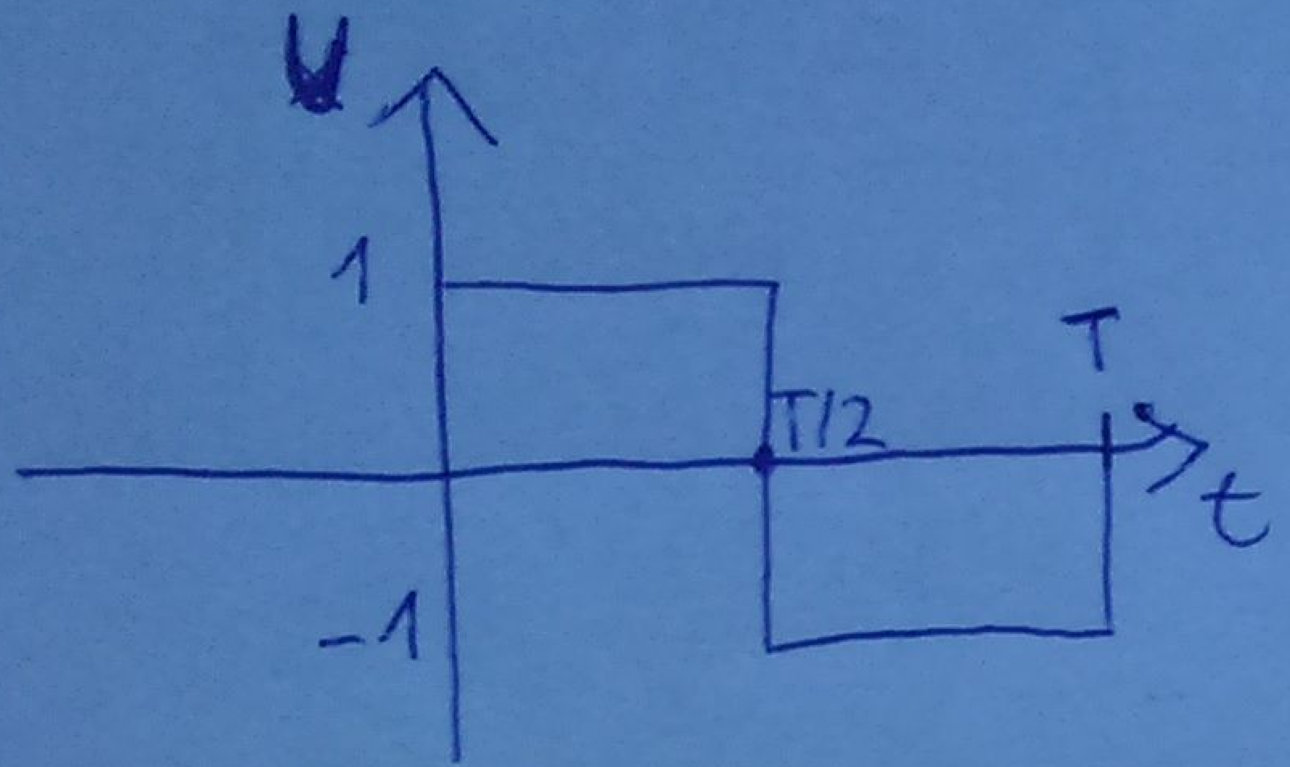
# Szimmetrikus négyszögjel:




$$\boxed{U_0 =} \frac{1}{T}\cdot\left(\int_0^{T/2} 1\cdot dt + \int_{T/2}^{T} -1\cdot dt\right) = \frac{1}{T}\cdot\left(\frac{T}{2} - 0 - T + \frac{T}{2}\right)\boxed{= 0}$$

$$U_A^k = \frac{2}{T}\cdot\int_0^T U(t)\cdot\cos(k\cdot w_0\cdot t)\cdot dt = \frac{2}{T}\cdot\int_0^T U(t)\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)dt$$

