Kalkulus vizsga 1 MO

2021. jan.

$$(A-B)\cdot\frac{A+B}{A+B}=\frac{A^2-B^2}{A+B} \quad (2)$$

1)
$$\lim_{n\to+\infty}\left(\sqrt{n^4+3n^2}-\sqrt{n^4-7n^3}\right)\overset{!}{=}$$

$$=\lim_{n\to+\infty}\frac{n^4+3n^2-(n^4-7n^3)}{\sqrt{n^4+3n^2}+\sqrt{n^4-7n^3}} \ (2) =\lim_{+\infty}\frac{7n^3+3n^2}{\underbrace{\sqrt{n^4+3n^2}}_{\underbrace{\sqrt{n^4}}_{n^2}\cdot\sqrt{1+\frac{3}{n^2}}}+\underbrace{\sqrt{n^4-7n^3}}_{\sqrt{n^4}\cdot\sqrt{1-\frac{7}{n}}}}= \quad (2)$$

alak: (2)

$$=\lim_{n\to+\infty}\boxed{\frac{n^3}{n^2}}\cdot\frac{7+\frac{3}{n}}{\sqrt{1+\frac{3}{n^2}}+\sqrt{1-\frac{7}{n}}}=+\infty\cdot\frac{7}{2\cdot\sqrt{1}}=+\infty \quad (2)$$

$\frac{3}{n}\to 0$, $\frac{3}{n^2}\to 0$, $\frac{7}{n}\to 0$, $\frac{n^3}{n^2}=n$

2) a)
$$\lim_{x\to 0+}\frac{x^3+4x^2}{2x^5-x^3}\underset{\text{„}\frac{0}{0}\text{"}}{=}\lim_{x\to 0+}\boxed{\frac{x^2}{x^3}}\cdot\frac{x+4}{2x^2-1}=-\infty \quad (1)$$

„$\frac{0}{0}$" (2) kiemel!

alak: (3)

$x+4\to 4$ (1), $2x^2-1\to -1$ (1), $\frac{x+4}{2x^2-1}\to -4$

$\frac{x^2}{x^3}=\frac{1}{x}$ (1) $\to+\infty$ (1)

3.)
$$(-4i+4-(4-2i)\cdot z+z^2)\cdot(z+2)=0$$

„ha valamelyik $=0$" (2) $\to z=-2$ (1)

$a=1$
$b=-(4-2i)$
$c=-4i+4$

másodfokú megoldóképlet! (2)

$$z_{1,2}=\frac{4-2i\pm\sqrt{(4-2i)^2-4(-4i+4)}}{2} \ (4) =\frac{4-2i\pm 2i}{2} \begin{cases} 2 & (1,5)\\ 2-2i & (1,5)\end{cases}$$

$$\text{Diszkr}=16-16i+\underbrace{4i^2}_{-4}+16i-16=-4$$

$$\sqrt{-4}=\pm 2i \quad (3)$$

2.b)
$$\lim_{x\to 0}\frac{x^3}{\sin^3(5x)}=\lim_{x\to 0}\left(\frac{x}{\sin(5x)}\right)^3 \ (2) =\left(\frac{1}{5}\right)^3=\frac{1}{125} \quad (2)$$

$$\lim_{x\to 0}\frac{1}{5}\cdot\frac{5x}{\sin(5x)}=\frac{1}{5} \quad (5), \quad 5x\to 0, \ \frac{5x}{\sin(5x)}\to 1$$

$$\lim_{\alpha\to 0}\frac{\sin\alpha}{\alpha}=1 \quad (1)$$

4.) a) $(5x+\sinh(6x^2+1)+4)^5$

összetett fv. ①

① $\begin{cases} f(x)=x^5 & f'(x)=5x^4 \\ g(x)=5x+\sinh(6x^2+1)+4 & g'(x)=5+\underbrace{(\sinh(6x^2+1))'}_{①}+0 \end{cases}$

$\sinh(6x^2+1)$ összetett fv: ①

① $\begin{cases} a(x)=\sinh x & a'(x)=\cosh x \\ b(x)=6x^2+1 & b'(x)=12x \end{cases}$ ①

$(a\circ b)'(x)=a'(b(x))\cdot b'(x)=\cosh(6x^2+1)\cdot 12x$ ②

képletre jár 1 pont!

Végeredmény:

$(f\circ g)'(x)=f'(g(x))\cdot g'(x)=5(5x+\sinh(6x^2+1)+4)^4\cdot(5+12x\cdot\cosh(6x^2+1))$

összetett fv. der. szab. ② képlet is legyen leírva! (ha nincs: -1p)

4. b) $\left(\frac{\sqrt{4x^4+10}}{6x-e^x}\right)'=\frac{(\sqrt{4x^4+10})'\cdot(6x-e^x)-\sqrt{4x^4+10}\cdot\overbrace{(6x-e^x)'}^{6-e^x\ ①}}{(6x-e^x)^2}$ ①

hányados der. szab. $\left(\frac{f}{g}\right)'=\frac{f'\cdot g-f\cdot g'}{g^2}$ képlet: 1 p

részfl: $(\sqrt{4x^4+10})'=?$ összetett fv ①

① $\begin{cases} a(x)=\sqrt{x} & a'(x)=\frac{1}{2}\cdot x^{-1/2}=\frac{1}{2\sqrt{x}} \\ b(x)=4x^4+10 & b'(x)=16x^3 \end{cases}$ ①

