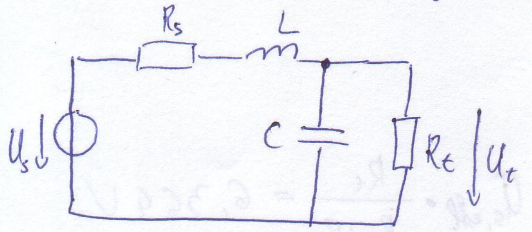
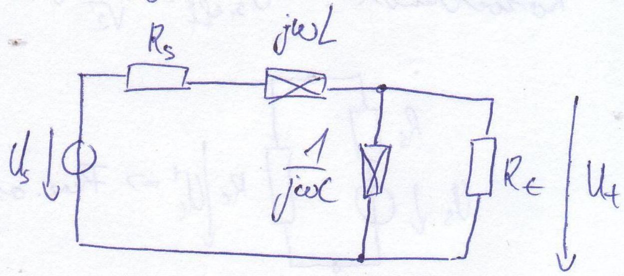
$R_s = 50\,\Omega \quad R_t = 450\,\Omega \quad \omega_0 = 10\,\frac{Mrad}{s}$

1



$\Rightarrow$



a) Fesz. osztóval:

$$U_t = U_s \frac{\frac{1}{j\omega C} \times R_t}{\frac{1}{j\omega C} \times R_t + j\omega L + R_s} = U_s \frac{\frac{\frac{R_t}{j\omega C}}{R_t + \frac{1}{j\omega C}}}{\frac{\frac{R_t}{j\omega C}}{R_t + \frac{1}{j\omega C}} + j\omega L + R_s} = \frac{\frac{R_t}{j\omega C}}{\frac{R_t}{j\omega C} + R_t j\omega L + \frac{j\omega L}{j\omega C} + R_t R_s + \frac{R_s}{j\omega C}}$$

$$H(j\omega) = \frac{U_t}{U_s} = \frac{R_t}{R_t + R_t (j\omega)^2 LC + Lj\omega + \cancel{R_t} R_s j\omega C + R_s}$$

$$\boxed{H(j\omega) = \frac{\frac{1}{LC}}{(j\omega)^2 + j\omega\left(\frac{1}{R_t C} + \frac{R_s}{L}\right) + \frac{1}{LC} + \frac{R_s}{R_t LC}}}$$

b)

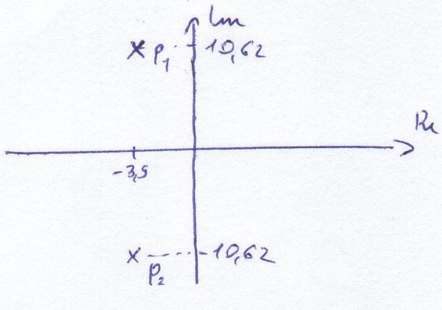
$$H(j\omega) = \frac{112{,}5}{(j\omega)^2 + 7j\omega + 125} \xrightarrow{\text{lousoli}} H(s) = \frac{112{,}5}{s^2 + 7s + 125}$$

$\Rightarrow$ Zérusa nincs a nevezőnek.

$\Rightarrow$ Pólusok: $s_{1,2} = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 125}}{2}$

$p_1 = -3{,}5 + j\,10{,}62 \ \frac{1}{\mu s}$

$p_2 = -3{,}5 - j\,10{,}62 \ \frac{1}{\mu s}$



c)

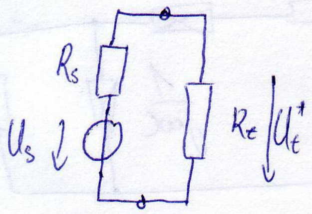
$$U_s(t) = 10\cos(\omega_0 t)\ V \Rightarrow H(j\omega)\Big|_{\omega=\omega_0 = 10\frac{Mrad}{s}} = 1{,}5135\,e^{-j1{,}228}$$

