INFO ANZ. ZH1. MINTAMEGOLDÁS —1— 2018.03.22.

α VARIÁNS

1, [15] $y' \cdot e^{5y} = \frac{\sqrt[3]{2+3x}}{y} \Longrightarrow \underbrace{\int y \cdot e^{5y}\, dy}_{I_1} = \underbrace{\int (2+3x)^{1/3}\, dx}_{I_2}$ (5)

$I_1 = \int y e^{5y}\, dy = y \cdot \frac{e^{5y}}{5} - \int \frac{e^{5y}}{5}\, dy = \frac{1}{5} y e^{5y} - \frac{1}{25} e^{5y} + C$ (5)

$I_2 = \int (2+3x)^{1/3}\, dx = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{4} \cdot (2+3x)^{4/3} + C$ (5)

$\left(\frac{y}{5} - \frac{1}{25}\right) e^{5y} = \frac{1}{4}(2+3x)^{4/3} + C$

2, Inhomogén lin. $y' + \frac{2}{x} y = 5x^2$; $y(1) = 3$

[20]

(8) (H): $y' = -\frac{2}{x} y \Rightarrow \int \frac{dy}{y} = -2 \int \frac{dx}{x} \Rightarrow \ln|y| = -2 \ln|x| + C$

$y_{H,\text{ált}}(x) = K \cdot x^{-2}$ $(K \in \mathbb{R})$

(8) Variálás: $y_{I,p}(x) = K(x) \cdot x^{-2}$; $y'_{I,p}(x) = K'(x) \cdot x^{-2} - 2K(x)\, x^{-3}$

Beírva: $\frac{K'(x)}{x^2} - \cancel{\frac{2K(x)}{x^3}} + \cancel{\frac{2}{x} \cdot \frac{K(x)}{x^2}} = 5x^2 \Rightarrow K'(x) = 5x^4$

$K(x) = \int 5x^4\, dx = x^5$; $y_{I,p}(x) = x^3$; $y_{I,\text{ált}}(x) = y_{I,p} + y_{H,á} = x^3 + K \cdot x^{-2}$

(4) Illesztés: $1^3 + K \cdot 1^{-2} = 3 \Rightarrow K = 2$;

$y(x) = x^3 + \frac{2}{x^2}$

3, $\boxed{15}$ $y'' - 4y' + 4y = 3e^{2x}$

7 (H): $\lambda^2 - 4\lambda + 4 = (\lambda-2)^2 = 0 \implies \lambda_{1,2} = +2$ Belső rez.

$y_{H,\text{ált}}(x) = C_1 e^{2x} + C_2 x e^{2x}$; $C_1, C_2 \in \mathbb{R}$

$\boxed{8}$ Külső rez. is van!

$y_{I,p}(x) = A x^2 e^{2x}$ 3 $/ \times 4$

$y'_{I,p}(x) = A e^{2x}(2x + 2x^2)$ $/ \times (-4)$

$+$ $y''_{I,p}(x) = A e^{2x}(2 + 4x + 4x + 4x^2) = A e^{2x}(2 + 8x + 4x^2)$

$3e^{2x} = A e^{2x}(\cancel{4x^2} - \cancel{8x} - \cancel{8x^2} + 2 + \cancel{8x} + \cancel{4x^2})$

$3 = 2A$, $A = \frac{3}{2}$ 3 ; $y_{I,\text{ált}}(x) = \frac{3}{2} x^2 e^{2x} + C_1 e^{2x} + C_2 x e^{2x}$ 2

4, a, $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$ 4

b, T.: Ha $\sum_{n=1}^{\infty} a_n$ konvergens, akkor $\lim_{n\to\infty} a_n = 0$ 4

B.: $a_n = S_n - S_{n-1} = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n-1} a_k \longrightarrow S - S = 0$ 6

5, a, $a_n = (2n+3)^{-3/2} = \frac{1}{(2n+3)^{3/2}} \leq \frac{1}{(2n)^{3/2}} = \left(\frac{1}{2}\right)^{3/2} \cdot \frac{1}{n^{3/2}}$,

7 és $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} < \infty$ $\left(\frac{3}{2} > 1\right)$, tehát a majoráns krit. értelmében a sor konvergens.

