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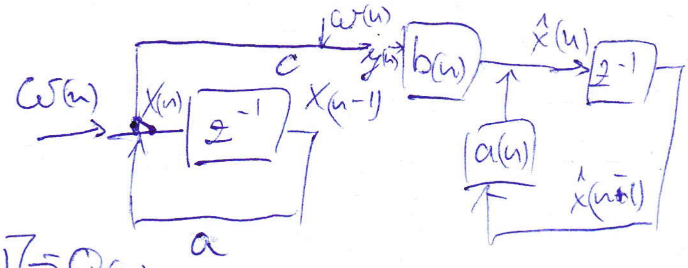
(1) $x(n) = a \cdot x(n-1) + w(n)$

$y(n) = c \cdot x(n) + n(n)$



$E[w(n)] = 0; \quad E[w(n)\, w^T(n)] = Q(n)$

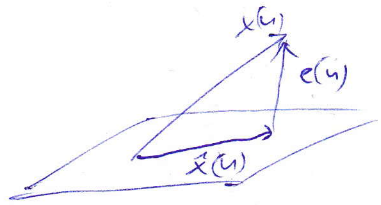
$E[n(n)] = 0; \quad E[n(n)\, n^T(n)] = R(n)$

$\hat{x}(n) = a(n)\, \hat{x}(n-1) + b(n)\, y(n)$

$$E\left[(x(n)-\hat{x}(n))^2\right] = E\Big[\underbrace{\big(x(n) - a(n)\,\hat{x}(n-1) - b(n)\,y(n)\big)}_{e(n)}{}^2\Big]$$

$$\frac{\partial}{\partial a(n)} = -2E\left[e(n)\, \hat{x}(n-1)\right] = 0$$

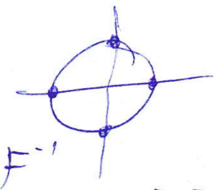
$$\frac{\partial}{\partial b(n)} = -2E\left[e(n)\, y(n)\right] = 0$$



(2) DFT, Walsh, matrixok N=4-re

$$Y(m) = \frac{1}{N}\sum_{n=0}^{N-1} y(n) \cdot e^{-j\frac{2\pi}{N}mn} \quad ; \; m = 0,1,2,3 \quad N = 4\text{-re}$$

$$e^{-j\frac{2\pi}{N}} = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}}$$



$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{bmatrix} = \frac{1}{N} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix}$$

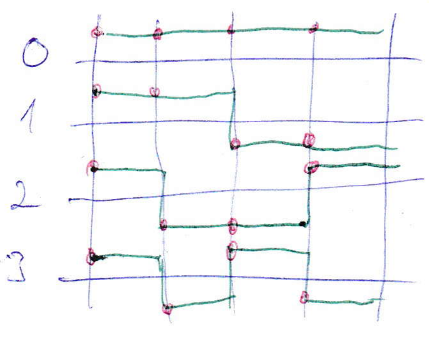
F

F^{-1}

$$y(n) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{bmatrix}$$

$$y(n) = \sum_{m=0}^{N-1} Y(m)\, e^{j\frac{2\pi}{N}mn}$$

$m = 0,1,2,3$



Walsh

$$W = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} \qquad W^{-1} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \end{bmatrix} = W^{-1} \cancel{Y} \begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \end{bmatrix}$$

$$W = UF \qquad W F^{-1} = U$$

$$U = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2+2j & 0 & 2-2j \\ 0 & 2-2j & 0 & 2-2j \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 \\ 2 \\ 0 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 1 \\ 1-j \\ j \\ 1+j \end{bmatrix}$$

$$1 e^{j\frac{2\pi}{4}0n} + (1-j)e^{j\frac{2\pi}{4}1n} + j e^{j\frac{2\pi}{4}2n} + (1+j)e^{j\frac{2\pi}{4}3n} =$$

$$1 + \underbrace{e^{j\frac{2\pi}{4}n} + e^{-j\frac{2\pi}{4}n} - j e^{j\frac{2\pi}{4}n} + j e^{-j\frac{2\pi}{4}n}}_{2\cos\left(\frac{2\pi}{4}n\right)} + (-1)^n$$