$$U_B^k = \frac{2}{T}\cdot\int_0^T U(t)\cdot\sin\left(\frac{2k\pi}{T}\cdot t\right)dt$$

$$\boxed{U_A^k =} \frac{2}{T}\cdot\left(\int_0^{T/2} 1\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)dt + \int_{T/2}^{T} -1\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)dt\right)$$

$$= \frac{2}{T}\cdot\underbrace{\left(\left[\frac{T}{2k\pi}\cdot\sin\left(\frac{2k\pi}{T}\right)\right]_0^{T/2} - \left[\frac{T}{2k\pi}\cdot\sin\left(\frac{2k\pi}{T}\right)\right]_{T/2}^{T}\right)}_{0}\boxed{= 0}$$

$$\boxed{U_B^k} = \frac{2}{T}\cdot\left(\int_0^{T/2} 1\cdot\sin\left(\frac{2k\pi}{T}\cdot t\right)dt + \int_{T/2}^{T} -1\cdot\sin\left(\frac{2k\pi}{T}\cdot t\right)dt\right)$$

$$= \frac{2}{T}\cdot\left(\left[\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\right)\right]_0^{T/2} + \left[\frac{T}{2k\pi}\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)\right]_{T/2}^{T}\right)$$

$$= \frac{2}{T}\cdot\frac{T}{2k\pi}\cdot\left(-\cos(k\pi)+1+1-\cos(k\pi)\right) =$$

