A

**1.**

a) $\sum \frac{n^n \cdot 3^n}{(2n)!}$ HÁNYADOS KRIT. (1P)

$$\lim_{\infty} \frac{(n+1)^{n+1} \cdot 3^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{n^n \cdot 3^n} = \lim_{\infty} \frac{(n+1)^n}{n^n} \cdot \frac{n+1}{2n+2} \cdot \frac{3}{2n+1} =$$ (1P)

$$= \lim_{\infty} \frac{\left(1+\frac{1}{n}\right)^n}{1} \cdot \frac{1}{2} \cdot \frac{3}{2n+1} = 0 < 1$$ (1P) . A SOR ABSZ. KONV (1P)

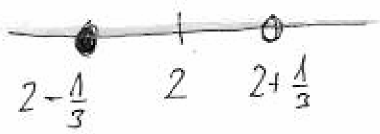
(1P) $\downarrow e$ $\quad \downarrow \frac{1}{2}$ $\quad \downarrow 0$

b)

$\sum \frac{(x-2)^n \cdot 3^n}{\sqrt{n}}$ GYÖK KRIT. (1P) $\quad \lim_{\infty} \frac{|x-2| \cdot 3}{(\sqrt[n]{n})^{1/2}} = 3|x-2| < 1$ (1P) (1P)

$|x-2| < \frac{1}{3}$ KONV. SUGÁR: $\frac{1}{3}$ (1P)

KONV. TART.




VÉGPONTOK:

$\sum \frac{1}{\sqrt{n}}$ DIV (1P)

$\sum \frac{(-1)^n}{\sqrt{n}}$

LEIBNIZ-SOR (1P) $\Rightarrow$ KONV (1P)

**12 PONT**

**2.**

$f(x) = \frac{1}{\sqrt{e^{4x}}}$ TAYLOR SORA

$\frac{1}{\sqrt{e^{4x}}} = e^{-\frac{4x}{2}} = e^{-2x}$ (1P) $\qquad e^x = \sum_0^{\infty} \frac{x^n}{n!}$ (1P)

$$e^{-2x} = \sum_0^{\infty} \frac{(-2x)^n}{n!} = \sum_0^{\infty} \frac{(-2)^n}{n!} \cdot x^n$$ (1P) (1P)

$\boxed{4P}$

A

b) $\sin 1$ $\quad x_0=0 \Rightarrow$ LAGRANGE-FÉLE MARADÉKTAG (1P)

$\pm\sin x$ VAGY $\pm\cos x$ KORLÁTOS (1P)

$$\frac{f^{(n+1)}(c)}{(n+1)!}(x-x_0)^{n+1} \leq \frac{1}{(n+1)!}\cdot 1 < \frac{1}{1000}$$ (1P)

$(n+1)! > 1000$

$6! = 720$

$7! = 5040$

TEHÁT $n=6$ JÓ LESZ (1P)

(SŐT MÁR AZ ÖTÖDFOKÚ)

4P

8 PONT

$\boxed{3.}$ $f(x)=\dfrac{5x^4}{4+x^5}$

$$5x^4\cdot\frac{1}{4+x^5} = \frac{5}{4}x^4\cdot\frac{1}{1+\frac{x^5}{4}} = \frac{5}{4}x^4\cdot\frac{1}{1-\frac{-x^5}{4}} = \frac{5}{4}x^4\cdot\sum_0^\infty\left(\frac{-x^5}{4}\right)^n =$$
(1P) (1P) (1P)

$\left|\frac{-x^5}{4}\right|<1$ (1P)

$\underline{|x|<\sqrt[5]{4}}$

$$= \underline{\sum_0^\infty \frac{5}{4}\left(\frac{-1}{4}\right)^n\cdot x^{5n+4}}$$ (2P)

$g(x)=\ln(4+x^5)$ $\quad g'=\dfrac{5x^4}{4+x^5}$ (1P) $\Rightarrow$ $g(x)=g(0)+\displaystyle\int_{t=0}^{x}g'(t)\,dt = \ln 4+\sum_0^\infty\frac{5}{4}\left(\frac{-1}{4}\right)^n\cdot\frac{x^{5n+5}}{5n+5}$ (2P)

MERT $\underline{|x|<\sqrt[5]{4}}$

9 PONT

$\boxed{4.}$ $f(x)=\dfrac{6x}{\sqrt[5]{32-x^2}} = 6x\cdot(32-x^2)^{-1/5}=$ (1P)

$$=6x\cdot 32^{-1/5}\cdot\left(1+\frac{-x^2}{32}\right)^{-1/5} = 6x\cdot\frac{1}{2}\cdot\sum_0^\infty\binom{-1/5}{n}\left(\frac{-x^2}{32}\right)^n = \sum_0^\infty 3\cdot\binom{-1/5}{n}\left(\frac{-1}{32}\right)^n x^{2n}$$
(1P) (2P) (2P)

$x^{2n+1}$

$\frac{-x^2}{32}=u \Rightarrow$ $\boxed{|u|<1 \quad |x|<\sqrt{32}}$ (1P)

7 PONT

A

$\boxed{5.}$ $f(x,y)=\dfrac{x^2y^3}{x^5+y^5}$

a)

$$\lim_{\substack{x\to 0\\ y\to 0}}\frac{x^2y^3}{x^5+y^5}=\lim_{\substack{x\to 0\\ \cancel{y\to 0}}}\frac{x^5\cdot m^3}{x^5+m^5x^5}=\lim_{x\to 0}\frac{m^3}{1+m^5}\cdot\frac{x^5}{x^5}=\frac{m^3}{1+m^5}$$

(1P) ~~(1P)~~ (1P)

$y=mx$ (1P)

A HATÁRÉRTÉK $m$-TŐL FÜGG $\Rightarrow$ $\nexists$ HATÁRÉRTÉK (1P)

b)

MIVEL NEM LÉTEZIK HATÁRÉRTÉK $(0,0)$-BAN EZÉRT A FV NEM TOT. DIFFHATÓ (1P)

c)

$$g(x,y)=(x^2+\cos x)^{3y}$$

$$g'_x=3y(x^2+\cos x)^{3y-1}\cdot(2x-\sin x)$$

~~(2P)~~ (1P) (1P)

$$g'_y=(x^2+\cos x)^{3y}\cdot\ln(x^2+\cos x)\cdot 3$$

(1P) (1P)

d)

$$h'_y=\frac{(\cos xy)\cdot x+e^{3x^2+y^2}\cdot 2y}{\sqrt{\operatorname{arctg} x^2+\operatorname{ch} x}}$$

(2P) (2P)

(1P)

$\boxed{\cancel{16}\ \text{PONT}}$

$\boxed{14\ \text{PONT}}$