gyakorlás 1. beiZH-ra

Orvosbiológiai méréstechnika

2015. 09. 28.

Képletek: Nernst-egyenlet: $\Delta U = -\frac{RT}{zF} \cdot \ln\left(\frac{c_1}{c_2}\right)$

~~ha csak~~ több ionra: $\Delta U = -\frac{RT}{zF} \cdot \ln\left(\frac{P_{Na^+}\cdot[Na^+]_1 + P_{K^+}\cdot[K^+]_1 + P_{Cl^-}\cdot[Cl^-]_2}{P_{Na^+}\cdot[Na^+]_2 + P_{K^+}\cdot[K^+]_2 + P_{Cl^-}\cdot[Cl^-]_1}\right)$

$\approx -58\,mV \cdot \lg(\,)$

P = permeabilitás

$[x]$ = x koncentrációja

F = Faraday konstans = $96500\ \frac{C}{mól}$

Fick-egyenlet: $j_d = -D \cdot \frac{dC}{dx}$ $\quad [j_d] = 1\ \frac{mól}{cm^2 \cdot s}$ $\quad \left[\frac{dC}{dx}\right] = 1\ \frac{mól}{cm^4}$

$[C] = 1\ \frac{mól}{cm^3}$ $\quad [D] = 1\ \frac{cm^2}{s}$

$D = \frac{R\cdot T}{f}$

ozmózis nyomás: $\Pi = R \cdot T \cdot \Delta C$

f = diffúziót akadályozó hatásos súrlódási érték, frikció

$Q = C\,U$ $\quad E = \frac{\Delta U}{d}$

$\Delta Q = C \cdot \Delta U$ $\quad C = \varepsilon_0 \varepsilon_r \cdot \frac{A}{d}$

$\left[\frac{C}{A}\right] = 1\ \frac{\mu F}{cm^2}$ fajlagos kapacitás $\quad \Delta U = U(I_1) - U(I_2)$

$\Delta Q = \left(\frac{C}{A}\right) \cdot A \cdot \Delta U$ $\quad Z = \frac{1}{j\omega C}$, $X_C = \frac{1}{\omega C}$

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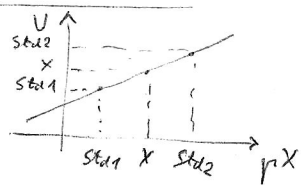
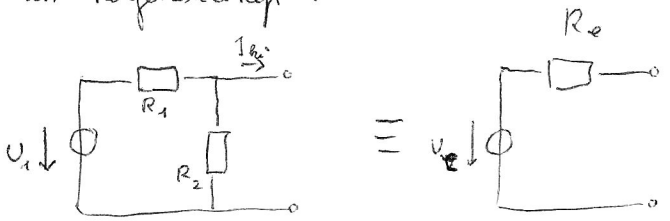
vérgáz analízis: $\frac{pX(x) - pX(Std1)}{U_{ki}(x) - U_{ki}(Std1)} = \frac{pX(Std2) - pX(Std1)}{U_{ki}(Std2) - U_{ki}(Std1)}$

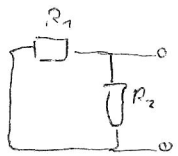


Thévenin-helyettesítőkép:



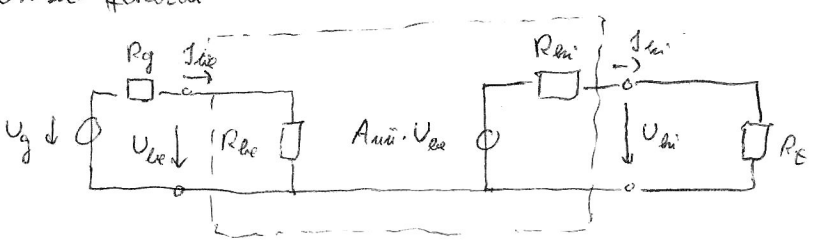
$U_{ki}\Big|_{I_{ki}=0} = U_1 \cdot \frac{R_2}{R_1+R_2}$

$\Rightarrow U_e = U_1 \cdot \frac{R_2}{R_1+R_2}$



$R_{ke} = \frac{R_1 R_2}{R_1+R_2} = R_e$ $\quad / I_1 = \frac{U_1}{R_1}$ rövidrezárás esetén / $\leftarrow \frac{U_e}{R_{ke}}$

