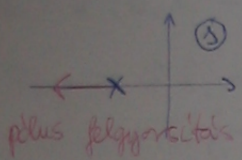
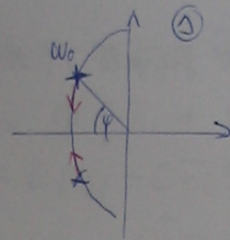
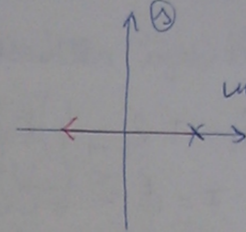
Szabályzott szakasz:

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$





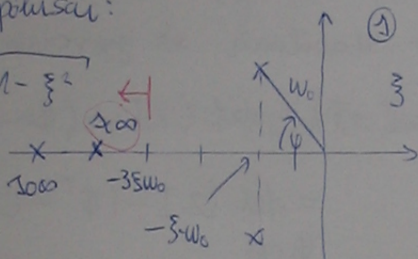
$\xi = \cos\varphi$



instabil pólus

Zárt Rendszer (ZR) pólusai:

$$s_1 = -\xi w_0 + j w_0 \sqrt{1-\xi^2}$$
$$s_2 = \bar{s}_1$$
$$s_{c\infty}$$



$\xi = \cos\varphi$

↑
legalább 3-szor messzebb az origótól, mint $-\xi w_0$.

T mintavételi idő:

$z_1 = e^{s_1 T}$; $z_2 = \bar{z}_1$

$z_{c\infty} = e^{s_{\infty} T}$

$z_{0\infty} = e^{s_0 T}$

Shanon tétel miatt:

$$|s_1|, |\bar{s}_1|, |s_{c\infty}|, |s_{0\infty}| < \frac{\pi}{T} = w_N$$

Szabályozó tervezés diszkrét időben

(szakasznál: $D = 0$)

$$(A, B, C, D) \xrightarrow[\text{'zoh'}, T]{c2d} (\Phi, \Gamma, C, D)$$

$$\left.\begin{aligned} x_{i+1} &= \Phi x_i + \Gamma u_i \\ y_i &= C x_i \end{aligned}\right\} \text{szakasz}$$

Állapot visszacsatolás (ÁV): $u_i = -K x_i$

ZR : $x_{i+1} = (\Phi - \Gamma K) x_i$

ZR karakterisztikus egyenlet (ZRKE):

$$\varphi_c(z) = \det(zI - (\Phi - \Gamma K)), \quad K = ?$$

Ackermann képlettel:

$$K = (0 \dots 0\,1)\, M_c^{-1}\, \varphi_c(\Phi) \quad \rightarrow \text{ SISO rendszereknél használható}$$

$$M_c = [\Gamma \;\; \Phi\Gamma \dots \Phi^{n-1}\Gamma] \quad \rightarrow \text{ Kvadratikus mátrix}$$

Diszkrét idejű aktuális állapot figyelő (ÁM):

$$\hat{x}_i = F\hat{x}_{i-1} + G y_i + H u_{i-1}$$

(1) $F = \Phi - GC\Phi$

(2) $H = \Gamma - GC\Gamma$

(3) $\tilde{x}_i = F\tilde{x}_{i-1}$ ⇐ stabilnak és gyorsnak kell tervezni

$$\varphi_0(z) = \det(zI - F) = \det(zI - (\Phi - GC\Phi)) =$$

$$= \det(zI - (\Phi^T - \Phi^T C^T G^T))$$

$$(\Phi, C)_I \rightarrow (\Phi^T, \Phi^T C^T)_{II} \xrightarrow[M_{cII}]{\varphi_0(z)} K_{II} \longrightarrow G = K_{II}^T$$