② (képlet nélk: 1p)

$(a\circ b)'(x)=a'(b(x))\cdot b'(x)=\frac{1}{2\cdot\sqrt{4x^4+10}}\cdot 16x^3$

A végeredmény: ②

$\frac{\frac{16x^3}{2\cdot\sqrt{4x^4+10}}\cdot(6x-e^x)-\sqrt{4x^4+10}\cdot(6-e^x)}{(6x-e^x)^2}$

(12p+8p)

5.) a) $\int \frac{2x+3}{\sqrt{2x+1}}\,dx = \int \frac{\overbrace{2\cdot\frac{t^2-1}{2}+3}^{②}}{t}\cdot t\,dt =$

$t=\sqrt{2x+1}$

$t^2=2x+1$ ①

$g(t) \overset{①}{=} \frac{t^2-1}{2} = x$

$g'(t) \overset{①}{=} t$ Hely. int. képlete: ①

$$\int f(x)\,dx \Big|_{x=g(t)} = \int f(g(t))\cdot g'(t)\,dt$$

$= \int \overbrace{t^2+2}^{①}\,dt = \overbrace{\frac{t^3}{3}+2t+C}^{②}$ (*) $C\in\mathbb{R}$ tetsz.

vmelyik végeredmény-re 3p és kész.

$= \frac{t^3}{3}+2t+C \underset{t=\sqrt{2x+1}}{=} \overbrace{\frac{(\sqrt{2x+1})^3}{3}+2\cdot\sqrt{2x+1}+C}^{③}$

(vagy: $t\cdot\left(\frac{t^2}{3}+2\right)+C = \frac{1}{3}t\cdot(t^2+6)+C \underset{t=\sqrt{2x+1}}{\overset{!}{=}}$

$= \overbrace{\frac{1}{3}\sqrt{2x+1}\cdot(2x+1+6)+C}^{③}$, $C\in\mathbb{R}$ tetsz.

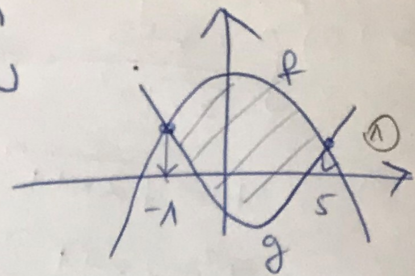
b) $\int_0^3 \frac{2x+3}{\sqrt{2x+1}}\,dx \underset{\text{N.-L. ③}}{=} \left[\frac{1}{3}\sqrt{2x+1}\cdot(2x+7)\right]_{x=0}^{3} =$

$= \frac{1}{3}\sqrt{7}\cdot 13 - \frac{1}{3}\cdot 1\cdot 7 = \frac{1}{3}(13\sqrt{7}-7)$ ⑤

6.) $f(x) = -x^2+4x+1$, $a<0 \Rightarrow \cap$

$g(x) = x^2-4x-9$, $a>0 \Rightarrow \cup$

Metszéspontok: $f(x)=g(x)$ ②



$-x^2+4x+1 = x^2-4x-9$

$0 = 2x^2-8x-10$

$0 = x^2-4x-5$

$x_{1,2} = \dots$ $\begin{matrix} 5 \\ -1 \end{matrix}$ ②

ha $x \in [-1, 5]$

$\Rightarrow f(x) \geq g(x)$ ①

$$T = \underbrace{\int_{-1}^{5} f(x)-g(x)\,dx}_{①} = \underbrace{\int_{-1}^{5} -2x^2+8x+10\,dx}_{②} =$$

$$= \underbrace{\left[-2\frac{x^3}{3}+4x^2+10x\right]_{x=-1}^{5}}_{③} = -\frac{2}{3}\cdot 125+4\cdot 25+50-\left(+\frac{2}{3}+4-10\right) =$$

N.-L.

$$= -\frac{2}{3}(\underbrace{125+1}_{126})+4\cdot 24+60 = -84+96+60 = \underline{\underline{72}}$$ ③

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Pótfel:

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 20 \\ 5 & 10 & 15 & 160 \\ 0 & -2 & 3 & 30 \end{array}\right) \underset{\frac{1}{5}\cdot \mathrm{II}}{\sim} ③ \left(\begin{array}{ccc|c} 0 & 1 & 2 & 20 \\ 1 & 2 & 3 & 32 \\ 0 & -2 & 3 & 30 \end{array}\right) \underset{\mathrm{I} \leftrightarrow \mathrm{II}}{\sim} ③$$

$$\left(\begin{array}{ccc|c} 1 & 2 & 3 & 32 \\ 0 & 1 & 2 & 20 \\ 0 & -2 & 3 & 30 \end{array}\right) \underset{\mathrm{III}+2\cdot\mathrm{II}}{\sim} ③ \left(\begin{array}{ccc|c} 1 & 2 & 3 & 32 \\ 0 & 1 & 2 & 20 \\ 0 & 0 & 7 & 70 \end{array}\right) \Rightarrow \begin{aligned} x+2y+3z &= 32 \\ y+2z &= 20 \\ 7z &= 70 \end{aligned}$$ ③

$\Rightarrow \underline{z=10} \Rightarrow y+2\cdot 10 = 20 \Rightarrow \underline{y=0} \rightarrow x+0+3\cdot 10 = 32 \Rightarrow \underline{\underline{x=2}}$

végeredmény: ③