VAGY: Integrál-kritérium: feltételek teljesülnek ✓

7 $\int_1^{\infty} (2x+3)^{-3/2} dx = \lim_{\Omega\to\infty} \left[-2 \cdot \frac{1}{2} \cdot (2x+3)^{-1/2}\right]_1^{\Omega} = 5^{-1/2} < \infty \implies$ a sor konv.

5/b, $\underbrace{\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n-4}{n^2}}_{a_n}$ Igazoljuk, hogy a sor Leibniz.

7

$a_n \to 0$ ✓ alternál ✓

$$|a_{n+1}| = \frac{n+1-4}{(n+1)^2} \overset{?}{\leq} |a_n| = \frac{n-4}{n^2}$$

$$n^2(n-3) \overset{?}{\leq} (n-4)(n+1)^2 = (n-4)(n^2+2n+1)$$

$$\cancel{n^3} - 3n^2 \overset{?}{\leq} \cancel{n^3} - \cancel{2n^2} - 7n - 4$$

$$0 \leq n^2 - 7n - 4$$ ✓ igen, ha $n > N_0$

Tehát a sor Leibniz, így konvergens.

c, $\sum_{n=1}^{\infty} \underbrace{\frac{(n+3)!}{\sqrt{(2n)!}}}_{0 < a_n}$ Hányados krit.:

7

$$\frac{a_{n+1}}{a_n} = \frac{(n+4)!}{\sqrt{(2n+2)!}} \cdot \frac{\sqrt{(2n)!}}{(n+3)!} =$$

$$= (n+4)\sqrt{\frac{1}{(2n+1)(2n+2)}} = \frac{1+4/n}{\sqrt{(2+1/n)(2+2/n)}} \longrightarrow \frac{1}{2} < 1$$

$\Rightarrow$ konvergens a sor.

6,

a, igen
b, nem
c, nem
d, nem
e, nem

$(\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos(2x))$

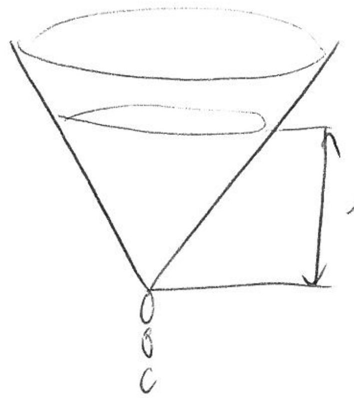
$$q^2 - 2q - 3 = (q-3)(q+1) \; ; \; q_1 = 3 \; ; \; q_2 = -1$$

$$a_{2k} = -\frac{1}{2k} \; ; \; a_{2k+1} = +\frac{1}{(2k+1)^2}$$

$$\frac{1}{1^2} - \frac{1}{2} + \frac{1}{3^2} - \frac{1}{4} + \frac{1}{5^2} - \frac{1}{6} + \frac{1}{7^2} - \frac{1}{8} \dots$$

$$(a_k) = \left(\frac{1}{1^2} - \frac{2}{2^2} + \frac{1}{3^2} - \frac{2}{4^2} + \frac{1}{5^2} - \frac{2}{6^2} + \frac{1}{7^2} - \frac{2}{8^2}\right)$$

IMSC:



a, $V(t) \sim h^3(t)$

$\dot{V}(t) \sim 3h^2(t)\,\dot{h}(t) \sim h^{1/2}(t)$

Tehát $3h^2(t)\cdot\dot{h}(t) = -K\cdot h^{+1/2}(t)$

4 $\dot{h}(t) = -\frac{K}{3}\cdot h^{-3/2}(t)$ Szeparálható

$$\int h^{3/2}\,dh = -\frac{K}{3}\int dt \Rightarrow \frac{2}{5}h^{5/2} = -\frac{K}{3}t + C$$

Így $h(t) = (A - Bt)^{2/5}$, $h(0) = H$ $\Rightarrow$ $h(t) = \left(H^{5/2} - Bt\right)^{2/5}$ 4

b, $\frac{H}{2} = \left(H^{5/2} - BT_1\right)^{2/5} \Rightarrow T_1 = \frac{H^{5/2} - \left(\frac{H}{2}\right)^{5/2}}{B}$