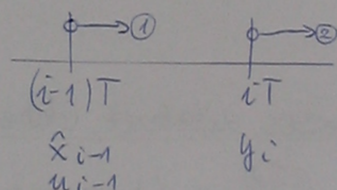
$$F = \Phi - GC\Phi$$

$$H = \Gamma - GC\Gamma$$

$$(\Phi, \Gamma) \xrightarrow[M_c]{\varphi_c(z)} K$$

(1) $\bar{x}_i = \Phi\hat{x}_{i-1} + \Gamma u_{i-1}$

(2) $\hat{x}_i = \bar{x}_i + G(y_i - C\bar{x}_i)$



Alapjel miatti korekció:

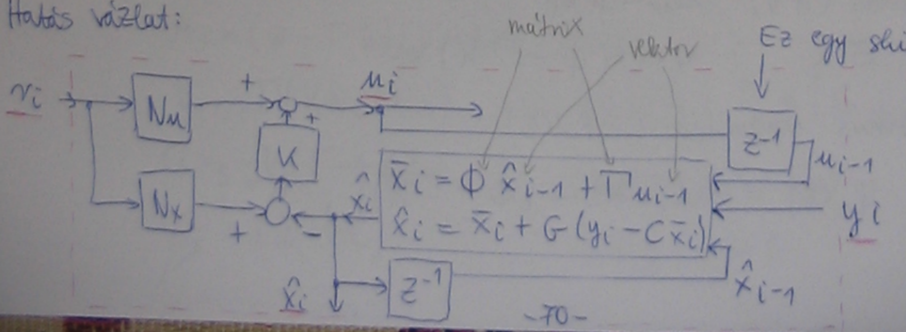
$$\text{ÁÁ}: \; x_\infty = \Phi x_\infty + \Gamma u_\infty$$

$$\begin{pmatrix} N_x \\ N_u \end{pmatrix} = \begin{bmatrix} \Phi - I & \Gamma \\ C & 0 \end{bmatrix}^{-1} \begin{pmatrix} 0_{n \times m} \\ I_m \end{pmatrix}$$

N_u – skalár

N_x – vektor

Hatás vázlat:



Szimulinkben megvalósítható!

XI.23. P.
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Az előző ábrán:

Két bemenet: $\underline{r}_i$, $\underline{y}_i$

Két kimenet: $\underline{u}_i$, $\underline{\hat{x}}_i$

Ezeket összelehet-e vonni? Igen:

$$u_i = K(N_x r_i - \hat{x}_i) + N_u r_i = -K\hat{x}_i + (KN_x + N_u) r_i$$

$$\bar{x}_{i+1} = \Phi \hat{x}_i + \Gamma u_i = \Phi\{\bar{x}_i + G(y_i - C\bar{x}_i)\} +$$
$$+ \Gamma\{-K[\bar{x}_i + G(y_i - C\bar{x}_i)] + (KN_x + N_u) r_i\}$$

$$\bar{x}_{i+1} = \{\underbrace{\Phi - \Phi G C}_{\Phi(I-GC)} - \Gamma K(I-GC)\}\bar{x}_i + \Gamma(KN_x + N_u) r_i + (\Phi - \Gamma K) G y_i$$

$$\bar{x}_{i+1} = \underbrace{(\Phi - \Gamma K)(I-GC)}_{A_c}\bar{x}_i + \underbrace{\Gamma(KN_x + N_u)}_{B_{cr}} r_i + \underbrace{(\Phi - \Gamma K)G}_{B_{cy}} y_i$$

$$u_i = -K\{\bar{x}_i + G(y_i - C\bar{x}_i)\} + (KN_x + N_u) r_i =$$
$$= \underbrace{-K(I-GC)}_{C_u}\bar{x}_i + \underbrace{(KN_x + N_u)}_{D_{ur}} r_i \underbrace{- KG}_{D_{uy}} y_i$$

$$\hat{x}_i = \underbrace{(I-GC)}_{C_{\hat{x}}}\bar{x}_i + \underbrace{0}_{D_{\hat{x}r}} \cdot r_i + \underbrace{G}_{D_{\hat{x}y}} y_i$$

