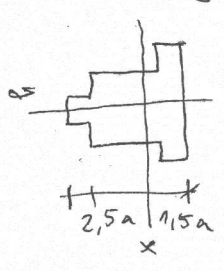
$$-12 \cdot 6 - 5 \cdot 2 \cdot 5 + 15 + 5 \cdot 4 \cdot 2 = -67$$

$$M_1 = -12 \cdot 5 \cdot 1.5 = -94.5 \text{ kNm}$$

$$M_{\max} = -68 \text{ kNm}$$

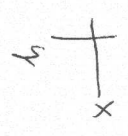


h)

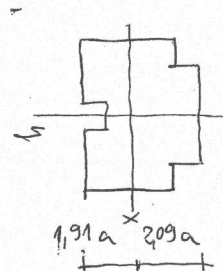


$$I_x = 13 a^4 = I_2$$

$$I_y = 15 a^4 = I_1$$



g)



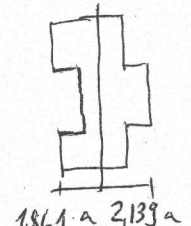
$$I_x = 19.53 a^4 = I_2$$

$$I_y = 33.42 a^4 = I_1$$

$$19.53432$$

$$33.41667$$

f)

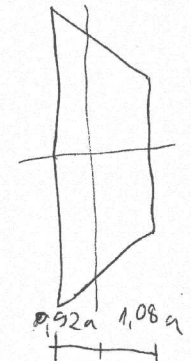


$$I_x = 18.65 a^4 = I_2$$

$$I_y = 7.2 a^4 = I_1$$

e)

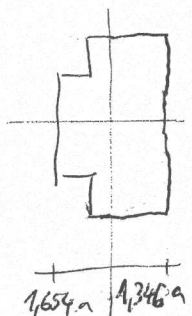
$$I_x = I_y = 189.7286 a^4$$



$$I_x = 2.6111 a^4 = I_2$$

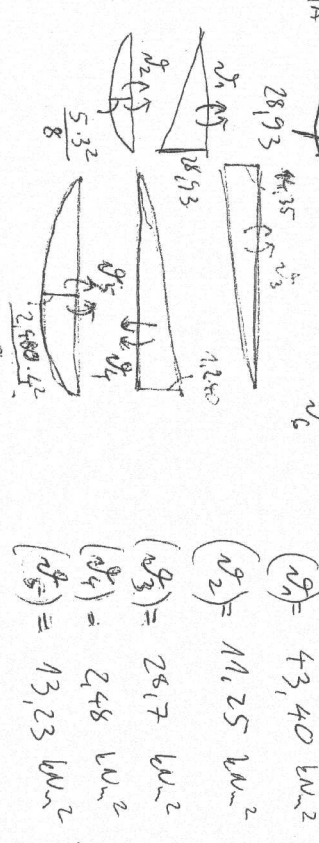
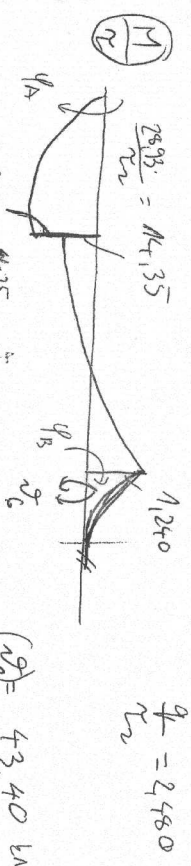
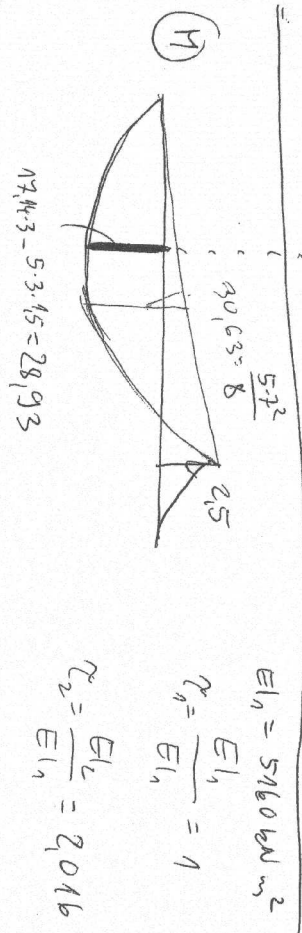
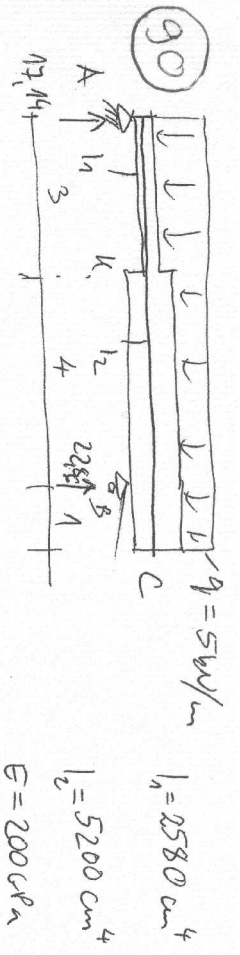
$$I_y = 11.3333 a^4 = I_1$$

d)



$$I_x = 8.776 a^4 = I_2$$

$$I_y = 23.08 a^4 = I_1$$



A → B

$$q_A \cdot 7 - 43,40 \cdot 0,5 - 11,25 \cdot 5,5 - 28,7 \cdot \frac{8}{3} + 248 \cdot \frac{4}{3} - 13,23 \cdot 2 = 0$$

(q_A) = +54,08 kNm² (↺)

B → A

$$-q_B \cdot 7 - 13,23 \cdot 5 + 248 \cdot \frac{17}{3} - 28,7 \cdot \frac{13}{3} - 11,25 \cdot 1,5 - 43,40 \cdot 2 = 0$$

(q_B) = -40,02 kNm² (↻)

ELL: $54,08 - 43,40 - 11,25 - 28,7 + 248 - 13,23 = -40,02$ ✓

A → C

(e_u) = +54,08 · 3 - 43,40 · 1 - 11,25 · 1,5 = 107,0 kNm³ (↓)

(q_u) = +54,08 - 43,40 - 11,25 = -0,57 kNm² (↻)

B → C

(e_c) = -40,02 · 1 + 13,23 · $\frac{3}{4}$ = -30,10 kNm³ (↑)

(q_c) = -40,02 + 13,23 = -26,79 kNm² (↻)

$q_A = +1048 \cdot 10^3 \text{ mm}^3$ (↺) $q_B = -7,756 \cdot 10^3 \text{ mm}^3$ (↻)

$q_U = +19,77 \text{ mm}^3$ (↓) $e_C = -5,833 \text{ mm}^3$ (↑)

$q_U = 0,110 \cdot 10^3 \text{ mm}^3$ (↻) $q_C = -5,192 \cdot 10^3 \text{ mm}^3$ (↻)