$$\boxed{= \frac{1}{k\pi}\cdot\left(2-2\cdot\cos(k\pi)\right)}$$

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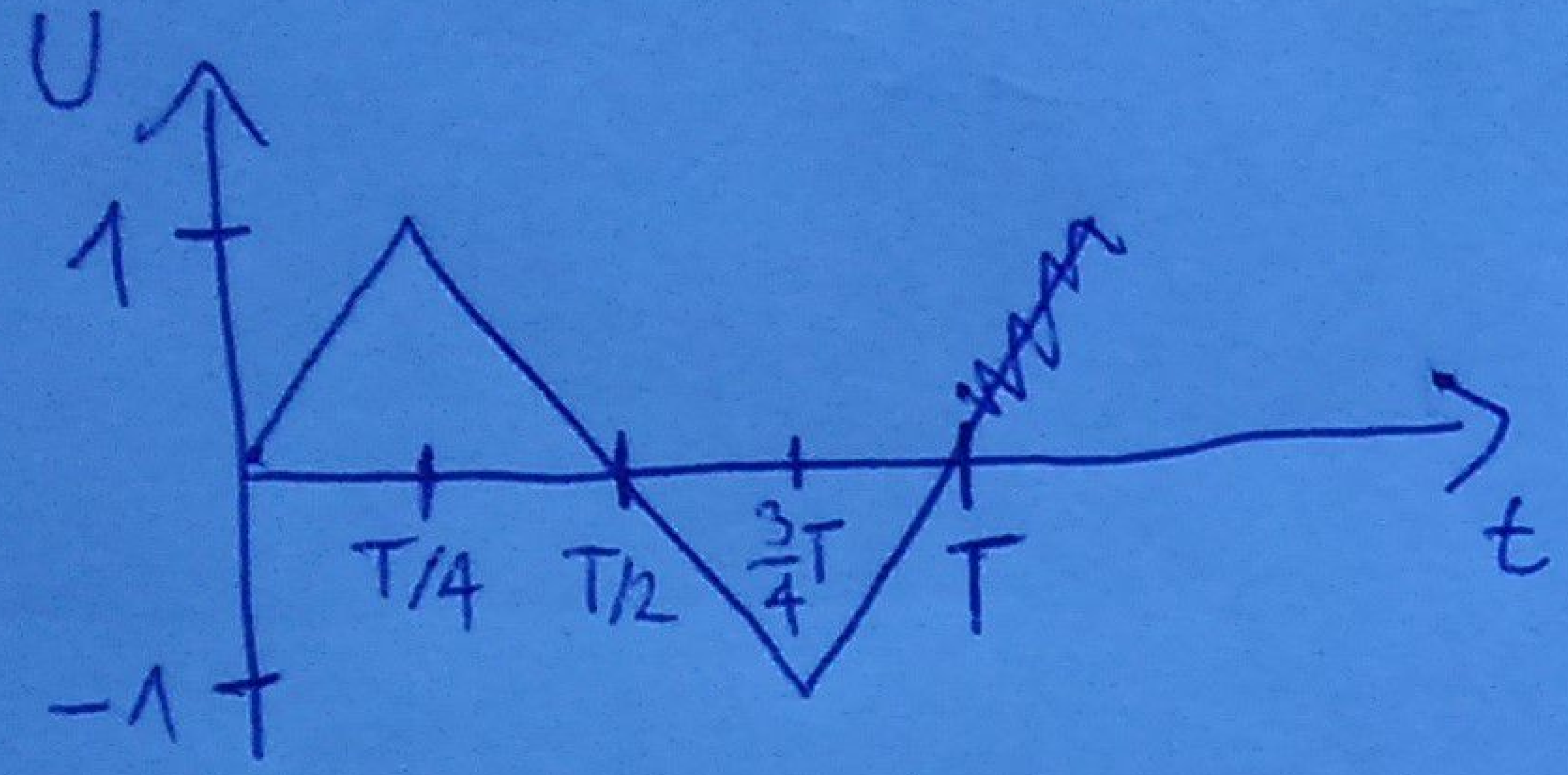
| K | $U_A^k$ | $U_B^k$ | $U_{B(dB)}^k = 20\cdot\lg(\lvert U_B^k\rvert) - 3\text{ dB}$ |
|---|---|---|---|
| 1 | Ø | $\frac{2}{\pi}$ | −6,92 dB |
| 2 | Ø | Ø | — |
| 3 | Ø | $\frac{2}{3\pi}$ | −16,46 dB |
| 4 | Ø | Ø | — |
| 5 | Ø | $\frac{2}{5\pi}$ | −20,90 dB |
| 6 | Ø | Ø | — |
| 7 | Ø | $\frac{2}{7\pi}$ | −23,82 dB |
| 8 | Ø | Ø | — |
| 9 | Ø | $\frac{2}{9\pi}$ | −26,00 dB |
| 10 | Ø | Ø | |

Szimmetrikus háromszögjel




$$\boxed{U(t)=\begin{cases}\frac{4}{T}\cdot t, & \text{ha } 0\leq t<\frac{T}{4}\\ 2-\frac{4}{T}\cdot t, & \text{ha } \frac{T}{4}\leq t<\frac{3}{4}\cdot T\\ -4+\frac{4}{T}\cdot t, & \text{ha } \frac{3}{4}T\leq t\leq T\end{cases}}$$

$$\boxed{U_0=}\frac{1}{T}\cdot\left(\int_0^{T/4}\left(\frac{4}{T}\cdot t\right)dt+\int_{T/4}^{\frac{3}{4}T}\left(2-\frac{4}{T}\cdot t\right)dt+\int_{\frac{3}{4}T}^{T}\left(-4+\frac{4}{T}\cdot t\right)dt\right)=\frac{1}{T}\cdot\left(\left[\frac{2}{T}\cdot t^2\right]_0^{T/4}+\left[2t-\frac{2}{T}\cdot t^2\right]_{T/4}^{3/4T}+\left[-4t+\frac{2}{T}\cdot t^2\right]_{3/4T}^{T}\right.$$

$$=\frac{1}{T}\cdot\left(\underbrace{\frac{T}{8}-0+\frac{6}{4}T-\frac{18}{16}T-\frac{T}{2}+\frac{T}{8}-4T+2T+3T-\frac{18}{16}T}\right)\boxed{=0}$$