$\hookrightarrow U_t(t) = 15{,}135 \cos(10t - 1{,}228)\ V$

$\hookrightarrow U_{t,eff} = \frac{15{,}135}{\sqrt{2}} = 10{,}7021\ V$

$$P_t = \frac{U_{t,eff}^2}{R_t} = \underline{\underline{0{,}2545\ W}}$$

d) Közvetlenül: $U_{s,eff} = \frac{10}{\sqrt{2}} V = 7,071 V$



$\Rightarrow$ Fesz. osztó: $U_t^* = U_{s,eff} \cdot \frac{R_t}{R_s + R_t} = 6,364 V$

$$P_t^* = \frac{{U_t^*}^2}{R_t} = \underline{\underline{0,09 W}}$$

(2)

$$y[k] = c_1\left(u[k-1] - y[k-1]\right) + c_2\left(u[k] - y[k-2]\right) + u[k-2]$$

a,

$$Y = c_1\left(Uz^{-1} - Yz^{-1}\right) + c_2\left(U - Yz^{-2}\right) + Uz^{-2}$$

$$Y = c_1 U z^{-1} - c_1 Y z^{-1} + c_2 U - c_2 Y z^{-2} + U z^{-2}$$

$$H(z) = \frac{Y}{U} = \underline{\underline{\frac{c_1 z^{-1} + c_2 + z^{-2}}{1 + c_1 z^{-1} + c_2 z^{-2}}}}$$

Jury-krit.: $|c_1| < 1+c_2 \quad c_2 < 1$

$c_1 = 1{,}4 \Rightarrow 1{,}4 < 1 + c_2 \Rightarrow 0{,}4 < c_2$

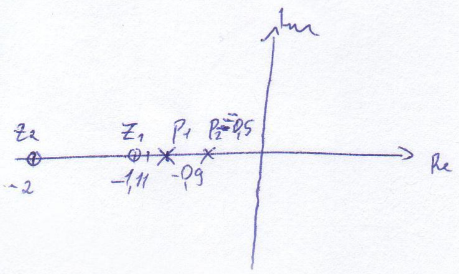
$\Rightarrow \underline{\underline{0{,}4 < c_2 < 1}}$

b,

$$H(z) = \frac{0{,}45 + 1{,}4 z^{-1} + z^{-2}}{1 + 1{,}4 z^{-1} + 0{,}45 z^{-2}} = \frac{0{,}45 z^2 + 1{,}4 z + 1}{z^2 + 1{,}4 z + 0{,}45}$$

Zérusok: $z_{1,2} = \dfrac{-1{,}4 \pm \sqrt{1{,}4^2 - 4 \cdot 0{,}45}}{2 \cdot 0{,}45}$ $z_1 = -\frac{10}{9} \approx -1{,}11$, $z_2 = -2$

Pólusok: $p_{1,2} = \dfrac{-1{,}4 \pm \sqrt{1{,}4^2 - 4 \cdot 0{,}45}}{2}$ $p_1 = -0{,}5$, $p_2 = -0{,}9$



c.) $u[k] = \delta[k]$ és $k<0$ energiamentes → imp. válasz?

$$H(z) = 0,45\,\frac{z^2 + \frac{28}{9}z + \frac{20}{9}}{z^2 + 1,4z + 0,45} = 0,45\,\frac{z^2 + 1,4z + 0,45 + \frac{77}{45}z + \frac{319}{180}}{z^2 + 1,4z + 0,45}$$

$$H(z) = 0,45 + 0,45\,\frac{\frac{77}{45}z + \frac{319}{180}}{(z+0,9)(z+0,5)} = \frac{A}{z+0,5} + \frac{B}{z+0,9} + 0,45$$

$$A = \lim_{z\to -0,5} 0,45\cdot\frac{\frac{77}{45}z + \frac{319}{180}}{z+0,9} = 1,0313$$

$$B = \lim_{z\to -0,9} 0,45\cdot\frac{\frac{77}{45}z + \frac{319}{180}}{z+0,5} = -0,2613$$

$$H(z) = 0,45 + \frac{1,0313}{z+0,5} + \frac{-0,2613}{z+0,9}$$

$$\hookrightarrow h[k] = 0,45\,\delta[k] + \varepsilon[k-1]\left(1,0313(-0,5)^{k-1} + (-0,2613)(-0,9)^{k-1}\right)$$

d.) $$H(z) = \frac{0,45 + 1,4z^{-1} + z^{-2}}{1 + 1,4z^{-1} + 0,45z^{-2}} \Rightarrow Y + 1,4Yz^{-1} + 0,45Yz^{-2} = 0,45U + 1,4Uz^{-1} + Uz^{-2}$$

$$y[k] = -1,4y[k-1] - 0,45y[k-2] + 0,45u[k] + 1,4u[k-1] + u[k-2]$$

$y[-2] = y[-1] = 1$

↓ behelyettesítéssel

$y[0] = -1,4$

$y[1] = 2,91$

$y[\infty] = 0$, mert G-V stabil a rendszer (pólusok egységkörön belül vannak)