A kapott rendszer összefoglalva:

$$\bar{x}_{i+1} = A_c \bar{x}_i + [B_{cr} \mid B_{cy}] \begin{pmatrix} r_i \\ y_i \end{pmatrix} \rightarrow \underline{u_i}$$

$$\begin{pmatrix} u_i \\ \hat{x}_i \end{pmatrix} = \begin{bmatrix} C_u \\ C_{\hat{x}} \end{bmatrix} \bar{x}_i + \begin{bmatrix} D_{ur} & D_{uy} \\ 0 & D_{\hat{x}y} \end{bmatrix} \begin{pmatrix} r_i \\ y_i \end{pmatrix} \rightarrow \underline{\hat{x}_i}$$

$\uparrow \underline{r_i}$ $\uparrow \underline{y_i}$

$(\Phi, \Gamma, C) \rightarrow K$ (ÁV)

$(\Phi, \Gamma, C) \rightarrow F, G, H$ (ÁM)

$(\Phi, \Gamma, C) \rightarrow N_x, N_u$

$A_c = (\Phi - \Gamma K)(I - GC)$

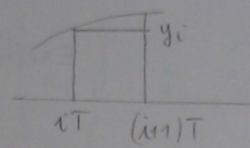
$B_{cr} = \Gamma(KN_x + N_u)$ $\quad B_{cy} = (\Phi - \Gamma K) G$

$C_u = -K(I - GC)$ $\quad D_{ur} = (KN_x + N_u)$ $\quad D_{\hat{x}y} = G$

$C_{\hat{x}} = (I - GC)$ $\quad D_{uy} = -KG$

Integráló szabályozás

$$x_I = \int_0^t y\,dt$$



$$x_{I,i+1} = x_{I,i} + \overbrace{C x_i}^{y_i}\, T$$

Bővített rendszer

$$\begin{pmatrix} x_{i+1} \\ x_{I,i+1} \end{pmatrix} = \begin{bmatrix} \Phi & 0 \\ CT & I \end{bmatrix} \begin{pmatrix} x_i \\ x_{I,i} \end{pmatrix} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} u_i$$

$$y_i = \begin{bmatrix} C & 0 \end{bmatrix} \begin{pmatrix} x_i \\ x_{I,i} \end{pmatrix}$$

Átnevezve:

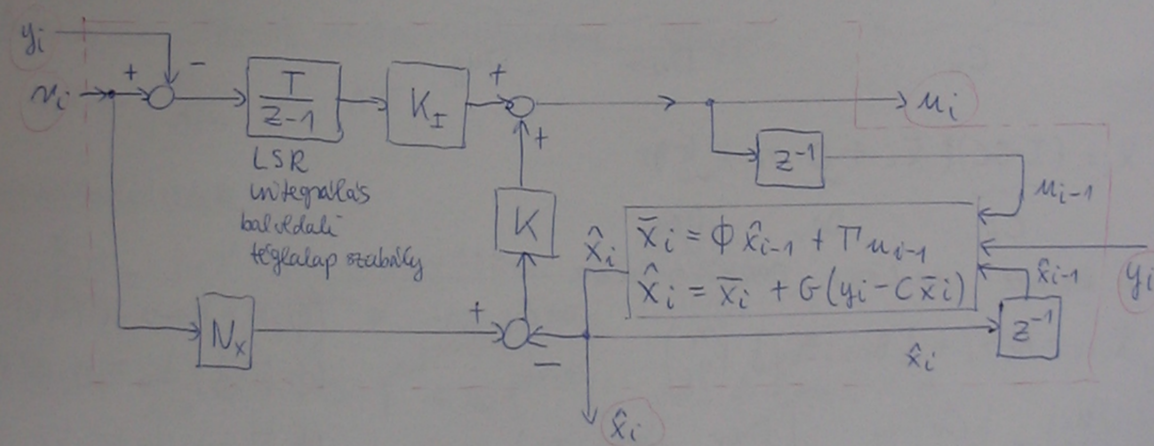
$$\tilde{x}_{i+1} = \tilde{\Phi}\,\tilde{x}_i + \tilde{\Gamma}\,u_i$$

$$y_i = \tilde{C}\,\tilde{x}_i$$

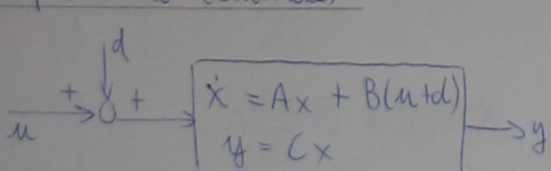
$(\tilde{\Phi}, \tilde{\Gamma}, \tilde{C}) \longrightarrow \tilde{K} = [K \;\; K_I]$ (ÁV)