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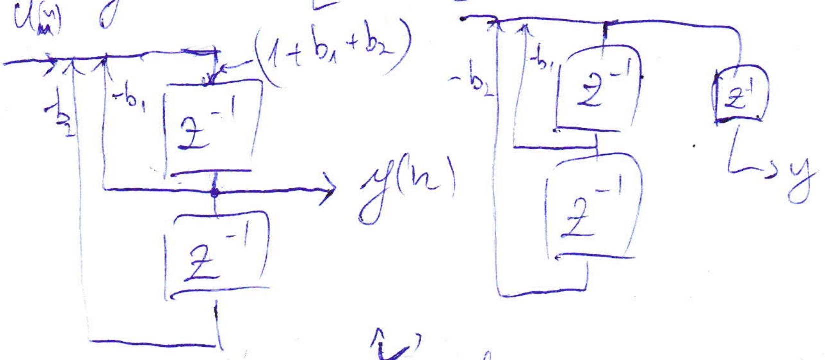
$$H(z)=\frac{(1+b_1+b_2)z^{-1}}{1+b_1z^{-1}+b_2z^{-2}}=\frac{Y(z)}{U(z)}$$

$$\cancel{U(z)(1+b_1z^{-1}+b_2z^{-2})=}$$

$$U(z)(1+b_1+b_2)z^{-1}=Y(z)(1+b_1z^{-1}+b_2z^{-2})$$

$$[1+b_1+b_2]u(n-1)=y(n)+b_1\,y(n-1)+b_2\,y(n-2)$$

$$y(n)=?=[1+b_1+b_2]u(n-1)-b_1\,y(n-1)-b_2\cdot y(n-2)$$



→ 3 késleltető fölösleges, elég 2

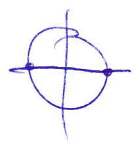
⇕

ekvivalens

Rezonátor

$z=1$

$z=-1$



$$\frac{r_m z_m z^{-1}}{1-z_m z^{-1}}$$

$$\frac{r_0 z^{-1}}{1-z^{-1}} \qquad \frac{-r_1 z^{-1}}{1+z^{-1}}$$

$$w_0=1 \qquad w_1=\frac{1+b_1+b_2}{1-b_1+b_2}$$

$$\frac{\frac{r_0z^{-1}}{1-z^{-1}}\cdot 1-\frac{r_1z^{-1}}{1+z^{-1}}w_1}{1+\frac{r_0z^{-1}}{1-z^{-1}}-\frac{r_1z^{-1}}{1+z^{-1}}}=\frac{r_0z^{-1}+r_0z^{-2}*-(r_1z^{-1}-r_1z^{-2})w_1}{1-z^{-2}+r_0z^{-1}+r_0z^{-2}+r_1z^{-1}+r_1z^{-2}}=$$

$$=\frac{(1+b_1+b_2)z^{-1}}{1+b_1z^{-1}+b_2z^{-2}}$$

$$\left.\begin{aligned}(r_0-r_1)z^{-1}&=b_1z^{-1}\\(r_0+r_1-1)z^{-2}&=b_2z^{-2}\end{aligned}\right\}$$

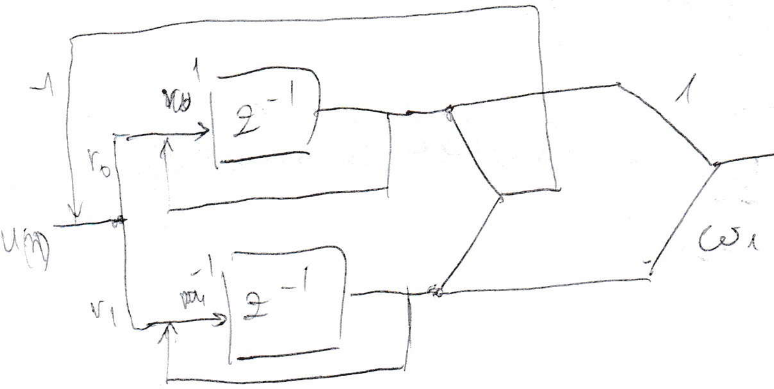
$$(r_0 - r_1 \omega_1) z^{-1} = (1 + b_1 + b_2) z^{-1}$$

$$(r_0 + r_1 \omega_1) z^{-2} = 0$$

$$\left.\begin{aligned} r_0 - r_1 &= b_1 \\ r_0 + r_1 &= b_2 + 1 \end{aligned}\right\}$$

$$r_0 = \frac{1 + b_1 + b_2}{2}$$

$$r_1 = \frac{1 - b_1 + b_2}{2}$$



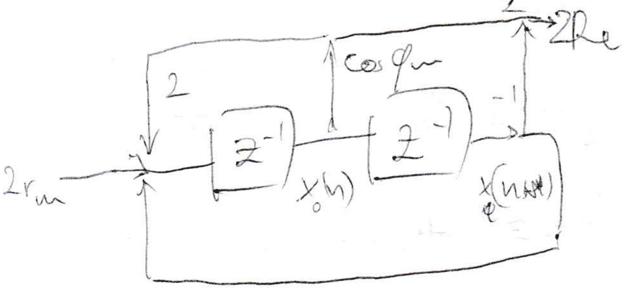
Ugyanannyi tárolót és szorzót igényel, de DC-n és $\frac{f_m}{2}$-n jobb

