

4 vizsga/β varián

1. a)  $\int_0^{\sqrt{2}} \sin^2 x \cos^3 x dx = (u = \sin x, du = \cos x dx) = \int u^2 (1-u^2) du$  (2)  
 $= \frac{u^3}{3} - \frac{u^5}{5} = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} \Big|_0^{\sqrt{2}} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$  (2)

b)  $\int_{-\infty}^0 \frac{dx}{x^2 - 6x + 12} = \int_{-\infty}^0 \frac{dx}{(x-3)^2 + 3} = \frac{1}{3} \int_{-\infty}^0 \frac{dx}{(\frac{x-3}{\sqrt{3}})^2 + 1} = \frac{1}{\sqrt{3}} \arctan \frac{x-3}{\sqrt{3}} \Big|_{-\infty}^0 = \frac{\pi}{6\sqrt{3}}$  (2)

c)  $\int \frac{x+3}{x^2+4x+4} = \int \frac{x+2+1}{(x+2)^2} = \int \frac{1}{x+2} + \int \frac{1}{(x+2)^2} = \ln|x+2| - \frac{1}{x+2} + C$  (2)

d)  $\int x^2 \ln x dx = \int \ln x d(\frac{x^3}{3}) = \frac{x^3}{3} \ln x - \int \frac{x^3}{3} \cdot \frac{1}{x} = \frac{x^3}{3} \ln x - \frac{x^3}{9} + C$  (2)

2. a)  $f \in R[a, b] \wedge \exists F' = f \Rightarrow \int_a^b f = F(b) - F(a)$  (2)

8 b)  $\int_{-\infty}^{-1} x^{-(2p+3)} dx = \lim_{N \rightarrow -\infty} \frac{x^{-2p-2}}{-2p-2} \Big|_N^{-1}$ , feltét  $p > -1$  (2)

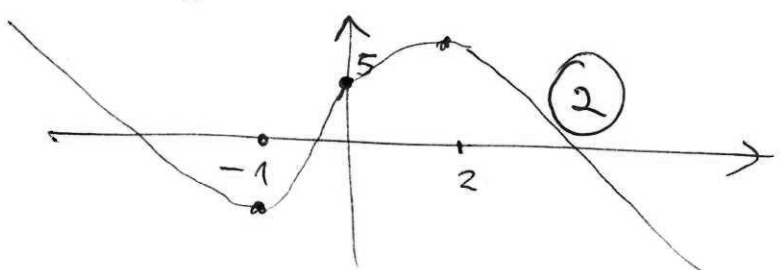
3.  $l = \int_1^2 \sqrt{1+(6x^2)^2} = \int_1^2 \sqrt{1+36x^4} = \frac{1}{54} (1+36x^4)^{3/2} \Big|_1^2 = \frac{1}{54} (73^{3/2} - 37^{3/2})$  (4)

4.  $(iz)^3 = 27 \Rightarrow z^3 = 27i = 27e^{i\pi/2}$ ,  $z_0 = 3e^{i\pi/6}$ ,  $z_0 = \frac{3\sqrt{3}}{2} + \frac{3i}{2}$  (2)  
 $z_1 = 3e^{5\pi i/6}$ ,  $z_2 = 3e^{3\pi i/2}$ ,  $z_1 = -\frac{3\sqrt{3}}{2} + \frac{3i}{2}$ ,  $z_2 = -3i$  (2)

5.  $(fg)' = f'g + fg'$  (2) + biz. (6) b)  $\ln y = x^3 \ln x$ ,  $\frac{y'}{y} = 3x^2 \ln x + x^2$  (2)

6.  $\sqrt{x^2+2x}-x = \frac{2x}{\sqrt{x+1}} \rightarrow 1$ ,  $\cos \pi k = (-1)^k$  (2)  
 $\sqrt{x^2+2}-x = \frac{2}{\sqrt{x+1}} \rightarrow 0$ ,  $|\cos| \leq 1$ ,  $\lim = 0$  (2)

