2. PPZH. 2018 május 25 megoldások

1. gyök krit. => $|x^2-1|<4$ (2), $x^2 \le 5$ (2) => $(-\sqrt{5},\sqrt{5})$ (2)

Ha $x=\pm\sqrt{5} \Rightarrow \sum \frac{1}{n^3} < \infty \Rightarrow \underline{[-\sqrt{5},\sqrt{5}]}$ (2)

2a. $f(x)=\frac{1}{2}\cdot\frac{1}{1+x^2/2}=\frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{x^2}{2}\right)^n$ (2) $=\sum(-1)^n\frac{x^{2n}}{2^{n+1}}$ (2), $\frac{x^2}{2}<1$, $R=\sqrt{2}$ (3)

2b. $g(x)=\frac{1}{2}\left(e^{3x}+e^{-x}\right)$ (2), $e^{3x}=e^{3(x-3)}e^{9}=e^{9}\sum\frac{3^n(x-3)^n}{n!}$ (2)

$e^{-x}=e^{3-x}e^{-3}=e^{-3}\sum(-1)^n\frac{(x-3)^n}{n!}$ (2), $R=\infty$ (1)

3a. $f(x,x^2)\equiv 1/6$ nem folytonos $(0,0)$-ban

b. $f_x=\frac{2xy(4x^4+2y^2)-x^2y\cdot 16x^3}{(4x^4+2y^2)^2}$ (3), $f_y=\frac{x^2(4x^4+2y^2)-x^2y\cdot 4y}{(4x^4+2y^2)^2}$ (3), $f_x=f_y=0$ (1) $(0,0)$-ban

c. f_x, f_y folytonosak $\forall (x,y)\neq(0,0)$ esetén tehát itt diffható (3)

$(0,0)$-ban f nem folytonos, tehát nem deriválható (2)

d. $f\left(3t/5, 4t/5\right)=\frac{(3t/5)^2\, 4t/5}{4(3t/5)^4+2(4t/5)^2}=\frac{36/125\, t^3}{At^4+32/25\, t^2}=\frac{36/125\, t}{At^2+32/25}$ (3)

$\frac{d}{dt}f(\dots)=\lim_{t\to 0}\frac{36/125}{At^2+32/25}=\frac{36}{32\cdot 5}=\frac{9}{40}$ (2)

4. $f_x=8x^3-2x$ (2), $x=0$, $x=\pm 1/2$ (2), $f_y=4y^3-4y$ (2), $y=0$, $y=\pm 1$ (2)

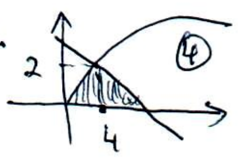
$P_1=(0,0)$, $P_2=(0,1)$, $P_3=(0,-1)$, $P_4=(-1/2,0)$, $P_5=(-1/2,1)$, $P_6=(-1/2,-1)$

$P_7=(1/2,0)$, $P_8=(1/2,1)$, $P_9=(1/2,-1)$ (4) kritikus pontok

$f_{xx}=24x^2-2$, $f_{yy}=12y^2-4$, $f_{xy}\equiv 0$ (2)

$D(P_i)>0$, $i=5,6,8,9$ és itt $f_{xx}>0$ tehát ezek lok. min. (2)

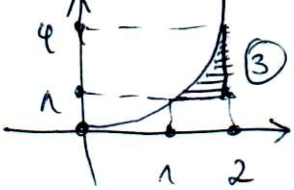
$D(P_i)<0$, $i=2,3,4,7$ nyeregpontok (2), $D(P_1)>0$, $f_{xx}(P_1)<0$ min. (1)

5. 

(4)

$\iint_T xy=\int_0^2\int_{y^2}^{6-y}xy\,dx\,dy$ (5) (3) $=\int_0^2 y\,\frac{(6-y)^2-y^4}{2}\,dy$ (3) (2)

$=\frac{1}{2}\int_0^2\left(36y-12y^2+y^3-y^5\right)dy=\left(9y^2-2y^3+\frac{y^4}{8}-\frac{y^6}{12}\right)\Big|_0^2=\frac{50}{3}$ (1)

6. 

(3)

$\iint\dots=\int_1^2\int_1^{x^2}\sin\left(\frac{x^3}{3}-x\right)dy\,dx=$ (4)

$=\int_1^2\left(x^2-1\right)\sin\left(\frac{x^3}{3}-x\right)dx$ (2)

$=-\cos\left(\frac{x^3}{3}-x\right)\Big|_1^2=-\cos\frac{2}{3}+\cos\left(-\frac{2}{3}\right)=0$ (4) (4)