Erősítő fokozat:



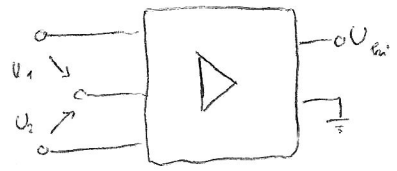
$U_{ke} = U_g \cdot \frac{R_{ke}}{R_g + R_{ke}}$

$R_{ke} = \frac{U_{ke}}{I_{ke}}$

$U_{ki} = A_{ü} \cdot U_{ke} \cdot \frac{R_t}{R_{ki} + R_t}$

$R_{ki} = -\frac{U_{ki\ üresjárási}}{I_{ki\ rövidzár.}}$ $\rightarrow [R_t \rightarrow \infty]$ $\rightarrow (R_t = 0)$

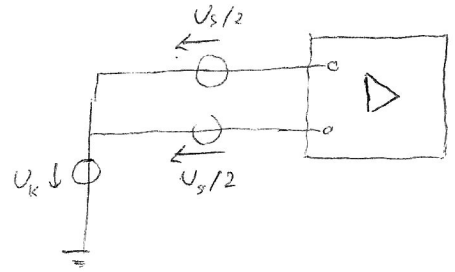
Szimmetrikus erősítő :



$$U_S = U_1 - U_2$$

$$U_K = \frac{U_1 + U_2}{2}$$

$\approx$



$$U_{ki} = A_{u_S} \cdot U_{be_S} + A_{u_K} \cdot U_{be_K} = A_{u_S}\left(U_{be_S} + \frac{A_{u_K}}{A_{u_S}} \cdot U_{be_K}\right)$$

$E_{K_u} = \frac{A_{u_K}}{A_{u_S}}$ közösjelelnyomás

$$k_x = \frac{\partial F}{\partial x} \cdot \frac{\Delta x}{F}$$

Mechanikai érzékelők : $R = \rho \cdot \frac{\ell}{A}$

$$\Delta R = \frac{\partial R}{\partial \rho} \cdot \Delta \rho + \frac{\partial R}{\partial \ell} \cdot \Delta \ell + \frac{\partial R}{\partial A} \cdot \Delta A$$

$$\left.\begin{aligned} \frac{\Delta R}{R} &\approx \frac{\Delta \rho}{\rho} + \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A} \\ (V &= \ell \cdot A) \\ \frac{\Delta V}{V} &= \frac{\Delta \ell}{\ell} + \frac{\Delta A}{A} \end{aligned}\right\} \quad \frac{\Delta R}{R} \approx \frac{\Delta \rho}{\rho} + 2 \cdot \frac{\Delta \ell}{\ell}$$

$$G = \frac{\left(\frac{\Delta R}{R}\right)}{\left(\frac{\Delta \ell}{\ell}\right)} = 2$$ gauge-faktor

1. $P_K : P_{Na} : P_{Cl} = 1 : 0{,}03 : 0{,}1$

kint	10	460	540
kext	400	50	40
ion	K	Na	Cl

megoldás:

$$\Delta U = -58\,mV \cdot \lg\left(\frac{1\cdot 400 + 0{,}03\cdot 50 + 0{,}1\cdot 540}{1\cdot 10 + 0{,}03\cdot 460 + 0{,}1\cdot 40}\right)$$

$$\Delta U = -70{,}44\,mV$$ ✓

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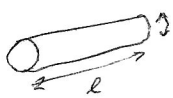
2. $\Delta U = +90\,mV$

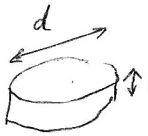
$\frac{C}{A} = 1\,\mu F/cm^2$ sejtmembrán fajlagos kapacitása