$|H(z)| = 1$ feltételezett: $\dfrac{b_2 + b_1 z^{-1} + z^{-2}}{1 + b_1 z^{-1} + b_2 z^{-2}} = z^{-2} \dfrac{1 + b_1 z + b_2 z^2}{1 + b_1 z^{-1} + b_2 z^{-2}}$

↳ mindentáteresztő

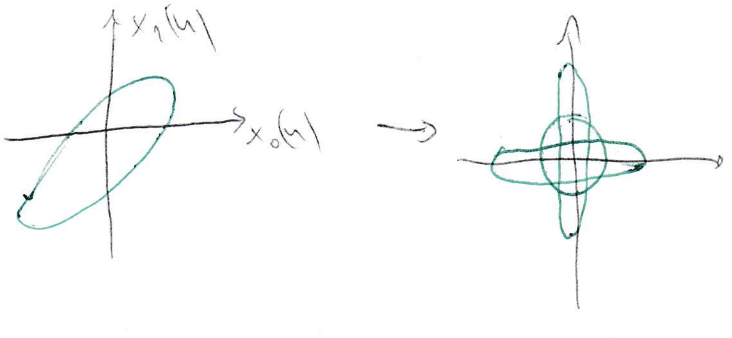
$$\frac{r_m z_m z^{-1}}{1 - z_m z^{-1}} + \frac{r_m z_m^{-1} z^{-1}}{1 - z_m^{-1} z^{-1}} = 2 r_m \frac{z^{-1} \cos\varphi_m - z^{-2}}{1 - 2 z^{-1} \cos\varphi_m + z^{-2}} \Rightarrow 2\,\mathrm{Re}$$

$$j\, 2 r_m \frac{z^{-1} \sin\varphi_m}{1 - 2 z^{-1} \cos\varphi_m + z^{-2}} \Rightarrow 2\,\mathrm{Im}$$



$$\begin{bmatrix} x_0(n+1) \\ x_1(n+1) \end{bmatrix} = \begin{bmatrix} 2\cos\varphi_m & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_0(n) \\ x_1(n) \end{bmatrix} + \begin{bmatrix} 2 r_m \\ 0 \end{bmatrix} u(n)$$

$$2\,\mathrm{Re} = y(n) = \begin{bmatrix} \cos\varphi_m & -1 \end{bmatrix} \begin{bmatrix} x_0(n) \\ x_1(n) \end{bmatrix}$$



$A' = T A T^{-1}$

$B' = T B$

$c' = c T^{-1}$

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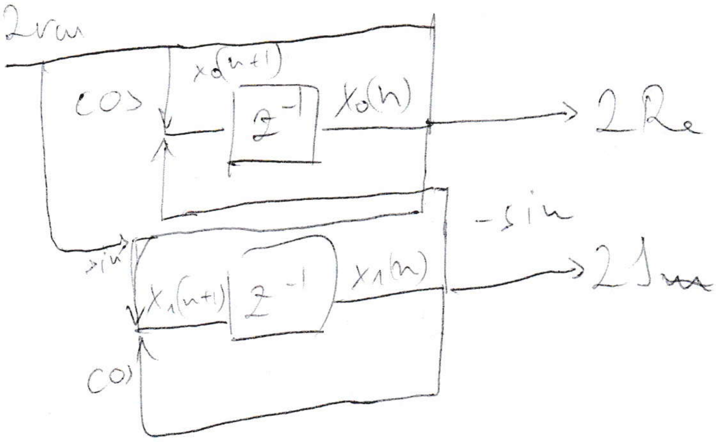
$$A = \begin{bmatrix} \cos\varphi_m & -\sin \\ \sin & \cos \end{bmatrix}, \quad \cancel{B} = G = \begin{bmatrix} 2r_m \cos \\ 2r_m \sin \end{bmatrix}$$

$$C_{Re} = [1 \; 0] \quad C_{Im} = [0 \; 1]$$



$$z X_0(z) = X_0(z)\cos - X_1(z)\sin + 2r_m U(z)\cos$$

$$z X_1(z) = X_0(z)\sin + \overset{\cos}{\cancel{X_1(z)\sin}} + 2r_m U(z)\sin$$

$$y(z) = x_0(z)$$

$$x_1(z) = \frac{1}{z-\cos}\left(x_0(z)\sin + 2r_m U(z)\sin\right)$$

$$z X_0(z) = X_0(z)\cos + \frac{1}{z-\cos}\left(x_0(z)\sin + 2r_m U(z)\sin\right)\sin + 2r_m U(z)\cos$$