$(\Phi, \Gamma, C) \longrightarrow F, G, H$ (ÁM)

$(\Phi, \Gamma, C) \longrightarrow N_x, \cancel{N_u}$



Állapot becslő (zavarás)



Zavar: $x_d = d$

Konstans zavarás: $\dot{x}_d = 0$ $\qquad x_{d,i+1} = x_{d,i}$

Diszkrét időben

$$\begin{pmatrix} x_{i+1} \\ x_{d,i+1} \end{pmatrix} = \begin{bmatrix} \Phi & \Gamma \\ 0 & I \end{bmatrix} \begin{pmatrix} x_i \\ x_{di} \end{pmatrix} + \begin{pmatrix} \Gamma \\ 0 \end{pmatrix} u_i$$

$$y_i = [C \;\; 0] \begin{pmatrix} x_i \\ x_{di} \end{pmatrix}$$

Nem irányítható, de megfigyelhető

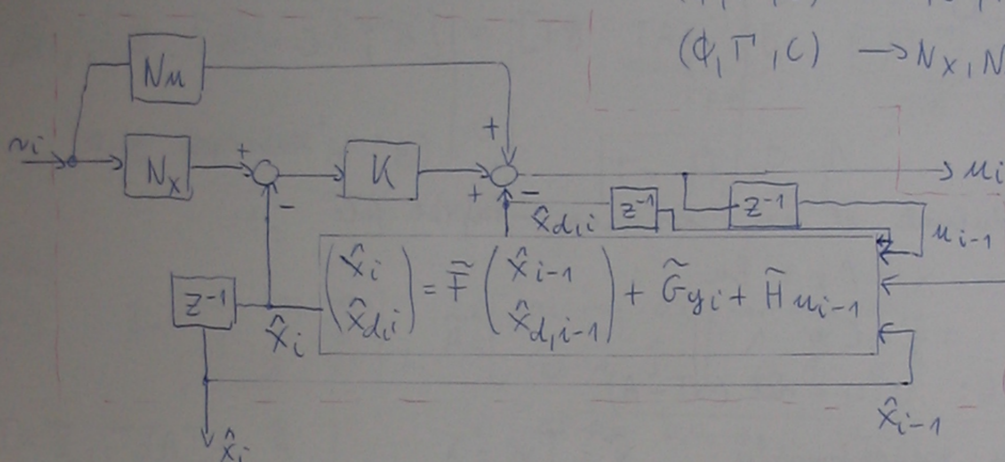
$$\tilde{x}_{i+1} = \tilde{\Phi} \tilde{x}_i + \tilde{\Gamma} u_i$$

$$y_i = \tilde{C} \tilde{x}_i$$

$(\Phi, \Gamma, C) \rightarrow K$ (A'V)

$(\tilde{\Phi}, \tilde{\Gamma}, \tilde{C}) \rightarrow \tilde{F}, \tilde{G}, \tilde{H}$ (A'M)