$Q = ?$

a.) $\ell = 10\,cm$ hosszú, $d = 640\,\mu m$ átmérőjű → $r = 320\,\mu m$



b.) $d = 20\,\mu m$ átmérőjű, $h = 5\,\mu m$ magas → $r = 10\,\mu m$



$F = 96500\,\frac{C}{mól}$

megoldás: $Q = C \cdot U$

$$\Delta Q = C \cdot \Delta U = \left(\frac{C}{A}\right) \cdot A \cdot \Delta U$$

$$\Delta Q = 1\,\frac{\mu F}{cm^2} \cdot A \cdot 90\,mV = 90 \cdot \frac{10^{-6}}{10^{-4}} \cdot 10^{-3} \cdot A\,[C] = 90 \cdot 10^{-5} \cdot A\,[C] = 900\,A\,[\mu C]$$

a.) $A = 0{,}1 \cdot 2 \cdot 320 \cdot 10^{-6}\,\pi + \underbrace{2\cdot(320\cdot 10^{-6})^2\,\pi}_{\approx 0} = 201{,}062 \cdot 10^{-6}\,m^2$

$= 201{,}062\,mm^2$

b.) $A = 20 \cdot 10^{-6} \cdot 5 \cdot 10^{-6} + 2\cdot(10\cdot 10^{-6})^2\,\pi = (100 + 400\pi)\cdot 10^{-12}$

$A = 9{,}425 \cdot 10^{-10}\,m^2$

$A = 728{,}32 \cdot 10^{-12}\,m^2 = 728{,}32 \cdot 10^{-6}\,mm^2 = 728{,}32\,\mu m^2$

$$\Delta Q_{a.} = 900 \cdot 10^{-6} \cdot 201{,}062 \cdot 10^{-6} = 180955{,}8 \cdot 10^{-12}\,[C] = 180{,}9558 \cdot 10^{-9}\,[C] \cong 180{,}96\,nC$$

$$\Delta Q_{b.} = 900 \cdot 10^{-6} \cdot 9{,}425 \cdot 10^{-10} = 8{,}48 \cdot 10^{-13}\,C \cong 0{,}85\,pC$$ ✓

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3. sejtmembrán vastagsága: $12\,nm$

$\varepsilon_r = ?$ $\left(\frac{C}{A} = 1\,\frac{\mu F}{cm^2}\right)$

$$C = \varepsilon_0 \varepsilon_r \cdot \frac{A}{d} = 8{,}85 \cdot 10^{-12} \cdot \varepsilon_r \cdot \frac{A}{12\cdot 10^{-9}}\,m$$

$$\frac{C}{A} = \frac{1\cdot 10^{-6}}{10^{-4}} = 10^{-2} = 8{,}85 \cdot 10^{-12}\,\varepsilon_r \cdot \frac{1}{12\cdot 10^{-9}}$$

$\Rightarrow \varepsilon_r = 13{,}56$ ✓

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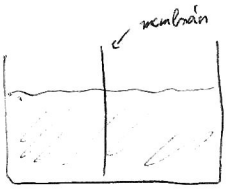
4. $\Delta U = 70\,mV$, $d = 12\,nm$ → $E = ?$

$$E = \frac{\Delta U}{d} = \frac{70\cdot 10^{-3}}{12\cdot 10^{-9}} = 5{,}83\cdot 10^{6}\,\frac{V}{m} \cong 5833333\,\frac{V}{m}$$

5.



KCl

$T = 298\,K$

$\Delta U = 10\,mV$

csak a K mozdulna átjárható!

a.) koncentrációkülönbség? $\frac{c_1}{c_2}$

$$\Delta U = -58\,mV \cdot \lg\frac{c_1}{c_2} \rightarrow \frac{c_1}{c_2} = 10^{\frac{10\,mV}{+58\,mV}} = 1{,}4873 \cong 1{,}49$$

b.) $c_1 = 0{,}1\,\frac{mól}{\ell}$, $h_T = \pm 1\,K$ → $h_{c_2} = ?$

$$\Delta U = -\frac{RT}{zF} \cdot \ln\frac{c_1}{c_2} \rightarrow e^{(-)\,10mV\,:\,\frac{8{,}314\cdot T}{1\cdot 96500}} = \frac{c_1}{c_2}$$

