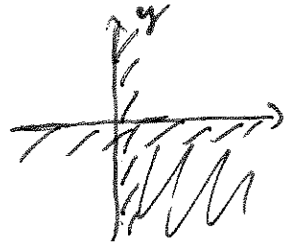
α VARIÁNS:

Henger - koordinátákkal



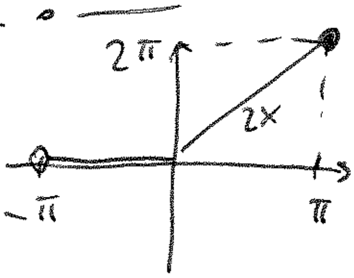
1* / $\boxed{12}$

$$V:\begin{cases} x^2+y^2\le 4 \\ x\ge 0,\ y\le 0 \\ 0\le z\le x \end{cases} \iff \begin{array}{l} 0\le r\le 2\ ; \\ -\pi/2\le\varphi\le 0 \\ 0\le z\le r\cos\varphi \end{array}$$

$$I=\int_{r=0}^{2}\int_{\varphi=-\pi/2}^{0}\int_{z=0}^{r\cos\varphi}\underbrace{r\cos\varphi}_{x}\ \underbrace{r\sin\varphi}_{y}\cdot\underbrace{r}_{\text{Jac.}}\,dz\,d\varphi\,dr= \quad (6)$$

$$=\int_{r=0}^{2}\int_{\varphi=-\pi/2}^{0} r^4\cos^2\varphi\sin\varphi\,d\varphi\,dr \overset{(2)}{=}\left[\frac{r^5}{5}\right]_0^2\cdot\left[-\frac{\cos^3\varphi}{3}\right]_{-\pi/2}^{0} \overset{(3)}{=} \underline{\underline{\frac{32}{5}\cdot\left(\frac{-1}{3}\right)}} \quad (1)$$

2* / $\boxed{12}$



$$a_0=\frac{1}{\pi}\int_0^{\pi}2x\,dx=\frac{1}{\pi}\left[x^2\right]_0^{\pi}=\pi \quad (3)$$

$$a_1=\frac{1}{\pi}\int_0^{\pi}2x\cdot\cos x\,dx=\frac{1}{\pi}\underbrace{\left[2x\sin x\right]_0^{\pi}}_{0}-\frac{1}{\pi}\int_0^{\pi}2\sin x\,dx=\frac{2}{\pi}\left[\cos x\right]_0^{\pi}=\underline{\underline{-\frac{4}{\pi}}} \quad (4)$$

$$b_1=\frac{1}{\pi}\int_0^{\pi}2x\sin x\,dx=\frac{1}{\pi}\underbrace{\left[-2x\cos x\right]_0^{\pi}}_{2\pi}+\frac{2}{\pi}\underbrace{\int_0^{\pi}\cos x\,dx}_{[\sin x]_0^{\pi}=0}=\underline{\underline{2}} \quad (4)$$

$$T_3(x)=\frac{a_0}{2}+a_1\cos x+b_1\sin x=\underline{\underline{\frac{\pi}{2}-\frac{4}{\pi}\cos x+2\sin x}} \quad (1)$$

3* / $\boxed{12}$

T.: $\mathcal{F}[f(x+h)](\omega)=e^{i\omega h}\,\mathcal{F}[f](\omega)$ (4)

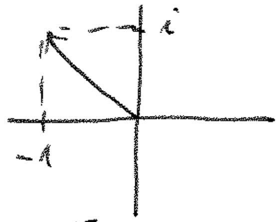
(8)

$$\text{B.: } \mathcal{F}[f(x+h)](\omega)=\int_{-\infty}^{\infty}e^{-i\omega x}f(x+h)\,dx \underset{\substack{t=x+h \\ x=t-h \\ dx=dt}}{=} \int_{-\infty}^{\infty}e^{-i\omega(t-h)}f(t)\,dt$$

$$=e^{i\omega h}\,\mathcal{F}[f](\omega)$$

4, $\boxed{10}$

$$z^3 = \frac{-2}{1+i} = \frac{-2(1-i)}{(1+i)(1-i)} = -1+i = \sqrt{2}\cdot e^{i\frac{3}{4}\pi} \quad (3)$$