$$X_0(z)\cancel{z\cos}\left[z^2 - z\cos - z\cos + \cos^2 + \sin^2\right] = 2r_m U(z)\cancel{(\sin^2+\cos)}$$

$$(z\cos - \cos^2 - \sin^2)$$

$$y(z) = X_0(z) = \frac{2r_m U(z)(z\cos - 1)}{z^2 - 2z\cos + 1} = 2r_m \frac{z^{-1}\cos\varphi_m - z^{-2}}{1 - 2z^{-1}\cos\varphi_m + z^{-1}} U(z)$$

$$\left|\frac{a+jb}{1+a+jb}\right| = 1 \qquad A - GC \qquad \begin{bmatrix} 2r_m\cos \\ 2r_m\sin \end{bmatrix}[1 \; 0] = \underbrace{\begin{bmatrix} 2r_m\cos & 0 \\ 2r_m\sin & 0 \end{bmatrix}}_{\begin{bmatrix} \cos & 0 \\ \sin & 0 \end{bmatrix}}$$

$$\sum_{m=0}^{N-1} r_m = 1 \qquad [A - GC] = \begin{bmatrix} 0 & -\sin \\ 0 & \cos \end{bmatrix}$$

$$\begin{bmatrix} x(n+1) \\ y(n) \end{bmatrix} = T\begin{bmatrix} x(n) \\ u(n) \end{bmatrix}, \quad T^T = T^{-1}$$

$$x^T(n+1)\,x(n+1) - x^T(n)\,x(n) = -y^T(n)\,y(n) \qquad u(n) = 0$$

$$T = \begin{bmatrix} 0 & -\sin & \cos \\ 0 & \cos & \sin \\ 1 & 0 & 0 \end{bmatrix} \xrightarrow{\text{ortogonális-e?}} T^T_2 \begin{bmatrix} 0 & 0 & 1 \\ -\sin & \cos & 0 \\ \cos & \sin & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \checkmark$$

$x^T(n+1)\,x(n+1) - x^T(n)\,x(n)$

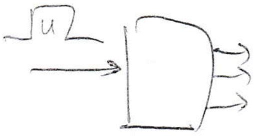
$x^T(n+1)\,P\,x(n+1) \Rightarrow y^T(n)\,y(n) \Rightarrow$ $y(n) = Cx(n)$, $y(n+1) = CAx(n)$

$x^T(n)\,C^TC\,x(n) \Rightarrow x^T(n)\left(\sum_{i=0}^{\infty} A^{T^i} C^T C A^i\right)x(n)$

$x^T(n+1)\,P\,x(n+1) - x^T(n)\,P\,x(n) = -y^T(n)\,y(n)$

$APA^T + C^TC = P$

Adatredukció: Főkomponens analízis



$x = Ty$, $T^T = T^{-1}$ (ortogonális transzformáció

$= [\varphi_0, \varphi_1, \dots, \varphi_{N-1}]$

$$y = T^T x = \sum_{i=0}^{N-1} x_i\varphi_i = \underbrace{\sum_{i=0}^{M-1} x_i\varphi_i}_{\text{redukált fölírás}} + \underbrace{\sum_{i=M}^{N-1} b_i\varphi_i}_{\text{konstans}}$$

$$E[(y-\hat{y})^2] = \varepsilon = \sum_{i=0}^{N-1}\varphi_i^T E[(x_i-b_i)(x_i-b_i)^T]\varphi_i$$

$$\frac{\partial}{\partial b_i} = E(x_i-b_i) = 0 \Rightarrow b_i = E(x_i) = \varphi_i^T E[y]$$

$$\varepsilon = \sum_{i=0}^{N-1}\varphi_i^T \underbrace{E\left((y-E(y))(y-E(y))^T\right)}_{C_{yy}}\varphi_i$$

$$\hat{\varepsilon} = \varepsilon - \sum_{i=0}^{N-1}\beta_i(\varphi_i^T\varphi_i - 1)$$

$$\frac{\partial}{\partial\varphi_i} = 2C_{yy}\varphi_i - 2\beta_i\varphi_i = 0$$

$$C_{yy}\varphi_i = \beta_i\varphi_i \qquad \varepsilon = \sum_{i=0}^{N-1}\varphi_i^T C_{yy}\varphi_i = \sum_{i=0}^{N-1}\beta_i$$