$(F = 96500\,\frac{C}{mól})$

$$\Rightarrow c_2 = 0{,}1\,\frac{mól}{\ell} \cdot e^{-10^{-2}\cdot\frac{8{,}314\cdot[298]}{96500}}$$

$$\frac{8{,}314\cdot[298]}{96500}\cdot 10^{-2}\,\left[\frac{mól}{\ell}\right]$$

$$c_{max} = 0{,}1\,\frac{mól}{\ell} \cdot e^{-\frac{0{,}01\cdot 8{,}314\cdot 299}{96500}} = 0{,}099974\,\frac{mól}{\ell}$$

$$c_{min} = 0{,}1\,\frac{mól}{\ell} \cdot e^{-\frac{0{,}01\cdot 8{,}314\cdot 297}{96500}} = 0{,}099974\,\frac{mól}{\ell}$$

$1{,}72\cdot 10^{-4}$ % eltérés

$$\Delta U = -\frac{RT}{zF}\ln\frac{c_1}{c_2} = -\frac{8{,}314\cdot[298\pm1]}{(1)\ 96500}\cdot\ln\frac{c_1}{c_2} = 10\,mV$$

$$= -\left(\frac{8{,}314\cdot298}{96500} \pm \frac{8{,}314}{96500}\right)\cdot\ln\frac{c_1}{c_2} = 10\,mV$$

$$\frac{c_1}{c_2} = \left[\frac{10\,mV}{-\left(\frac{8{,}314\cdot298}{96500} \pm \frac{8{,}314}{96500}\right)}\right] \Rightarrow c_2 = c_1\cdot e^{\pm\frac{10\,mV}{\frac{8{,}314\cdot298}{96500}\pm\frac{8{,}314}{96500}}}$$

$c_1 = 0{,}1\ \frac{mól}{l}$ e

$$c_2 = 0{,}1\cdot e^{\frac{10^{-2}}{\frac{8{,}314\cdot(298\pm1)}{96500}}} \Rightarrow \frac{c_{2\,max}}{c_{2\,min}} = \frac{e^{\frac{10^{-2}}{\frac{(8{,}314\cdot297)}{96500}}}}{e^{\frac{10^{-2}}{\frac{(8{,}314\cdot299)}{96500}}}} = \frac{e^{\frac{965}{8{,}314\cdot297}}}{e^{\frac{965}{8{,}314\cdot299}}}$$

$$\frac{c_{2\,max}}{c_{2\,min}} = e^{\left(\frac{965}{8{,}314\cdot297}-\frac{965}{8{,}314\cdot299}\right)} = 1{,}002617504 \equiv 0{,}26175\%\ \text{eltérés}$$ ✓

$$\approx \pm 0{,}13\% \Rightarrow 0{,}1\ \frac{mól}{l}\cdot 0{,}13\%\cdot \cancel{96500\ \frac{C}{mól}} = 0{,}13\ \frac{mmól}{l}$$ ✓

6.) Na-ra ionszelektív elektród:

$P_{Na} = 2\cdot10^{-4}\ \frac{cm}{s}$

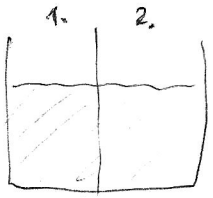
$P_K = 10^{-6}\ \frac{cm}{s}$

$P_{Cl} = 5\cdot10^{-7}\ \frac{cm}{s}$

$T = 25°C$

	1.	2.
K^+	20 mM	400 mM
Na^+	20 mM	40 mM → 48 mM
Cl^-	100 mM	50 mM

(M = mól)



a.) → $\Delta U = ?$

$$\Delta U = \overset{-58\,mV}{\cancel{-59{,}8}}\cdot\log\frac{10^{-6}\cdot(20)+2\cdot10^{-4}\cdot(20)+(50)\cdot5\cdot10^{-7}}{10^{-6}\cdot400+2\cdot10^{-4}\cdot40+100\cdot5\cdot10^{-7}} = \cancel{0{,}1856\ mV} \approx \cancel{0{,}19\ mV} = 18{,}56\ mV$$ ✓

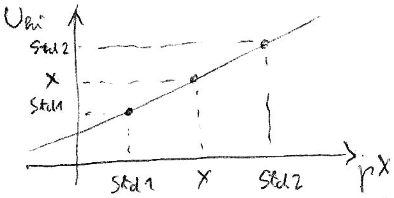
b.) a Na-koncentrációt a 2-es kamrában megnöveljük 20%-kal. Hogyan változik az eredmény?