1) $f(t) = 2 + 3\cos(\omega_0 t) - 4\cos(3\omega_0 t)$ V $\qquad \omega_0 = 100\pi$ rad/s

$$F_{eff} = \sqrt{2^2 + \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \underline{\underline{4{,}062\ V}}$$

2) $u(t) = 3 + 2\cos(\omega_0 t - 1{,}4)$ V $\qquad i(t) = 4\cos(\omega_0 t + 2{,}1) + 5\cos(2\omega_0 t)$ mA

Parseval tétel teljesítményre: $P = U_0 \cdot I_0 + \sum_{q=1}^{\infty} \frac{U_q \cdot I_q}{2} \cdot \cos(\varphi_q) \qquad \varphi_q = \alpha_q - \beta_q$

$$P = \frac{2 \cdot 4}{2} \cdot \cos(3{,}5) = \underline{\underline{-3{,}746\ mW}}$$

3) $X(j\omega) = \begin{cases} 1 - \frac{|\omega|}{2}, & |\omega| < 2 \\ 0, & |\omega| > 2 \end{cases} \qquad [\omega] = \frac{rad}{s}$

$a_{dB} = 20\ dB \Rightarrow 10^{-\frac{20}{20}} = 1 - \frac{|\omega|}{2}$

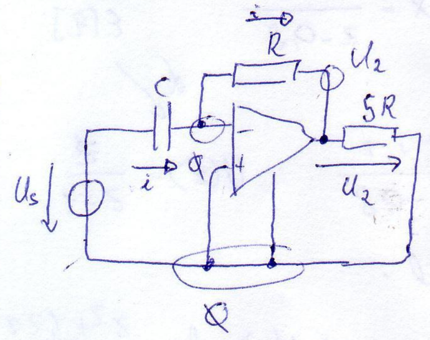
$\frac{1}{10} = 1 - \frac{|\omega|}{2} \Rightarrow 0{,}2 = 2 - |\omega| \Rightarrow |\omega| = 1{,}8$

$\underline{\underline{\omega_b = 1{,}8\ \frac{rad}{s}}}$

4) $f(t)$ spektrum $F(j\omega)$ $\qquad g(t) = 2f(t)\cos(4t) = \frac{1}{2}\left(e^{j4t} + e^{-j4t}\right) \cdot 2f(t)$

→ Modulációs tétellel $\qquad \underline{\underline{G(j\omega) = F(j(\omega - 4)) + F(j(\omega + 4))}}$

5)



$i = \frac{U_s}{\frac{1}{sC}} = sC \cdot U_s \qquad U_2 = -i \cdot R = -sC \cdot R \cdot U_s$

↳ $\underline{\underline{H(s) = -sCR}}$

6) $H(s) = \frac{9s^2}{s^2 + 0{,}6s + 0{,}05} \qquad U(s) = \frac{1}{s}\left(= \mathcal{L}(\varepsilon(t))\right) \qquad Y(s) = \frac{9s}{s^2 + 0{,}6s + 0{,}05}$

$Y(s) = \frac{A}{s + 0{,}1} + \frac{B}{s + 0{,}5} \qquad A = \lim_{s \to -0{,}1} \frac{9s}{s + 0{,}5} = -\frac{9}{4} = -2{,}25$

$B = \lim_{s \to -0{,}5} \frac{9 \cdot s}{s + 0{,}1} = 11{,}25$

$y(t) = \varepsilon(t)\left(11{,}25 e^{-j0{,}5t} - 2{,}25 e^{-j0{,}1t}\right) \Rightarrow \underline{\underline{y(0) = 9}}$

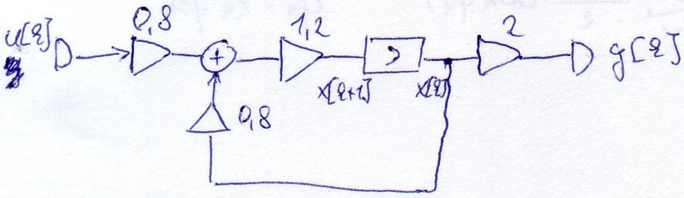
(7) $u(t)=\varepsilon(t-2) \Rightarrow y(t)=\varepsilon(t-2)\left[5+3e^{-2(t-2)}\right]$

lineáris, invariáns

$u(t)=\varepsilon(t) \Longrightarrow y(t)=\varepsilon(t)\left[5+3e^{-2t}\right]$

$\cancel{Y(j\omega)} = \mathcal{L}\{y(t)\} = Y(s) = \frac{5}{s}+\frac{3}{s+2} = \frac{5s+10+3s}{s^2+2s} = \underline{\underline{\frac{8s+10}{s^2+2s}}}$