$0 = \left(H^{5/2} - BT_2\right)^{2/5} \Rightarrow T_2 = \frac{H^{5/2}}{B}$

$\frac{T_2}{T_1} = \frac{1}{1 - (1/2)^{5/2}}$ 7

β VARIÁNS (tömör)

1, [15] Szeparálható: $\underbrace{\int y\, e^{4y}\, dy}_{} = \underbrace{\int (1+2x)^{1/5}\, dx}_{}$ (5)

$$\frac{y}{4} e^{4y} - \frac{1}{16} e^{4y} = \frac{5}{6}\cdot\frac{1}{2}\cdot(1+2x)^{6/5} + c$$

(5) (5)

2, [20] $y(1)=2$

(H): $y' = -\frac{2}{x} y \Rightarrow \ln|y| = -2\ln|x| + C \Rightarrow y_{H,ált}(x) = K\cdot x^{-2}$ (8)

$y_{I,p}(x) = K(x)\cdot x^{-2}$; Beírva: $\frac{K'(x)}{x^2} = 4x^3 \Rightarrow K(x) = \int 4x^5\,dx = \frac{2}{3}x^6$

$y_{I,ált}(x) = \frac{2}{3}x^4 + K\cdot x^{-2}$ (8)

Illesztés: $\frac{2}{3} + \underset{4/3}{\underset{\shortparallel}{K}} = 2 \Rightarrow y(x) = \frac{2}{3}x^4 + \frac{4}{3x^2}$ (4)

3, [15] $y'' + 6y' + 9y = 5e^{-3x}$; $\lambda^2 + 6\lambda + 9 = (\lambda+3)^2 = 0$

$y_{H,ált}(x) = C_1 e^{-3x} + C_2 x e^{-3x}$ (7) Belső és külső rezonancia!

$y_{I,p}(x) = Ax^2 e^{-3x}$ (3) Beírva:

$$Ae^{-3x}(2 - \cancel{12x} + \cancel{9x^2} + 6(\cancel{2x} - \cancel{3x^2}) + \cancel{9x^2}) \overset{!}{=} 5\cdot e^{-3x}$$

$A = 5/2$ (3); $y_{I,ált}(x) = \frac{5}{2}x^2 e^{-3x} + C_1 e^{-3x} + C_2 x e^{-3x}$ (2)

4, – mint α

5, a, Majorálással, vagy integrál kritériummal.

(7) $a_n = \frac{1}{(3n+2)^{7/5}} \le \frac{1}{(3n)^{7/5}} = 3^{-7/5}\cdot\frac{1}{n^{7/5}}$, és $\sum_{n=1}^{\infty}\frac{1}{n^{7/5}}$ $\overset{\text{konvergens}}{< \infty}$, mert $\frac{7}{5} > 1$

vagy $\int_1^{\infty} \frac{1}{(3x+2)^{7/5}}\,dx = \lim_{\Omega\to\infty}\left[-\frac{5}{2}\cdot\frac{1}{3}(3x+2)^{-2/5}\right]_1^{\Omega} = \frac{5}{6}\cdot 5^{-2/5} < \infty$

5/b, Leibniz. $a_n \to 0$ ✓, alternál ✓

$$|a_{n+1}| = \frac{n+1-3}{(n+1)^2} \overset{?}{\leq} \frac{n-3}{n^2}$$

$$\Updownarrow$$

$$n^2(n-2) \overset{?}{\leq} (n-3)(n^2+2n+1)$$

$$\cancel{n^3} - 2n^2 \overset{?}{\leq} \cancel{n^3} - n^2 - 5n - 3$$

$$0 \overset{?}{\leq} n^2 - 5n - 3 \quad ✓, \text{ ha } n > N_0$$

$\Rightarrow$ konvergens.

c, Hányados-kritérium:

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+2)!}{\sqrt{(2n+2)!}} \cdot \frac{\sqrt{(2n)!}}{(n+1)!} = \frac{n+1}{\sqrt{(2n+1)(2n+2)}} \longrightarrow \frac{1}{2} < 1$$

konvergens!

$$q^2 + 4q - 5 = (q+5)(q-1)$$

— : —

6, a, nem

b, nem

c, nem

d, nem

e, igen

— : —

IMSC – mint A.