$$\cancel{\Delta U = -2{,}298} \quad \Delta U = -58\,mV\cdot\log\left(\frac{10^{-6}\cdot20+2\cdot10^{-4}\cdot20+50\cdot5\cdot10^{-7}}{10^{-6}\cdot400+2\cdot10^{-4}\cdot48+500\cdot10^{-7}}\right) = 22{,}92\ mV$$ ✓

c.) $[Na]_2 = 48\,mM$ → $\Delta U = -58\,mV\cdot\log\frac{-\,''-}{10^{-6}\cdot480+2\cdot10^{-4}\cdot48+500\cdot10^{-7}} = 23{,}12\ mV$ ✓

$[K]_2 = 480\,mM$

7.) Végán-analízis



pH (Std 1.) = 6,838 ± 0,005
pH (Std 2.) = 7,382 ± 0,005
U_{ki} (Std 1.) = 1000 mV
U_{ki} (Std 2.) = 2000 mV

$$\frac{pX(x) - pX(Std1)}{U_{ki}(x) - U_{ki}(Std1)} = \frac{pX(Std2) - pX(Std1)}{U_{ki}(Std2) - U_{ki}(Std1)}$$

a.) $U_{ki}(x) = 2061$ mV

$$pX(x) = 6{,}838 + (2061\,mV - 1000\,mV) \cdot \frac{7{,}382 - 6{,}838}{2000 - 1000\ [mV]}$$

$pX(x) = 7{,}415$ ✓

b.) $U_{ki}(x) = 1900$ mV $pX(x) = 7{,}328$ ✓

c.) min - max ?

$$pX(x) = pX(Std1) + (U_{ki}(x) - U_{ki}(Std1)) \cdot \frac{pX(Std2) - pX(Std1)}{U_{ki}(Std2) - U_{ki}(Std1)}$$

$$pX(x) = pX(Std1) \cdot \left[1 - \frac{U_{ki}(x) - U_{ki}(Std1)}{U_{ki}(Std2) - U_{ki}(Std1)}\right] + \left[\frac{U_{ki}(x) - U_{ki}(Std1)}{U_{ki}(Std2) - U_{ki}(Std1)}\right] \cdot pX(Std2)$$

(a.) 1,061; b.) 0,9)

b.) min pX(x), ha min pX(Std 1) és min pX(Std 2)
max pX(x), ha max pX(Std 1) és max pX(Std 2)

a.) min pX(x), ha max pX(Std 1) és min pX(Std 2)
max pX(x), ha min pX(Std 1) és max pX(Std 2) ✓

a.) $pX(x)_{min} = (6{,}838 + 0{,}005) \cdot (1 - 1{,}061) + (7{,}382 - 0{,}005) \cdot 1{,}061 = 7{,}4096$

$pX(x)_{max} = (6{,}838 - 0{,}005) \cdot (1 - 1{,}061) + (7{,}382 + 0{,}005) \cdot 1{,}061 = 7{,}4208$

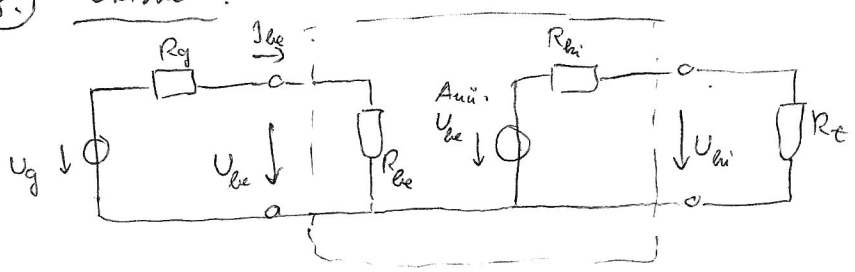
b.) $pX(x)_{min} = (6{,}838 - 0{,}005) \cdot (1 - 0{,}9) + (7{,}382 - 0{,}005) \cdot 0{,}9 = 7{,}3226$