(4)

$$z_k = 2^{1/6}\cdot e^{i\left(\frac{\pi}{4}+k\frac{2\pi}{3}\right)}, \quad k=0,1,2$$

$$\underline{\underline{z_0 = 2^{1/6}\cdot e^{i\frac{\pi}{4}}}} \ (1); \quad \underline{\underline{z_1 = 2^{1/6}\cdot e^{i\frac{11}{12}\pi}}} \ (1); \quad \underline{\underline{z_2 = 2^{1/6}\cdot e^{i\frac{19}{12}\pi}}} \ (1)$$

— o —

5, a, $\boxed{12}$ D.: $f:\mathbb{R}^n \to \mathbb{R}$ totálisan diff.-ható az $x_0 \in \text{int}\, D_f \subset \mathbb{R}^n$-ben, ha $\exists\, \underline{v}\in\mathbb{R}^n$, és $\underline{\varepsilon}(\underline{x})$: $\forall\, \underline{x}\in D_f$:

$$\Delta f = f(x)-f(x_0) = \langle \underline{v}, \underline{x}-\underline{x}_0\rangle + \langle \underline{\varepsilon}(\underline{x}), \underline{x}-\underline{x}_0\rangle, \text{ ahol}$$

$$\lim_{\underline{x}\to \underline{x}_0} \underline{\varepsilon}(\underline{x}) = 0. \text{ Ekkor } \underline{v} = \operatorname{grad} f(\underline{x}_0) \quad (3)$$

b, T.: Ha f tot. diff.-ható $\underline{x}_0$-ban akkor f folytonos x_0-ban. (3)

B.: $$f(x) = f(x_0) + \langle \underline{v}, \underbrace{\underline{x}-\underline{x}_0}_{\downarrow\, 0}\rangle + \langle \underbrace{\underline{\varepsilon}(x)}_{\downarrow\, 0}, \underbrace{\underline{x}-\underline{x}_0}_{\downarrow\, 0}\rangle \xrightarrow{\underline{x}\to\underline{x}_0} f(x_0) \checkmark \quad (6)$$

— o —

6, $\boxed{12}$

$$\int_{x=0}^{\infty} \frac{1}{\sqrt{x+1}\,(x+4)}\,dx = \lim_{K\to\infty}\int_{u=1}^{\infty} \frac{1}{\cancel{u}(u^2+3)}\cdot 2\cancel{u}\,du \overset{(5)}{=} \lim_{K\to\infty}\frac{2}{3}\int_{u=1}^{\infty}\frac{du}{1+\left(\frac{u}{\sqrt{3}}\right)^2} =$$

$u=\sqrt{x+1}$
$x = u^2-1$;
$dx = 2u\,du$

$$= \frac{2}{3}\lim_{K\to\infty}\left[\sqrt{3}\arctan\left(\frac{u}{\sqrt{3}}\right)\right]_1^K = \frac{2}{\sqrt{3}}\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \underline{\underline{\frac{2\pi}{3\sqrt{3}}}} \quad (5)\ (2)$$

— o —

7, $\boxed{10}$ Szeparálható:

