1, a, $\boxed{6}$

$$z_1=\frac{3+2i}{3i-4}=\frac{(3+2i)(-3i-4)}{(3i-4)(-3i-4)}\ 3=\frac{-12+6+i(-9-8)}{16+9}=-\frac{6}{25}+i\cdot\left(\frac{-17}{25}\right)\ 3$$

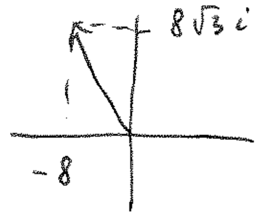
b, $\boxed{9}$

$$|z_2|=|1-i|^{13}=(\sqrt{2})^{13}\ 4=2^6\cdot\sqrt{2}=\underline{\underline{64\cdot\sqrt{2}}}$$

$$\arg(1-i)=-\frac{\pi}{4}\ 2\ ;\quad \arg z_2=13\cdot\left(-\frac{\pi}{4}\right)=-\frac{13}{4}\pi\ 3=\underline{\underline{\frac{3\pi}{4}}}\pmod{2\pi}$$

2, $\boxed{10}$

$$|-8+i8\sqrt{3}|=\sqrt{8^2+8^2\cdot 3}=16\quad 3$$



$$\arg(-8+i\cdot 8\sqrt{3})=\varphi$$

$$\operatorname{tg}\varphi=\frac{8\sqrt{3}}{-8}=-\sqrt{3}\implies\varphi=\frac{2\pi}{3}\quad 3$$

$$z=\sqrt[4]{16}\cdot e^{i\left(\frac{\varphi}{4}+k\frac{\pi}{2}\right)}=\underline{\underline{2\cdot e^{i\left(\frac{\pi}{6}+k\frac{\pi}{2}\right)},\quad k=0,1,2,3}}\quad 4$$

3, a, $\boxed{5}$ $\lim\limits_{n\to\infty}a_n=A$, ha $\forall\varepsilon>0$ esetén $\exists N(\varepsilon)\in\mathbb{N}$, melyre $\forall n>N(\varepsilon)$ esetén $|a_n-A|<\varepsilon$.

b, $\boxed{8}$

$$\left|\sqrt{4+\frac{3}{n}}-2\right|=\frac{\cancel{4}+\frac{3}{n}-\cancel{2^2}}{\sqrt{4+\frac{3}{n}}+2}\ 3\leq\frac{3}{2n}\ 3<\varepsilon,\text{ ha } n>\underline{\underline{\frac{3}{2\varepsilon}=N(\varepsilon)}}\ 2$$

4, a, $\boxed{7}$

$$a_n=\sqrt{n^2+2n+4}-\sqrt{n^2+1}=\frac{n^2+2n+4-(n^2+1)}{\sqrt{n^2+2n+4}+\sqrt{n^2+1}}=$$

$$=\frac{2n+3}{\sqrt{n^2+2n+4}+\sqrt{n^2+1}}\ 3=\frac{2+\overset{\to 0}{3/n}}{\sqrt{1+\underset{\downarrow 0}{2/n}+\underset{\downarrow 0}{4/n^2}}+\sqrt{1+\underset{\downarrow 0}{1/n^2}}}\ 2\longrightarrow\frac{2}{\sqrt{1}+\sqrt{1}}=\underline{\underline{1}}\ 2$$

4/b, $\boxed{7}$ $b_n = \dfrac{4^n+n^2}{n^6+2^{2n+3}} = \dfrac{4^n+n^2}{n^6+8\cdot 4^n}$ (2) $= \dfrac{1+\underbrace{n^2/4^n}_{\to 0}}{\underbrace{n^6/4^n}_{\to 0}+8}$ (4) $\longrightarrow \dfrac{1}{8}$ (2)

c, $\boxed{7}$ $c_n = \left(\dfrac{2n+1}{2n+4}\right)^n = \dfrac{\left(1+\frac{1/2}{n}\right)^n}{\left(1+\frac{2}{n}\right)^n}$ (4) $\longrightarrow \dfrac{e^{1/2}}{e^2} = e^{-3/2}$ (3)

d, $\boxed{9}$ $d_n = c_n^n$. Láttuk, hogy $c_n \to e^{-3/2} < 1$, tehát $n > N_0$ esetén

$$0 < c_n < q < 1 \quad (6) \quad / \uparrow n$$

$$0 < c_n^n = d_n < q^n$$

Rendőr elv alapján $d_n \to 0$. (3)

$$\underset{0}{\overset{\|}{0}} \qquad d_n \downarrow 0 \qquad q^n \downarrow 0$$

5, a, Teljes indukcióval. $a_1 = 3 \ge 3$ ✓ T.f.h. $a_n \ge 3$. Ekkor

$\boxed{6}$ $a_{n+1} = a_n^2 + 3a_n - 8 \ge 3^2 + 3\cdot 3 - 8 = 10 \ge 3$ ✓

$\boxed{9}$ b, Teljes indukcióval. $a_1 = 3 \le a_2 = 3^2 + 3\cdot 3 - 8 = 10 = a_2$ (2)

T.f.h. $a_n \le a_{n+1}$ (1) A rekurziós függvény $f(x) = x^2 + 3x - 8 = (x+\frac{3}{2}) - \frac{9}{4} - 8$

az $x > -\frac{3}{2}$ félegyenesen szigorúan monoton nő. (3) Tehát

$-\frac{3}{2} < 3 \le a_n \le a_{n+1}$ esetén $f(a_n) = a_{n+1} \le f(a_{n+1}) = a_{n+2}$ ✓ (3)

$\boxed{7}$ c, Ha $\exists \lim_{n\to\infty} a_n = A \in \mathbb{R}$, akkor $A = f(A)$, tehát

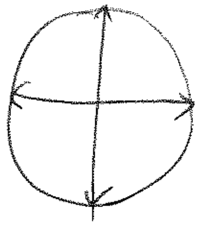
$$A = A^2 + 3A - 8 \Rightarrow A^2 + 2A - 8 = (A+4)(A-2) = 0$$

$A_1 = -4$, $A_2 = +2$ lehetséges határértékek. (4)

Ezek közül egyik sem jöhet szóba, hiszen $a_n \ge 3$, a_n monoton nő. Tehát a_n divergens. (3) $(a_n \to \infty)$

6, $\boxed{12}$

$$\cos(n\pi/2) = \begin{cases} 1, & \text{ha } n = 4k \quad (k \in \mathbb{Z}) \\ 0, & \text{ha } n = 2l+1 \quad (l \in \mathbb{Z}) \\ -1, & \text{ha } n = 4k+3 \quad (k \in \mathbb{Z}) \end{cases}$$



Így

$$\left.\begin{aligned} a_{4k} &= e^{4k} \longrightarrow \infty \\ a_{2l+1} &= e^{(2l+1)\cdot 0} = 1 \longrightarrow 1 \\ a_{4k+3} &= e^{(4k+3)\cdot(-1)} \longrightarrow 0 \end{aligned}\right\} \quad 6$$

Tehát a torlódási pontok: $\{0, 1, +\infty\}$ 3

$\liminf a_n = 0$, $\limsup a_n = +\infty$; $\nexists \lim a_n$. 3