$pX(x)_{max} = (6{,}838 + 0{,}005) \cdot (1 - 0{,}9) + (7{,}382 + 0{,}005) \cdot 0{,}9 = 7{,}3326$

a.) pX(x) = 7,410 ... 7,421 ✓

b.) pX(x) = 7,323 ... 7,333 ✓

(8.) ideális:



$$R_{be} = \frac{U_{be}}{I_{be}} \qquad U_{be} = U_g \cdot \frac{R_{be}}{R_{be}+R_g}$$

$$R_{ki} = -\frac{U_{ki\ üresjárási}}{I_{ki\ rövidzár}} \qquad U_{ki} = A_{ui} \cdot U_{be} \cdot \frac{R_t}{R_{ki}+R_t}$$

a.) $R_t = \infty \rightarrow U_{ki} = 1V$

$R_t = 10k\Omega \rightarrow U_{ki} = 0{,}5V$

$R_{ki} = ?$

$$U_{ki_1} = A_{ui} \cdot U_{be} \cdot 1$$

$$U_{ki_2} = A_{ui} \cdot U_{be} \cdot \frac{10k\Omega}{R_{ki}+10k\Omega} = \frac{U_{ki_1}}{2} \Rightarrow R_{ki} = 10k\Omega$$

($U_{ki_2} = \frac{1}{2} U_{ki_1}$)

b.) $R_g = 0 \rightarrow U_{ki_1} = 2V$

$R_g = 100k\Omega \rightarrow U_{ki_2} = 1V$

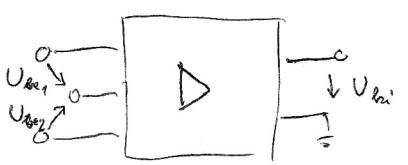
$R_{be} = ?$

$$U_{ki_1} = \underbrace{A_{ui} \cdot U_{be} \cdot \frac{R_t}{R_{ki}+R_t}}_{K_1} = \underbrace{K_1 \cdot U_g \cdot \frac{R_{be}}{R_{be}+R_g}}_{K_2} = 2V$$

$$U_{ki_2} = K_2 \cdot \frac{R_{be}}{R_{be}+R_g} = 1V$$

$$\frac{U_{ki1}}{U_{ki2}} = \frac{\frac{R_{be}}{R_{be}+0}}{\frac{R_{be}}{R_{be}+100k\Omega}} = 2 = \frac{R_{be}}{R_{be}+100k\Omega} \Rightarrow R_{be} = 100k\Omega$$

(9.) nemszimmetrikus erősítő



$U_S = U_{be_1} - U_{be_2}$ „jó"

$U_K = \frac{U_{be_1} + U_{be_2}}{2}$ „zaj"

feladat:	$U_1 = 9{,}95V$	$U_1 = 5{,}1V$
A_{u_S}, A_{u_K} ?	$U_2 = 10{,}05V$	$U_2 = 5V$
E_{ku} ?	$U_{ki} = -0{,}9V$	$U_{ki} = 1{,}0505V$

$$U_{ki} = A_{u_S} \cdot U_S + A_{u_K} \cdot U_K$$

$$E_{ku} = \frac{A_{u_S}}{A_{u_K}}$$

a.) $-0{,}9 = A_{u_S} \cdot \underbrace{(9{,}95 - 10{,}05)}_{-0{,}1} + A_{u_K} \cdot \underbrace{\frac{9{,}95+10{,}05}{2}}_{10V}$

b.) $+1{,}0505 = A_{u_S} \cdot (+0{,}1V) + A_{u_K} \cdot 5{,}05$

$$A_{u_K} = \frac{1{,}0505 + 0{,}1 A_{u_S}}{10}$$

$$-0{,}9 = A_{u_S} \cdot (-0{,}1) + \left(\frac{1{,}0505}{10} + 0{,}01 A_{u_S}\right) \cdot 10$$

$$-0{,}9 = A_{u_S}(1{,}0505 + 0{,}1 - 0{,}1) + 1{,}0505$$

$$A_{u_S} = \frac{-0{,}9}{1{,}0505} = -0{,}8567$$

$$A_{u_K} = 0{,}0965$$

$$\Rightarrow \begin{aligned} -0{,}9V &= -0{,}1 A_{u_S} + 10 A_{u_K}\ [V] \\ 1{,}0505 &= 0{,}1 A_{u_S} + 5{,}05 A_{u_K}\ [V] \end{aligned}$$

$\Rightarrow A_{u_S} = 10$; $A_{u_K} = 0{,}01$; $E_{ku} = 1000 \rightarrow E_{ku}^{dB} = 60dB$