$$\boxed{U_A^K=}\ \frac{2}{T}\cdot\left(\int_0^{T/4}\left(\frac{4}{T}\cdot t\right)\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)dt+\int_{T/4}^{\frac{3}{4}T}\left(2-\frac{4}{T}\cdot t\right)\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)dt+\int_{\frac{3}{4}T}^{T}\left(-4+\frac{4}{T}\cdot t\right)\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right)dt\right)\boxed{=0}$$

$$\boxed{U_B^k} = \frac{2}{T}\cdot\Bigg(\underbrace{\int_{0}^{T/4}\overset{f}{\left(\tfrac{4}{T}\cdot t\right)}\cdot\overset{g'}{\sin\left(\tfrac{2k\pi}{T}\cdot t\right)}dt}_{a} + \underbrace{\int_{T/4}^{3/4T}\overset{f}{\left(2-\tfrac{4}{T}\cdot t\right)}\overset{g'}{\sin\left(\tfrac{2k\pi}{T}\cdot t\right)}dt}_{b} + \underbrace{\int_{3/4T}^{T}\overset{f}{\left(-4+\tfrac{4}{T}\cdot t\right)}\overset{g'}{\sin\left(\tfrac{2k\pi}{T}\cdot t\right)}dt}_{c}\Bigg)$$

Felhasználom az azonosságot: $\boxed{\int f\cdot g' = f\cdot g - \int f'\cdot g}$

$$a = \left[\left(\frac{4}{T}\cdot t\right)\cdot\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\cdot t\right) - \int\frac{4}{T}\cdot\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\right)dt\right]_{0}^{T/4}$$

$$= \left[\frac{-2}{k\pi}\cdot t\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right) + \frac{2}{k\pi}\cdot\frac{T}{2k\pi}\cdot\sin\left(\frac{2k\pi}{T}\cdot t\right)\right]_{0}^{T/4} =$$

$$= \frac{2}{k\pi}\cdot\left(\frac{T}{2k\pi}\cdot\sin\left(\frac{k}{2}\pi\right) - \frac{T}{4}\cdot\cos\left(\frac{k}{2}\pi\right)\right) \boxed{= \frac{T}{(k\pi)^2}\cdot\sin\left(\frac{k}{2}\pi\right) - \frac{T}{2k\pi}\cdot\left(\cos\left(\frac{k}{2}\pi\right)\right)}$$

$$b = \left[\left(2-\frac{4}{T}\cdot t\right)\cdot\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\cdot t\right) - \int -\frac{4}{T}\cdot\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\cdot t\right)dt\right]_{\frac{1}{4}T}^{3/4T}$$

$$= \left[\frac{\left(\frac{4}{T}\cdot t-2\right)\cdot T}{2k\pi}\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right) - \frac{1}{2k\pi}\cdot\frac{T}{2k\pi}\cdot\sin\left(\frac{2k\pi}{T}\cdot t\right)\cancel{dt}\right]_{1/4T}^{3/4T}$$

$$= \frac{T}{2k\pi}\cdot\cos\left(\frac{3}{2}k\pi\right) - \frac{T}{(2k\pi)^2}\cdot\sin\left(\frac{3}{2}k\pi\right) + \frac{T}{2k\pi}\cdot\left(\cos\left(\frac{k}{2}\pi\right)\right) + \frac{T}{(2k\pi)^2}\cdot\sin\left(\frac{1}{2}k\pi\right)$$

$$\boxed{= \frac{T}{2k\pi}\cdot\left(\cos\left(\frac{3}{2}k\pi\right) + \cos\left(\frac{1}{2}k\pi\right)\right) + \frac{T}{(2k\pi)^2}\cdot\left(\sin\left(\frac{1}{2}k\pi\right) - \sin\left(\frac{3}{2}k\pi\right)\right)}$$

$$\textcircled{c} = \left[\left(-4+\frac{4}{T}\cdot t\right)\cdot\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\cdot t\right) - \int \frac{4}{T}\cdot\frac{T}{2k\pi}\cdot -\cos\left(\frac{2k\pi}{T}\cdot t\right)dt\right]_{3/4T}^{T}$$

