1. ZH MEGOLDÁSOK
A CSOPORT

2009.03.24.

1.) Az integrátor bemenetén $e=0$.

$$r = 3y^2 + 4$$

Tehát $$y = \sqrt{\frac{r-4}{3}}$$

2.) $\tau > T$ esetén a rendszer stabilis.
$\tau < T$ esetén a rendszer labilis.

Indoklás

A kar. egyenlet:

$$1 + L(s) = 1 + \frac{1+s\tau}{s^2(1+sT)} = 0$$

$$s^3 T + s^2 + s\tau + 1 = 0$$

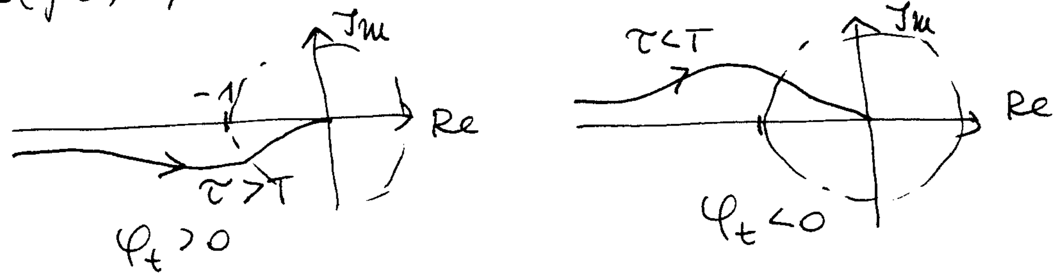
Hurwitz det.:

$$\begin{vmatrix} 1 & 1 & 0 \\ T & \tau & 0 \\ 0 & 1 & 1 \end{vmatrix} \qquad \Delta_2 = \tau - T > 0$$

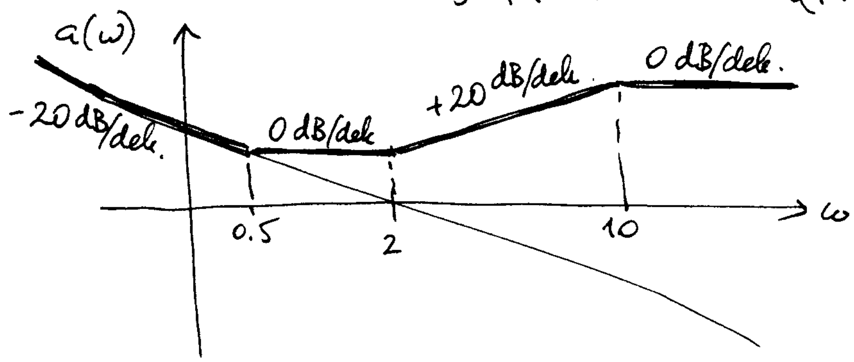
Stab. feltétele $\boxed{\tau > T}$

Vagy a Nyquist stab. kritérium alapján:

$L(j\omega)$, ha $\tau > T$ $\qquad$ $\tau < T$



3.) $$L(s) = 20\,\frac{s^2+2.5s+1}{s^2+10s} = \frac{2(s+2)(s+0.5)}{s(1+0.1s)} = \frac{2(1+0.5s)(1+2s)}{s(1+0.1s)}$$



4.) $L(s) = \dfrac{1}{s(1+s)^2}$; $L(j\omega) = \dfrac{1}{j\omega(1+j\omega)^2}$

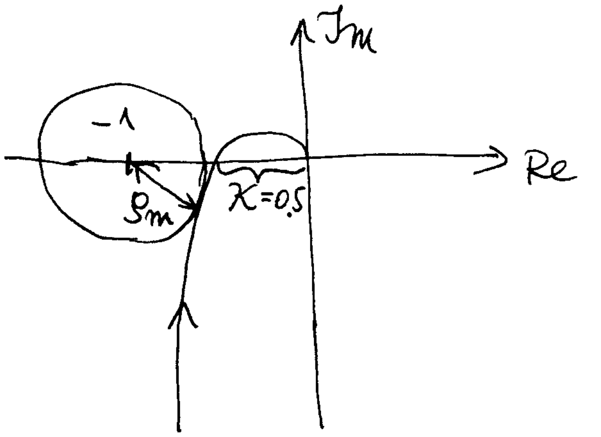
$\varphi = -180^\circ$, ha

$$-90^\circ - 2\,\mathrm{arctg}\,\omega_0 = -180^\circ.$$

$$\mathrm{arctg}\,\omega_0 = 45^\circ \Rightarrow \omega_0 = 1.$$

$$|L(j\omega_0)| = \frac{1}{\omega_0\left(\sqrt{1+\omega_0^2}\right)^2} = \frac{1}{1\cdot 2} = 0.5$$

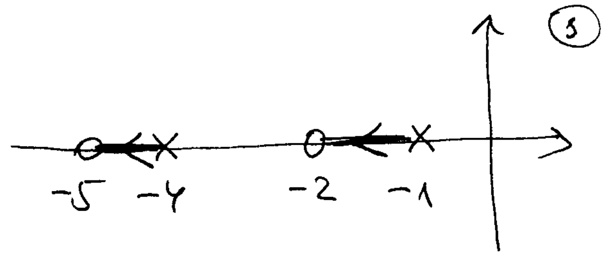
Tehát a Nyquist diagramtól a $-1+j0$ pont balkéz felé esik, tehát a rendszer stabilis.



