(α) június 20

1. $\begin{cases} x = 2r\cos t \ (3) \\ z = 3r\sin t \\ y = y \end{cases}$ $J = 6r$ (3) $\iiint\limits_V \ldots = \int_0^1 \int_0^2 \int_0^{2\pi} 2y \cdot 6r\, e^{r^2}\, dt\, dr\, dy$ (5)

$= 12 \int_0^{2\pi} 1\, dt \cdot \int_0^2 r e^{r^2} dr \cdot \int_0^1 y\, dy$

$= 12 \cdot 2\pi \cdot \frac{1}{2} \cdot \frac{1}{2} \int_0^4 e^u du = 6\pi(e^4 - 1).$ (5)

2. $f(x) = \begin{cases} 2x, & -2 < x < 0 \ (2) \\ 0, & x \geq 0, x \leq -2 \end{cases}$ $F(t) = \int_{-2}^{0} 2x e^{-ixt} dx$ (4)

$= \frac{1}{-it} \int_{-2}^{0} 2x\, d(e^{-ixt}) = \frac{i}{t}\left(2x e^{-ixt}\Big|_{-2}^{0} - 2\int_{-2}^{0} e^{-ixt} dx\right)$

$= \frac{i}{t}\left(4e^{2it} + \frac{2}{it} e^{-ixt}\Big|_{-2}^{0}\right) = \frac{4i}{t} e^{2it} + \frac{2}{t^2}\left(1 - e^{2it}\right)$ (6)

3. $a_n = \frac{1}{\pi}\int f(x)\cos nx\, dx$, $b_n = \frac{1}{\pi}\int f(x)\sin nx\, dx$, $S = \frac{a_0}{2} + \sum a_n \cos nx + b_n \sin nx$ (3)

$3 + 4\sin x - 2\cos 3x$ : önmaga! (5)

4. $f_x = 2x + \ln y$, $f_y = \frac{x}{y} \Rightarrow (0,1)$ (4) – kr. pont $D = \begin{pmatrix} 2 & \frac{1}{y} \\ \frac{1}{y} & -\frac{x}{y^2} \end{pmatrix}$

$= \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = -1 < 0$ (4) nyeregpont. (2)

5. $f(a) - f(b) = f'(c)(a-b)$, (4) $a < c < b$. Ha $f' \equiv 0 \Rightarrow$ (6) $f(x) = f(y)$, $\forall x, y$

6. Hom: $\lambda^2 + 9 = 0$, $\lambda = \pm 3i$ (2), $y_h = c_1 \cos 3x + c_2 \sin 3x$ (2)

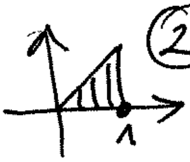
$y_p = (A\cos 3x + B\sin 3x)x$ (2) $y'' + 9y = -6A\sin 3x + 6B\cos 3x$

$A = 0$, $B = 3$ (4)

$y = 3x\sin 3x + c_1\cos 3x + c_2 \sin 3x$ (2)

7. $\lim\limits_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \begin{cases} <1: \text{konv.} \\ >1: \text{div} \\ =1: ? \end{cases}$ (4) b.1. $\left|\frac{a_{n+1}}{a_n}\right| = \left(\frac{n}{n+1}\right)^n$ (2) $\to \frac{1}{e} < 1$ (3) absz. konv.

b.2. $\sqrt{n+1} - \sqrt{n} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$ (2), Leibniz (3) feltételesen konv.

8. 

(2) $I = \int_0^1 \int_0^x e^{x^2} dy\, dx$ (4) $= \int_0^1 x e^{x^2} dx = \frac{1}{2} e^{x^2}\Big|_0^1$ (4) $= \frac{e-1}{2}$ (2)

9. $f'' = 10 + 9\sin 3x$ (3) $\geq 10 - 9 = 1 > 0$ (3) konvex!

(β) június 20

1. $\begin{cases} x = 2r\cos t \\ z = \dfrac{r\sin t}{3} \\ y = y \end{cases}$ (3) $\quad J = \frac{2}{3} r$ (3) $\quad \iiint\limits_V \ldots = \int_0^1 \int_0^2 \int_0^{2\pi} 2y \cdot \frac{2}{3} r \cdot e^{r^2} \, dt\, dr\, dy$ (5)

$= \frac{4}{3} \int_0^{2\pi} 1 \, dt \cdot \int_0^2 r e^{r^2} dr \cdot \int_0^1 y \, dy = \frac{4}{3} \cdot 2\pi \cdot \frac{1}{2} \cdot \frac{1}{2}(e^4 - 1) = \frac{2\pi(e^4-1)}{3}$ (5)

2. $f(x) = \begin{cases} 2x, & -5 < x < 0 \\ 0, & x \geq 0,\ x \leq -5 \end{cases}$ (2) $\quad F(t) = \int_{-5}^{0} 2x e^{-ixt} \, dx$ (4)

$= -\frac{1}{it}\left( 2x e^{-ixt} \Big|_{-5}^{0} - 2\int_{-5}^{0} e^{-ixt} \, dx \right) = \frac{i}{t}\left( 10 e^{5it} + \frac{2}{it} e^{-ixt} \Big|_{-5}^{0} \right)$

$= \frac{10i}{t} e^{5it} + \frac{2}{t^2}\left(1 - e^{5it}\right)$ (6)

3. u.a. mint α!

4. $f_x = 2x + 2\ln y, \quad f_y = \frac{2x}{y}, \quad \underline{\underline{(0,1)}}$ (4) $\quad D = \begin{vmatrix} 2 & 2 \\ 2 & 0 \end{vmatrix} = -4 < 0$ (4)

<u>nyeregpont</u> (2)

5. u.a. mint α!

6. Hom.: $\lambda^2 + 9 = 0, \ \lambda = \pm 3i$ (2), $y_h = C_1 \cos 3x + C_2 \sin 3x$ (2)

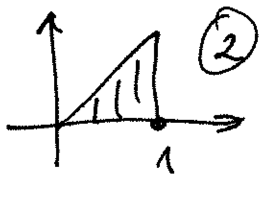
$y_p = x(A\cos 3x + B\sin 3x)$ (2), $y'' + 9y = -6A\sin 3x + 6B\cos 3x$

$B = 0, \ A = -3$ (4)

$y = -3x\cos 3x + C_1 \cos 3x + C_2 \sin 3x$ (2)

7. (a) u.a. mint α (4) $\quad$ b.1 $\frac{a_{n+1}}{a_n} = \left(\frac{n+1}{n}\right)^n$ (2) $\to e > 1$ <u>div.</u> (3)

$\sqrt{n+\sqrt{n}} - \sqrt{n}$ (2) $= \frac{\sqrt{n}}{\sqrt{n+\sqrt{n}} + \sqrt{n}} \to \frac{1}{2} \neq 0$ <u>div.</u> (3)

8. 
(2) $\quad I = \int_0^1 \int_0^x e^{2x^2} \, dy\, dx$ (4) $= \int_0^1 x e^{2x^2} dx$

$\stackrel{(4)}{=} \frac{1}{4} e^{2x^2} \Big|_0^1 = \frac{e^2 - 1}{4}$ (2)

9. $f'' = 20 - 18\sin 3x$ (3) $\geq 20 - 18 = 2 > 0$ konvex! (3)