(8)



$$\boxed{\begin{aligned} x[k+1] &= 0,96\, x[k] + 0,96\, u[k] \\ y[k] &= 2\, x[k] \end{aligned}}$$

(9) $x[0]=x[3]=1 \qquad x[1]=x[2]=0 \qquad L=4 \qquad \vartheta=0$ frekv. tag

$X_0 = \frac{1}{4}(1+0+1+0) = \underline{\underline{0,5}}$

(10) $h[k]=\delta[k-2]+\delta[k+2]$ spektrum: Fourier-transzf.

$H(e^{j\vartheta}) = \sum_{k=-\infty}^{\infty} h[k]\, e^{-j\vartheta k} = e^{-j2\vartheta}+e^{j2\vartheta} = \underline{\underline{2\cos(2\vartheta)}}$

(11) $x[k+1]=0,8x[k]-u[k]$ $\Rightarrow$ $Xz=0,8X-U \Rightarrow X=\frac{-U}{z-0,8}$

$y[k]=-2x[k]+u[k]$ $\qquad Y=-2X+U$

$Y=\frac{2U}{z-0,8}+U = \frac{2U-0,8U+Uz}{z-0,8} \Rightarrow H(z)=\frac{1,2+z}{z-0,8}$ $\qquad \varepsilon[k] \xrightarrow{\mathcal{Z}} U(z)=\frac{z}{z-1}$

$Y(z)=H(z)\cdot U(z) = \frac{z^2+1,2z}{(z-0,8)(z-1)}$

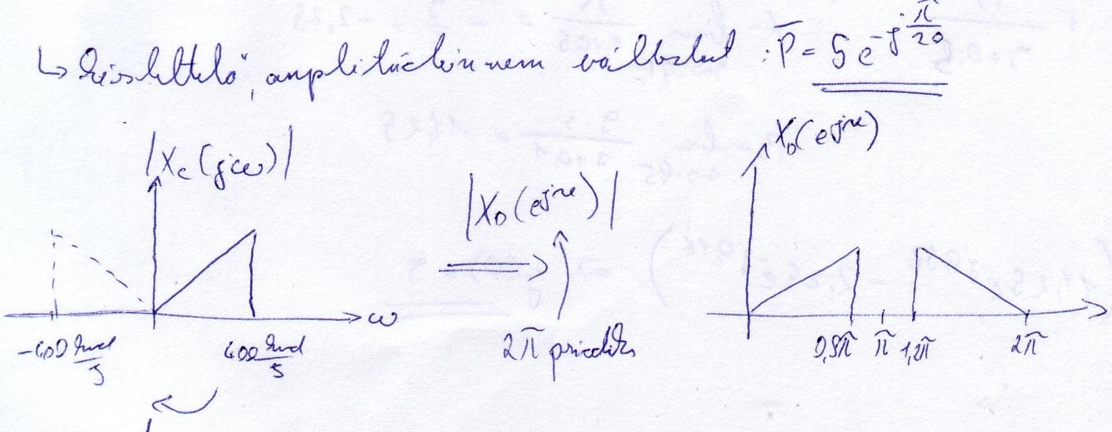
Végértéktétel:

$y[\infty] = \lim_{z\to 1}(z-1)\cdot Y(z) = \lim_{z\to 1}\frac{z^2+1,2z}{z-0,8} = \underline{\underline{11}}$

(12) $y[k]=5\cos\left(\frac{\pi}{20}k-\frac{\pi}{10}\right)$

↳ késleltető, amplitúdón nem változtat: $\bar{P}=\underline{\underline{5e^{-j\frac{\pi}{20}}}}$

(13)



(14) AM-DSB-SC esetben a modulált jel (Ω_s) sávsz. 2x-ese az eredetinek → $400 \frac{\text{rad}}{s}$ $\Rightarrow$ $\Omega_s = 800 \frac{\text{krad}}{s}$

(15)

$$H_D(z) = H_C(s) \Big|_{s = \frac{1}{T} \frac{z-1}{z+1}}$$