$\boxed{1.}$ $\sum \frac{5^n \cdot n!}{n^n}$ HÁNYADOS KRIT.

$$\lim_{\infty} \frac{5^{n+1}(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{5^n \cdot n!} = \lim_{\infty} \frac{5 \cdot 5^n \cdot (n+1) \cdot n! \, n^n}{(n+1)^n (n+1) \cdot 5^n \cdot n!} =$$

(1P) (1P)

$$= \lim_{\infty} \frac{n^n}{(n+1)^n} \cdot 5 = \lim_{\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n} \cdot 5 = \frac{5}{e} > 1 \Rightarrow \underline{\text{DIV}}$$

(1P) (1P) (1P)

$\boxed{5P}$

$\sum \frac{3^n (x+2)^{2n}}{n}$ $\quad \lim \sqrt[n]{\left|\frac{3^n (x+2)^{2n}}{n}\right|} = 3 \cdot (x+2)^2 < 1$

(1P) (1P)

$|x+2| < \frac{1}{\sqrt{3}}$ (1P)

$R = \frac{1}{\sqrt{3}}$ (1P)

$-2-\frac{1}{\sqrt{3}}$ $\quad -2 \quad$ $-2+\frac{1}{\sqrt{3}}$

VÉGPONTOK:

$\underline{\underline{\sum \frac{1}{n} \text{ DIV}}}$ (1P) $\quad$ $\underline{\underline{\sum \frac{1}{n} \text{ DIV}}}$ (1P)

$\boxed{6P}$

$\boxed{11 \text{ PONT}}$

$\boxed{2.}$ $$f(x) = \frac{x^2}{\sqrt{e^{2x}}} = x^2 \cdot e^{-x} = x^2 \cdot \sum_0^\infty \frac{(-x)^n}{n!} = \sum \frac{(-1)^n}{n!} x^{n+2}$$

(1P) (1P) (2P)

$$f(x) = \operatorname{ch} x^3 = \sum_{n=0}^\infty \frac{(x^3)^{2n}}{(2n)!} = \sum_{n=0}^\infty \frac{x^{6n}}{(2n)!}$$

(1P) (1P)

$$h(x) = x^2 \cdot \sin x \cdot \cos x = \frac{x^2 \cdot \sin 2x}{2} = \frac{x^2}{2} \cdot \sum_0^\infty \frac{(-1)^n (2x)^{2n+1}}{(2n+1)!} =$$

(1P) (1P)

$$= \sum_0^\infty \frac{4^n (-1)^n}{(2n+1)!} \cdot x^{2n+3}$$ (2P)

$\boxed{10 \text{ PONT}}$

3.

$$f(x)=\frac{4x}{5+x^4}=\frac{4x}{5}\cdot\frac{1}{1-\left(-\frac{x^4}{5}\right)}=\frac{4x}{5}\cdot\sum_0^\infty\left(-\frac{x^4}{5}\right)^n=$$

1P 1P

$$=\sum_0^\infty \frac{4}{5}\left(\frac{-1}{5}\right)^n\cdot x^{4n+1}$$

1P

$$\left|-\frac{x^4}{5}\right|<1$$

$$\underline{\underline{|x|<\sqrt[4]{5}}}$$ 1P

$\boxed{4P}$

$$g(x)=\frac{1}{x^2+2x+2}=\frac{1}{(x+1)^2+1}=\frac{1}{1-\left(-(x+1)^2\right)}=\sum_0^\infty\left(-(x+1)^2\right)^n=$$

1P 1P 1P

$$=\sum_0^\infty(-1)^n\cdot(x+1)^{2n}$$

1P

$|x+1|<1$ 1P

$\underline{\underline{-2<x<0}}$ 1P

$\boxed{10\ \text{PONT}}$

4.

$$f(x)=\sqrt{x^5+4x^4}=2x^2\sqrt{1+\frac{x}{4}}=2x^2\left(1+\frac{x}{4}\right)^{1/2}=$$

1P 1P

$$=2x^2\cdot\sum_0^\infty\binom{1/2}{n}\left(\frac{x}{4}\right)^n=\sum_0^\infty\binom{1/2}{n}\left(\frac{1}{4}\right)^n\cdot 2\cdot x^{n+2}$$

1P 1P

$$\left|\frac{x}{4}\right|<1$$

$$\underline{\underline{|x|<4}}$$

1P $\boxed{5\ \text{PONT}}$

5.

$$f(x,y)=\frac{e^{xy}}{x^2+y^2}$$

1P

NEM LÉTEZIK VÉGES HATÁRÉRTÉK MERT PL $y=0$ ESETÉN 1P $f(x)=\frac{1}{x^2}$ 1P $\lim f=+\infty$ 1P

NEM DIFFHATÓ, MERT NINCS VÉGES HATÁRÉRT $\Rightarrow$ NEM FOLYT 1P

$$f'_x=\frac{e^{xy}\cdot y\cdot(x^2+y^2)-e^{xy}\cdot 2x}{(x^2+y^2)^2}$$

2P 1P

$$g'_x=4y\left(x^3+\operatorname{ch}x\right)^{4y-1}\cdot\left(3x^2+\operatorname{sh}x\right)$$ 3P

$$g'_y=\left(x^3+\operatorname{ch}x\right)^{4y}\cdot\ln\left(x^3+\operatorname{ch}x\right)\cdot 4$$ 3P

$\boxed{14\ \text{PONT}}$