$$\int\frac{\cos y}{\sin y}\,dy = \int\frac{x^3}{2x^2+5}\,dx \quad (2)$$

$$\int\frac{\cos y}{\sin y}\,dy = \underline{\ln|\sin y| + C} \quad (3)$$

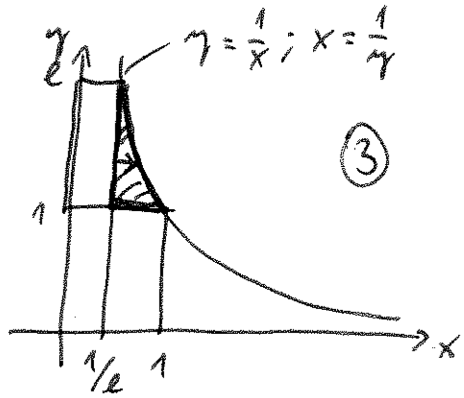
$$\int\frac{x^3}{2x^2+5}\,dx = \int\frac{\frac{x}{2}(2x^2+5)-\frac{5}{2}x}{2x^2+5}\,dx = \int\frac{x}{2}\,dx - \frac{5}{8}\int\frac{4x}{2x^2+5}\,dx =$$

$$= \underline{\frac{x^2}{4} - \frac{5}{8}\ln(2x^2+5) + C} \ (4) \Rightarrow \underline{\underline{\ln|\sin y| = \frac{x^2}{4} - \frac{5}{8}\ln(2x^2+5) + C}} \ (1)$$

8, Tudjuk, hogy $\operatorname{ch} x = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}$, így ③ —3—

$\boxed{8}$

$$\sum_{n=0}^{\infty} \frac{3^n}{(2n)!} \cdot x^{2n+3} = x^3 \cdot \sum_{n=0}^{\infty} \frac{(\sqrt{3}x)^{2n}}{(2n)!} = \underline{\underline{x^3 \operatorname{ch}(\sqrt{3}x)}} \quad ⑤$$



9, $\boxed{12}$

$$I = \int_{y=1}^{e} \int_{x=1/e}^{1/y} \cos(x - \ln x)\, dx\, dy =$$

④
$$= \int_{x=1/e}^{1} \int_{y=1}^{1/x} \cos(x - \ln x)\, dy\, dx =$$

② ②
$$= \int_{x=1/e}^{1} \left(\frac{1}{x} - 1\right) \cos(x - \ln x)\, dx = \Big[-\sin(x - \ln x)\Big]_{x=1/e}^{1} = \underline{\underline{-\sin(1) + \sin\left(\frac{1}{e} + 1\right)}} \quad ①$$

$-\varphi' \cdot \cos\varphi$ alak

β VARIÁNS (TÖMÖR)

1* $\boxed{12}$

$$I = \int_{r=0}^{3} \int_{\varphi=0}^{\pi/2} \int_{z=0}^{r\sin\varphi} r^2 \sin\varphi \cos\varphi \cdot r \, dz\, d\varphi\, dr = \left(\int_{r=0}^{3} r^4\, dr\right) \cdot \left(\int_{\varphi=0}^{\pi/2} \sin^2\varphi \cos\varphi\, d\varphi\right) \quad ⑥$$

$$= \frac{3^5}{5} \cdot \left[\frac{\sin^3\varphi}{3}\right]_0^{\pi/2} ⑤ = \underline{\underline{\frac{3^4}{5}}} \quad ①$$

2* $\boxed{12}$

$$f(x) = \begin{cases} 2x, & \text{ha } x \in (-\pi, 0] \\ 0, & \text{ha } x \in [0, \pi] \end{cases} ; \quad a_0 = \frac{1}{\pi} \int_{-\pi}^{0} 2x\, dx = \frac{-\pi^2}{\pi} = \underline{\underline{-\pi}} \quad ③$$

$$a_1 = \frac{1}{\pi} \int_{-\pi}^{0} 2x \cos x\, dx = \frac{2}{\pi} \Big[x \sin x\Big]_{-\pi}^{0} - \frac{2}{\pi} \int_{-\pi}^{0} \sin x\, dx = \underline{\underline{\frac{4}{\pi}}} \quad ④$$

$$b_1 = \frac{1}{\pi} \int_{-\pi}^{0} 2x \sin x\, dx = \frac{2}{\pi} \Big[x(-\cos x)\Big]_{-\pi}^{0} + \frac{2}{\pi} \int_{-\pi}^{0} \cos x\, dx = \underline{\underline{2}} \quad ④$$

$$\underline{\underline{T_3(x) = \frac{\pi}{2} + \frac{4}{\pi} \cos x + 2 \sin x}} \quad ①$$

3* $\boxed{12}$ Mint az α variáns.