$(\Phi, \Gamma, C) \rightarrow N_x, N_u$



Ackermann képlet rendszertechnikai alapja

$$W(s) = \frac{B(s)}{A(s)} \rightarrow \text{A'E}$$

Szabályozó alak

$$\frac{B(s)}{A(s)} = \frac{b_1 s^{n-1} + \ldots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n} = \frac{Y(s)}{U(s)}$$

$$\xi(s) = \frac{1}{A(s)} U(s) \Rightarrow \xi^{(n)} + a_1 \xi^{(n-1)} + \ldots + a_{n-1} \xi' + a_n \xi = u$$

$$x = \begin{pmatrix} \xi^{(n-1)} \\ \xi^{(n-2)} \\ \vdots \\ \xi' \\ \xi \end{pmatrix}$$

deriváltak

$$\frac{dx_1}{dt} = \frac{d}{dt} \xi^{(n-1)} = \xi^{(n)} = -a_1 \xi^{(n-1)} - \ldots - a_{n-1} \xi' - a_n \xi + u =$$
$$= -a_1 x_1 - \ldots - a_{n-1} x_{n-1} - a_n x_n$$

$$\frac{dx_2}{dt} = \frac{d}{dt} \xi^{(n-2)} = \xi^{(n-1)} = x_1$$

$$\vdots$$

$$\frac{dx_n}{dt} = \frac{d\xi}{dt} = \xi' = x_{n-1}$$

→

$Y(s) = B(s)\,\xi(s)$

$y = b_1 \xi^{(n-1)} + \dots + b_{n-1}\xi' + b_n \xi = b_1 x_1 + \dots + b_{n-1} x_{n-1} + b_n x_n$

$$\dot{x} = Ax + Bu \qquad W(s) = \frac{b_1 s^{n-1} + \dots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \dots + a_{n-1}s + a_n}$$
$$y = Cx$$

$$A = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_n \\ 1 & 0 & \dots & 0 \\ & \ddots & 0 & \vdots \\ 0 & & 1 & 0 \end{bmatrix}; \quad B = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}; \quad C = [b_1 \dots b_{n-1},\, b_n]$$

"szabályozó alak"

Most indulunk el az Ackermann-képlet felé

Szakasz: $\dot{x} = Ax + Bu$

$y = Cx$

SZ.KE: $\varphi(s) = \det(sI - A)$

Koordináta transzformáció: $\tilde{x} = Tx$ | $\tilde{A} = TAT^{-1}$, $\tilde{B} = TB$

$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u$ | $\tilde{C} = CT^{-1}$

$y = \tilde{C}\tilde{x}$

Keletkező rendszer: szabályozó alak

$\varphi(s) = \det(sI - A) = \det(sI - \tilde{A})$ invariáns

ÁV: $u = -\tilde{K}\tilde{x} = -\underbrace{\tilde{K}T}_{K}x = -Kx$; $\tilde{K} \xrightarrow{T} K = \tilde{K}T$

ZR: $\dot{\tilde{x}} = (\tilde{A} - \tilde{B}\tilde{K})\tilde{x}$

ZRKE: $\varphi_c(s) = \det(sI - (\tilde{A} - \tilde{B}\tilde{K})) = s^n + p_1 s^{n-1} + \dots + p_{n-1}s + p_n$

$$\tilde{A} - \tilde{B}\tilde{K} = \begin{bmatrix} -a_1 & -a_2 & \dots & -a_n \\ 1 & 0 & \dots & \vdots \\ & \ddots & 0 & \vdots \\ 0 & & 1 & 0 \end{bmatrix} - \underbrace{\begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} [\tilde{k}_1\ \tilde{k}_2\ \dots\ \tilde{k}_n]}_{\begin{bmatrix} \tilde{k}_1 & \tilde{k}_2 & \dots & \tilde{k}_n \\ 0 & 0 & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}} =$$

$$\tilde{A} - \tilde{B} * \tilde{K} = \begin{bmatrix} -(a_1+\tilde{k}_1) & -(a_2+\tilde{k}_2) & \dots & -(a_n+\tilde{k}_n) \\ 1 & 0 & & 0 \\ & \ddots & & \vdots \\ 0 & & 1 & 0 \end{bmatrix}$$

$p_1 \quad p_2 \quad p_n$

$\tilde{k}_1 = p_1 - a_1, \quad \tilde{k}_2 = p_2 - a_2, \quad \dots, \quad \tilde{k}_n = p_n - a_n$

