

1. $\sum \frac{5^n \cdot n!}{n^n}$ HÁNYADOS KRIT.

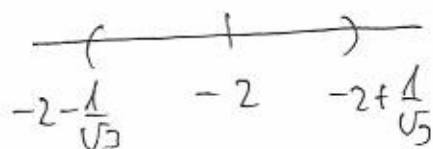
$$\lim_{\infty} \frac{5^{n+1} (n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{5^n \cdot n!} = \lim_{\infty} \frac{5 \cdot 5^n \cdot (n+1) \cdot n! \cdot n^n}{(n+1)^n (n+1) \cdot 5^n \cdot n!} =$$

$$= \lim_{\infty} \frac{n^n}{(n+1)^n} \cdot 5 = \lim_{\infty} \frac{1}{\left(1 + \frac{1}{n}\right)^n} \cdot 5 = \frac{5}{e} > 1 \Rightarrow \text{DIV}$$

$\sum \frac{3^n \cdot (x+2)^{2n}}{n}$ $\lim \sqrt[n]{\left| \frac{3^n (x+2)^{2n}}{n} \right|} = \frac{3 \cdot (x+2)^2}{1} < 1$

$$|x+2| < \frac{1}{\sqrt{3}}$$

$$R = \frac{1}{\sqrt{3}}$$



VÉGPONTOK:



$$\sum \frac{1}{n} \text{ DIV}$$

$$\sum \frac{1}{n} \text{ DIV}$$

6P

11 PONT

2. $f(x) = \frac{x^2}{\sqrt{e^{2x}}} = x^2 \cdot e^{-x} = x^2 \cdot \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} x^{n+2}$

$$g(x) = \text{ch } x^3 = \sum_{n=0}^{\infty} \frac{(x^3)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{x^{6n}}{(2n)!}$$

$$h(x) = x^2 \cdot \sin x \cdot \cos x = \frac{x^2 \cdot \sin 2x}{2} = \frac{x^2}{2} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n \cdot (2x)^{2n+1}}{(2n+1)!} =$$

$$= \sum_{n=0}^{\infty} \frac{4^n (-1)^n}{(2n+1)!} \cdot x^{2n+3}$$

10 PONT

3. $f(x) = \frac{4x}{5+x^4} = \frac{4x}{5} \cdot \frac{1}{1 - (-\frac{x^4}{5})} = \frac{4x}{5} \cdot \sum_0^{\infty} \left(-\frac{x^4}{5}\right)^n =$
 $= \sum_0^{\infty} \frac{4}{5} \left(-\frac{1}{5}\right)^n \cdot x^{4n+1}$
 (1P) (4P) $\left|-\frac{x^4}{5}\right| < 1$
 $|x| < \sqrt[4]{5}$ (1P)

4. $g(x) = \frac{1}{x^2+2x+2} = \frac{1}{(x+1)^2+1} = \frac{1}{1 - (-(x+1)^2)} = \sum_0^{\infty} (-(x+1)^2)^n =$
 $= \sum_0^{\infty} (-1)^n \cdot (x+1)^{2n}$
 (1P) (1P) $|x+1| < 1$ (1P)
 $-2 < x < 0$ (1P) 10 POINT

4. $f(x) = \sqrt{x^5+4x^4} = 2x^2 \sqrt{1+\frac{x}{4}} = 2x^2 \left(1+\frac{x}{4}\right)^{1/2} =$
 $= 2x^2 \cdot \sum_0^{\infty} \binom{1/2}{n} \left(\frac{x}{4}\right)^n = \sum_0^{\infty} \binom{1/2}{n} \left(\frac{1}{4}\right)^n \cdot 2 \cdot x^{n+2}$
 (1P) (1P) $\left|\frac{x}{4}\right| < 1$
 $|x| < 4$ (1P) 5 POINT

5. $f(x,y) = \frac{e^{xy}}{x^2+y^2}$ (1P)
 NEM LÉTERIK VÉGES HATÁRENTÉK
 MERT PL $y=0$ ESETÉN $f(x) = \frac{1}{x^2} \lim_{y \rightarrow 0} f = +\infty$
 (1P) (1P)
 NEM DIFFHATÓ, MERT NINC VÉGES HATÁRENTÉK \Rightarrow NEM FOLYT (1P)

$f'_x = \frac{e^{xy} \cdot y \cdot (x^2+y^2) - e^{xy} \cdot 2x}{(x^2+y^2)^2}$
 (1P) (1P)

$g'_x = 4y(x^3+4yx)^{4y-1} \cdot (3x^2+4yx)$ (3P)

$g'_y = (x^3+4yx)^{4y} \cdot \ln(x^3+4yx) \cdot 4$ (3P)

14 POINT