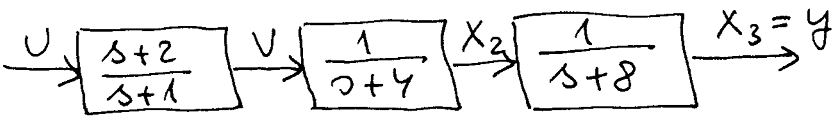
Erősítési tartalék: $1/\kappa = \frac{1}{0,5} = 2$

5.) A gyökhelygörbe a zárt rendszer karakterisztikus egyenletének gyökeit adja meg miközben valamelyik paraméter (rendszerint a huroktényező (körerősítés)) 0 és ∞ között változik.

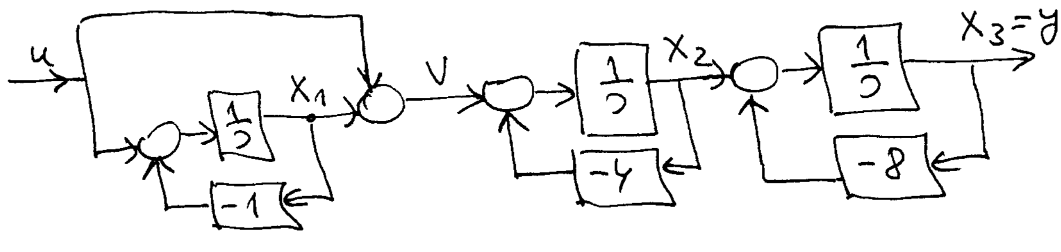
$$L(s) = K\,\frac{s^2+7s+10}{s^2+5s+4} = K\,\frac{(s+2)(s+5)}{(s+1)(s+4)}$$



6.)



ill.



$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \underbrace{\begin{bmatrix}-1 & 0 & 0\\ 1 & -4 & 0\\ 0 & 1 & -8\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \underbrace{\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}}_{b} u$$

$$y = \underbrace{[0 \quad 0 \quad 1]}_{c^T} X + \underbrace{0}_{d} \cdot u$$

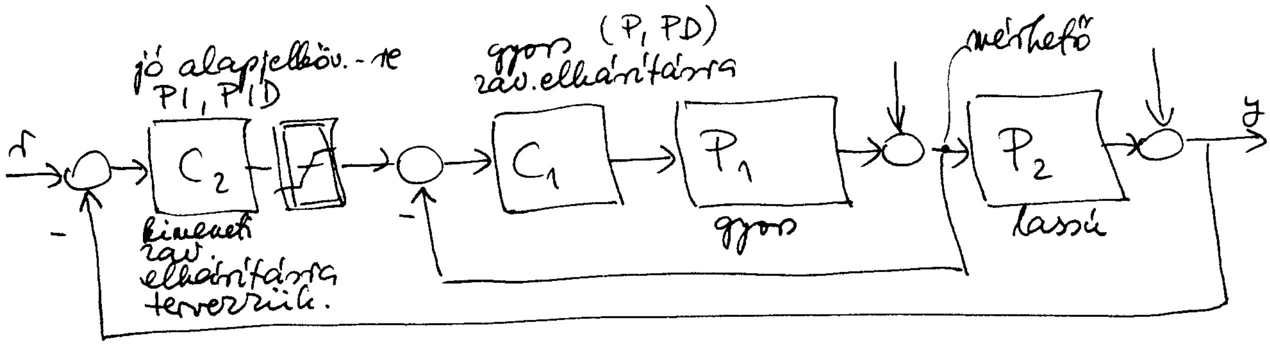
A statikus erősítés: $\cancel{8}\ \frac{2 \cdot}{1 \cdot 4 \cdot 8} = \frac{1}{16}$

7.) $$\phi(t) = e^{At} = e^{\begin{bmatrix}-1 & 0\\ 0.2 & -2\end{bmatrix}t} = \mathcal{L}^{-1}[\phi(s)] = \mathcal{L}^{-1}[(sI-A)^{-1}]$$

$$X(1) = e^{At} X(0) = e^{At} \cdot \begin{bmatrix}1\\ 1\end{bmatrix}$$

$$\phi(s) = \frac{\begin{bmatrix}s+2 & 0\\ 0.2 & s+1\end{bmatrix}}{(s+1)(s+2)} = \begin{bmatrix}\frac{1}{s+1} & 0\\ \frac{0.2}{(s+1)(s+2)} & \frac{1}{s+2}\end{bmatrix}$$

8.)