$\tilde{K} = [p_1 - a_1, \; p_2 - a_2, \dots, p_n - a_n]$

$$\tilde{M}_c = [\tilde{B} \;\; \tilde{A}\tilde{B} \; \dots \; \tilde{A}^{n-1}\tilde{B}] = [TB \;\; \underbrace{TA\cancel{T^{-1}T}B}_{TAB} \dots \underbrace{(TAT^{-1})^{n-1}TB}_{TA^{n-1}B}] =$$

$$= T[\underbrace{B \;\; AB \; \dots \; A^{n-1}B}_{M_c}] \quad ; \quad \boxed{T = \tilde{M}_c M_c^{-1}}$$

Példa:

$\underline{n=3}$: $\tilde{A} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$, $\tilde{A}^2 = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} =$

e_2^T

$I = \tilde{A}^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad$ $\tilde{A}^2 = \begin{bmatrix} a_1^2 - a_2 & a_1a_2 - a_3 & -a_1a_3 \\ -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \end{bmatrix}$

e_3^T $\quad$ e_1^T

e^T = egység vektor transzponáltja

$\tilde{M}_c = [\tilde{B} \;\; \tilde{A}\tilde{B} \;\; \tilde{A}^2\tilde{B}] = \begin{bmatrix} 1 & -a_1 & a_1^2 - a_2 \\ 0 & 1 & -a_1 \\ 0 & 0 & 1 \end{bmatrix}$; $\tilde{M}_c^{-1} = \begin{bmatrix} 1 & a_1 & a_2 \\ 0 & 1 & a_1 \\ 0 & 0 & 1 \end{bmatrix}$

$\tilde{M}_c$ és $\tilde{M}_c^{-1}$ szorzata egységmátrixot kell adjon! Ez igaz!

Általában: $\tilde{M}_c^{-1} = \begin{bmatrix} 1 & a_1 & a_2 & \dots & a_{n-1} \\ 0 & 1 & a_1 & \dots & a_{n-2} \\ \vdots & & & \ddots & \\ 0 & 0 & & \dots & 1 \end{bmatrix}$

e_n^T

I = egységmátrix

$\varphi_c(\tilde{A}) = \tilde{A}^n + p_1\tilde{A}^{n-1} + \dots + p_{n-1}\tilde{A} + p_n I$

$\varphi(\tilde{A}) = \tilde{A}^n + a_1\tilde{A}^{n-1} + \dots + a_{n-1}\tilde{A} + a_n I = 0$ (Cayley-Hamilton)

$$\tilde{A}^n = -a_1 \tilde{A}^{n-1} - \dots - a_{n-1}\tilde{A} - a_n I$$

$\downarrow$ Ezt helyettesítjük be, $\varphi_c(\tilde{A})$-ba, $\tilde{A}^n$ helyére:

$$\varphi_c(\tilde{A}) = \underbrace{(p_1 - a_1)}_{\tilde{k}_1}\tilde{A}^{n-1} + \underbrace{(p_2 - a_2)}_{\tilde{k}_2}\tilde{A}^{n-2} + \dots + \underbrace{(p_{n-1} - a_{n-1})}_{\tilde{k}_{n-1}}\tilde{A} + \underbrace{(p_n - a_n)}_{\tilde{k}_n} I$$

$$\left.\begin{aligned} e_n^T \cdot \varphi_c(\tilde{A}) &= [\tilde{k}_1\ \tilde{k}_2\ \dots \tilde{k}_n] = \tilde{K} \\ e_n^T \cdot \varphi_c(TAT^{-1}) &= e_n^T T \varphi_c(A) T^{-1} \end{aligned}\right\} \Rightarrow$$

$$\Rightarrow K = \tilde{K}T = e_n^T T \varphi_c(A) \cancel{T^{-1}T} = \underbrace{e_n^T \tilde{M}_c}_{e_n^T} M_c^{-1} \varphi_c(A) \Rightarrow$$ Ackermann-képlet

$$\boxed{K = (0 \dots 0\,1)\, M_c^{-1}\, \varphi_c(A)}$$