$$= \left[\frac{4T-4t}{2k\pi}\cdot\cos\left(\frac{2k\pi}{T}\cdot t\right) + \frac{2}{k\pi}\cdot\frac{T}{2k\pi}\cdot\sin\left(\frac{2k\pi}{T}\cdot t\right)\right]_{3/4T}^{T}$$

$$\boxed{= \frac{-T}{2k\pi}\cdot\cos\left(\frac{3}{2}k\pi\right) - \frac{T}{(k\pi)^2}\cdot\sin\left(\frac{3}{2}k\pi\right)}$$

$$\boxed{U_B^k =} \frac{2}{T}\cdot\left(a + b + c\right) = \frac{2}{T}\cdot\left(\frac{T}{(k\pi)^2}\cdot\sin\left(\frac{k}{2}\pi\right) - \frac{T}{2k\pi}\cdot\cos\left(\frac{k}{2}\pi\right) + \frac{T}{2k\pi}\cdot\left(\cos\left(\frac{3}{2}k\pi\right)+\cos\left(\frac{1}{2}k\pi\right)\right)\right.$$

$$\left. + \frac{T}{(k\pi 2)^2}\cdot\left(\sin\left(\frac{1}{2}k\pi\right) - \sin\left(\frac{3}{2}k\pi\right)\right) - \frac{T}{2k\pi}\cdot\cos\left(\frac{3}{2}k\pi\right) - \frac{T}{(k\pi)^2}\cdot\sin\left(\frac{3}{2}k\pi\right)\right)$$

$$= 2\cdot\left[\left(\frac{1}{(k\pi)^2}+\frac{1}{(2k\pi)^2}\right)\cdot\sin\left(\frac{1}{2}k\pi\right) - \left(\frac{1}{(k\pi)^2}+\frac{1}{(2k\pi)^2}\right)\cdot\sin\left(\frac{3}{2}k\pi\right)\right]$$

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$$\boxed{= 2\cdot\left(\frac{1}{(k\pi)^2}+\frac{1}{(2k\pi)^2}\right)\cdot\left(\sin\left(\frac{1}{2}k\pi\right)-\sin\left(\frac{3}{2}k\pi\right)\right)}$$

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| $k$ | $U_A^k$ | $U_B^k$ | $U_{B(dB)}^k = 20\cdot\lg(\lvert U_B^k\rvert) - 3\,dB$ |
|---|---|---|---|
| 1 | Ø | $2\cdot\left(\frac{1}{\pi^2}+\frac{1}{4\pi^2}\right)\cdot 2 = \frac{5}{\pi^2}$ | $-8{,}91\,dB$ |
| 2 | Ø | Ø | — |
| 3 | Ø | $2\cdot\left(\frac{1}{9\pi^2}+\frac{1}{36\pi^2}\right)\cdot -2 = \frac{-5}{9\pi^2}$ | $-27{,}99\,dB$ |
| 4 | Ø | Ø | — |
| 5 | Ø | $2\cdot\left(\frac{1}{25\pi^2}+\frac{1}{100\pi^2}\right)\cdot 2 = \frac{1}{5\pi^2} = \frac{5}{25\pi^2}$ | $-36{,}87\,dB$ |
| 6 | Ø | Ø | — |
| 7 | Ø | $2\cdot\left(\frac{1}{49\pi^2}+\frac{1}{196\pi^2}\right)\cdot -2 = \frac{-5}{49\pi^2}$ | $-42{,}71\,dB$ |
| 8 | Ø | Ø | — |
| 9 | Ø | $2\cdot\left(\frac{1}{81\pi^2}+\frac{1}{324\pi^2}\right)\cdot 2 = \frac{5}{81\pi^2}$ | $-47{,}08\,dB$ |
| 10 | Ø | Ø | — |