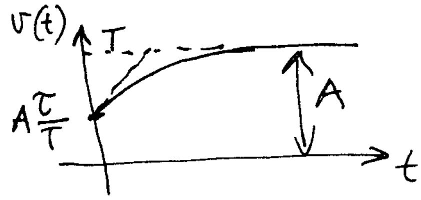
$$\phi(t) = \left[\begin{array}{c|c} e^{-t} & 0\\ \hline 0.2e^{-t} - 0.2e^{-2t} & e^{-2t}\end{array}\right]$$

$$x_1(1) = e^{-1} \cdot 1$$

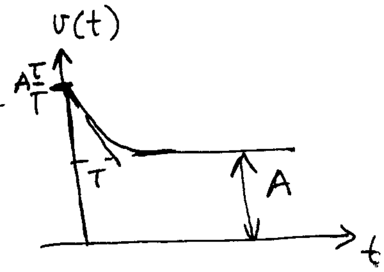
1. ZH MEGOLDÁSOK B CSOPORT 2009.03.24

1.) $H(s) = A\,\dfrac{1+s\tau}{1+sT}$

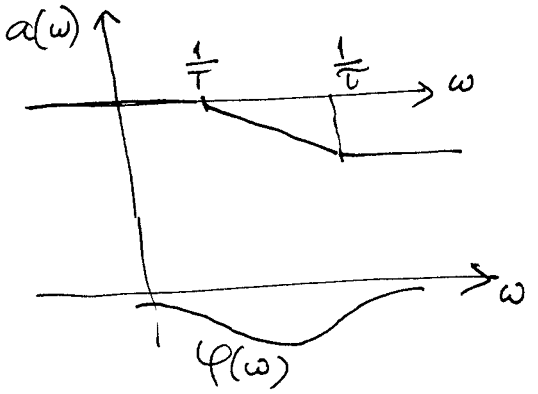
$\tau < T$

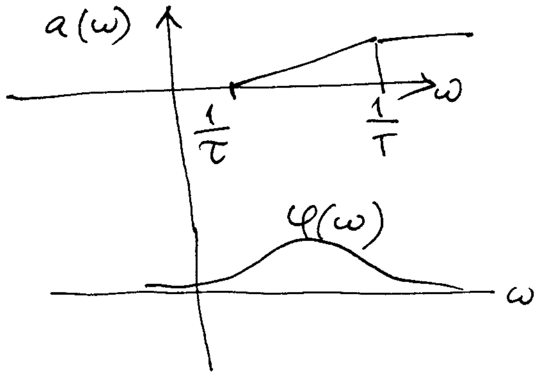


$\tau > T$

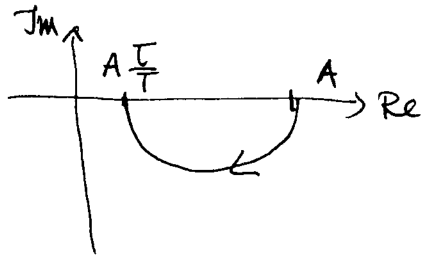


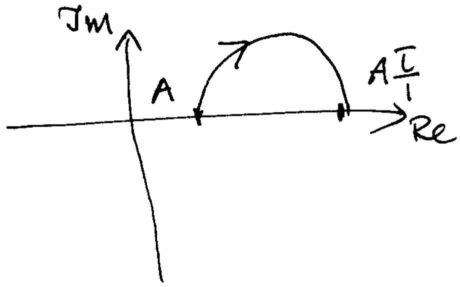
Bode:





Nyquist:





2.) $P(s) = \dfrac{5e^{-0.5s}}{(1+s)(1+2s)(1+3s)}$

$$a(\omega) = \frac{5}{\sqrt{1+\omega^2}\cdot\sqrt{1+4\omega^2}\cdot\sqrt{1+9\omega^2}}$$

$$\varphi(\omega) = -0.5\,\omega \cdot 180/\pi - \mathrm{arctg}\,\omega - \mathrm{arctg}\,2\omega - \mathrm{arctg}\,3\omega$$

3.) $L(s) = \dfrac{K}{(1+s)^3}$

Kar. egyenlet:

$$1 + \frac{K}{(1+s)^3} = 0$$

$$s^3 + 3s^2 + 3s + 1 + K = 0$$

Hurwitz det.:

$$\begin{vmatrix} 3 & 1+K & 0 \\ 1 & 3 & 0 \\ 0 & 3 & 1+K \end{vmatrix}$$

$$\Delta_2 = 9 - 1 - K > 0$$

$$K < 8$$

vagy a Nyquist stab. krit. alapján:

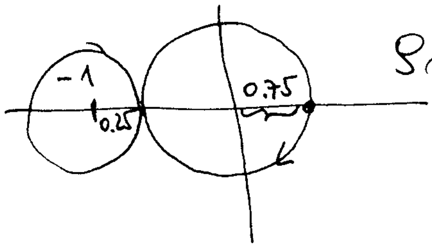
1. ZH MEGOLDÁSOK, B CSOPORT 1.folyt.

$$\varphi(\omega_0) = -3\,\mathrm{arctg}\,\omega_0 = -180^\circ \;; \quad \mathrm{arctg}\,\omega_0 = 60^\circ$$

ahol -180° a fázisszög $\qquad \omega_0 = \sqrt{3}$

$$|L(j\omega_0)| = \frac{K_{kr}}{\left(\sqrt{1+\omega_0^2}\right)^3} = 1 \;; \quad K_{kr} = \left(\sqrt{1+3}\right)^3 = 8$$

4.) A modulus tartalék a $-1+j0$ pont ~~és~~ középpontú, a felnyitott kör Nyquist diagramját érintő kör sugara. $\quad L(j\omega) = 0.75e^{-3j\omega}$



$\rho_m = 0.25$

5.)

$$|P(j3)| = \frac{5}{\sqrt{1+9}}$$

$$\varphi(\omega=3) = -\mathrm{arctg}\,3 - 0.2\cdot 3\cdot 180/\pi$$

$$y_{stac} = \frac{5}{\sqrt{10}}\sin(3t-\varphi)$$

6.)

$$\phi(t) = e^{At} \;; \quad \phi(s) = (sI-A)^{-1}$$

$$\phi(s) = \begin{bmatrix} s+1 & 0 \\ -0.2 & s+2 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} s+2 & 0 \\ 0.2 & s+1 \end{bmatrix}}{(s+1)(s+2)} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{0.2}{(s+1)(s+2)} & \frac{1}{s+2} \end{bmatrix}$$

$$\phi(t) = \begin{bmatrix} e^{-t} & 0 \\ 0.2e^{-t} - 0.2e^{-2t} & e^{-2t} \end{bmatrix}$$

$$x_2(1) = 0.2e^{-1} - 0.2e^{-2} + e^{-2} = 0.2e^{-1} + 0.8e^{-2}$$

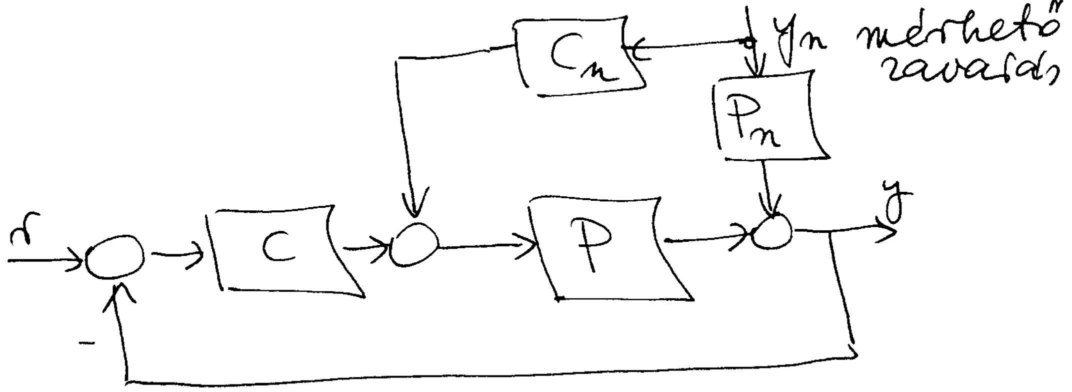
7.) $$T = \frac{CP}{1+CP}$$

$$\Delta T = \frac{\partial T}{\partial P}\,\Delta P = \frac{C}{(1+CP)^2}\,\Delta P$$

$$\frac{\Delta T}{T} = \frac{1}{1+CP}\,\frac{\Delta P}{P} = S\,\frac{\Delta P}{P}$$, ahol $S = \frac{1}{1+CP}$

az érzékenységi fv.

8.)



A zavarkompenzáció alkalmazható mérhető zavarás esetén. Előrecsatolás a zavarásról.

Ideális, ha

$$y_n\left(P_n/(1+CP) + \frac{C_n P}{1+CP}\right) = 0$$

Tehát $$P_n + C_n P = 0$$

$$C_n = -\frac{P_n}{P}$$

Akkor realizálható, ha a nevező fokszáma ≥ számláló fokszáma.

A holtidő nem kompenzálható.