7.  $f' = -6x^2 + 6x + 12 = -6(x+1)(x-2)$ ,  $f'' = -12x + 6$   $D_f = R$ ,  $R_f = R$  (1)  
 $f' > 0, -1 < x < 2$ ,  $f' < 0, x < -1, x > 2$  (2)  
 $x = 1/2$  inf. (2)  
 $x > 1/2$  konvex (2)  
 $x < 1/2$  konkáv (2)  
 $\lim_{x \rightarrow \pm\infty} = \mp \infty$  (1)



# 4. vizsga / 2 variáns

24) 1.\* a)  $\int_0^{\sqrt{2}} \sin^3 x \cos^2 x dx = \int_0^{\sqrt{2}} (1-u^2)u^2 du$  (2)  
 $= -\frac{u^3}{3} + \frac{u^5}{5} = -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} \Big|_0^{\sqrt{2}} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$  (2)

b)  $\int_0^{\infty} \frac{dx}{x^2+6x+12} = \int_0^{\infty} \frac{dx}{(x+3)^2+3} = \frac{1}{3} \int_0^{\infty} \frac{dx}{(\frac{x+3}{\sqrt{3}})^2+1} = \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{x+3}{\sqrt{3}} \Big|_0^{\infty} = \frac{\pi}{\sqrt{3}}$  (2)

c)  $\int \frac{x dx}{x^2+4x+4} = \int \frac{x-2+2 \cdot dx}{(x-2)^2} = \int \frac{dx}{x-2} + 2 \int \frac{dx}{(x-2)^2} = \ln|x-2| - \frac{2}{x-2}$  (2)

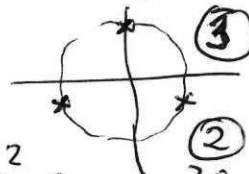
d)  $\int x \ln x dx = \int \ln x d(\frac{x^2}{2}) = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} dx = \frac{x^2}{2} \ln x - \frac{x^2}{4}$  (2)

2) 2.\* a)  $f \in R[a, b]$  és  $\exists F' = f \Rightarrow \int_a^b f(x) dx = F(b) - F(a)$  (2)

b)  $\int_{-\infty}^{-1} x^{-(p+3)} dx = \lim_{N \rightarrow -\infty} \frac{x^{-p+2}}{-p+2} \Big|_{-1}^{-1}$ , tehát  $p > 2$ . (2)

3.\*  $I = \int_1^2 \sqrt{1+(3x^{1/2})^2} dx = \int_1^2 \sqrt{1+9x} dx = \frac{2}{27} (1+9x)^{3/2} \Big|_1^2 = \frac{2(19^{3/2} - 10^{3/2})}{27}$  (4)

4.  $(-i)^3 z^3 = 8, +iz^3 = 8, z^3 = -8i = 8e^{3\pi/2 i}, z_0 = 2e^{\pi/2} = 2i$  (3)  
 $z_1 = 2e^{7\pi/6} = -(\sqrt{3}+i), z_2 = \sqrt{3}-i$  (2)



5. a)  $(f \cdot g)' = f'g + fg'$  (6) b)  $y = x^{x^2}, \ln y = x^2 \ln x, \frac{y'}{y} = 2x \ln x + x, y' = \dots$  (2)

6.  $\sqrt{x^2+x} - x = \frac{x}{\sqrt{x^2+x}} \rightarrow \frac{1}{2}, \cos \pi k = (-1)^k, \sqrt{x^2+1} - x = \frac{1}{\sqrt{x^2+1}} \rightarrow 0, \lim = 0$  (4)

7.  $f' = 6(x-1)(x+2), f'' = 6(2x+1), f' > 0: x > 1, x < -2, f' < 0: -2 < x < 1$   
 $-2: \max, 1: \min$  (4)  
 $x = -1/2$  inflex. (2)  
 $x < -1/2$ : konkáv (2)  
 $x > -1/2$ : konvex (2)