4, $\boxed{10}$ $z^3 = \frac{2(1+i)}{2} = 1+i = \underbrace{2^{1/2}\cdot e^{i\pi/4}}_{}$ ③ ; $z_k = 2^{1/6} e^{i\left(\frac{\pi}{4}+\frac{2\pi k}{3}\right)}$ ④ ; $k=0,1,2$

$z_0 = 2^{1/6} e^{i\pi/12}$ ① ; $z_1 = 2^{1/6}\cdot e^{i\frac{3\pi}{4}}$ ① ; $z_2 = 2^{1/6}\cdot e^{i\frac{19\pi}{12}}$ ①

5, $\boxed{12}$ Lásd a,

6, $\boxed{12}$ $\displaystyle\int_{x=0}^{\infty}\frac{dx}{(x+16)\underbrace{\sqrt{x+4}}_{u}} = \int_{u=2}^{\infty}\frac{2\cancel{u}\,du}{(u^2+12)\cdot\cancel{u}}$ ⑤ $= \frac{1}{6}\lim_{K\to\infty}\left[\sqrt{12}\,\mathrm{arctg}\left(\frac{u}{\sqrt{12}}\right)\right]_2^K$ ⑤ $=$

$x = u^2-4;\ dx = 2u\,du$

$= \frac{1}{\sqrt{3}}\left(\frac{\pi}{2}-\frac{\pi}{6}\right) = \frac{\pi}{3\sqrt{3}}$ ②

7, Szeparálható ②

$\boxed{10}$ bal old.: $\int \frac{\sin y}{\cos y}\,dy = -\ln|\cos(y)| + C$ ③

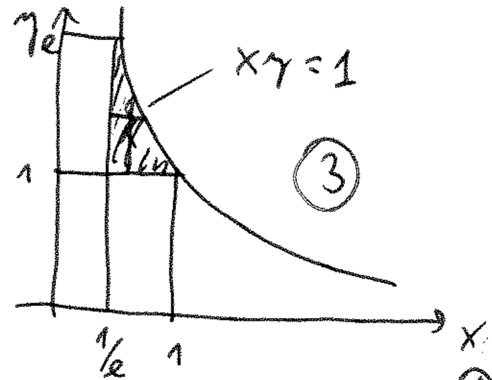
jobb old.: $\int \frac{x^3}{3x^2+2}\,dx = \int \frac{x}{3}\,dx - \frac{2}{3}\int\frac{x}{3x^2+2}\,dx = \frac{x^2}{6} - \frac{1}{9}\ln(3x^2+2) + C$ ④

Megoldás: $-\ln|\cos y| = \frac{x^2}{6} - \frac{1}{9}\ln(3x^2+2) + C$ ①

8, $\boxed{8}$ Ismert: $\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$ ③

$\sum_{n=0}^{\infty}\frac{(-5)^n\cdot\sqrt{5}}{(2n+1)!}x^{2n+4} = x^3\sum_{n=0}^{\infty}\frac{(-1)^n(\sqrt{5}x)^{2n+1}}{(2n+1)!} = x^3\sin(\sqrt{5}x)$ ⑤

9, $\boxed{12}$ $I = \int_{y=1}^{e}\int_{x=1/e}^{1/y}\sin(\ln x - x)\,dx\,dy =$



③

$= \int_{x=1/e}^{1}\int_{y=1}^{1/x}\sin(\ln x - x)\,dy\,dx =$ ④

$= \int_{x=1/e}^{1}\sin(\ln x - x)\left(\frac{1}{x}-1\right)dx$ ② $= \left[-\cos(\ln x - x)\right]_{1/e}^{1}$ ② $= -\cos 1 + \cos\left(1+\frac{1}{e}\right)$ ①