2008. 02. 12.

1.) Robotok orientációja

Vektortér: R^n

$\bar e_1, \bar e_2, \dots, \bar e_n$ bázis

$$\bar x = \sum_{j=1}^{n} x_j \cdot \bar e_j$$

$$\begin{array}{ccc} R^n & \xrightarrow{\mathcal{A}} & R^m \\ \bar e_1 & & \bar f_1 \\ \vdots & & \vdots \\ \bar e_n & & \bar f_n \\ R^n & \xrightarrow{\bar{\bar{A}}} & R^m \end{array}$$

Lineáris leképezés:

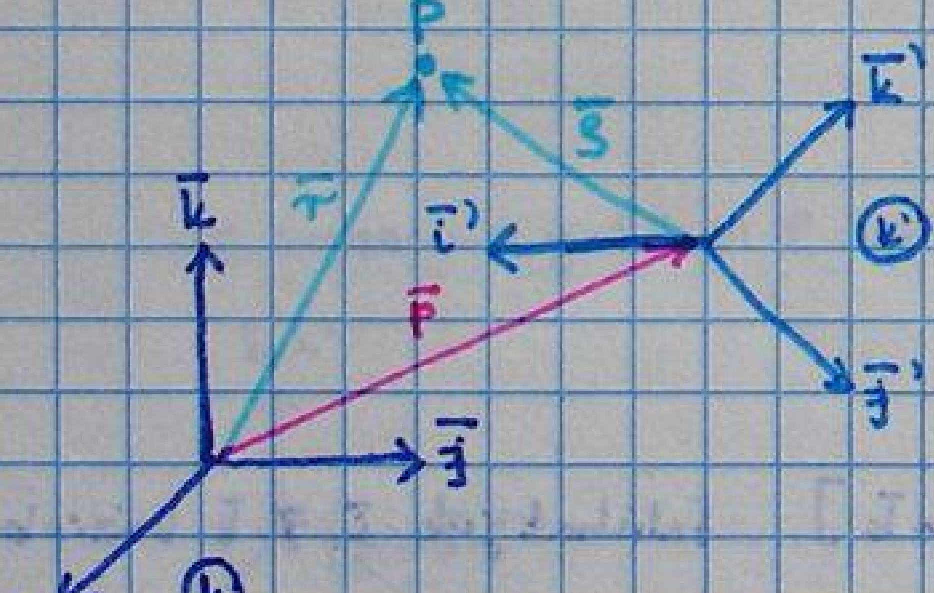
$$\mathcal{A}: R^n \longrightarrow R^n$$

$$\mathcal{A}(\alpha \bar x + \beta \bar y) = \alpha \mathcal{A}(\bar x) + \beta \mathcal{A}(\bar y)$$

$$R^n \rightarrow \bar x = \sum_{j=1}^{n} x_j \cdot \bar e_j$$

$$R^m \rightarrow \bar y = \mathcal{A}\bar x = \sum_{i=1}^{m} y_i \cdot \bar f_i = \sum_{j=1}^{n} x_j \cdot \mathcal{A}\bar e_j = \sum_{j=1}^{n} x_j \underbrace{\sum_{i=1}^{m} a_{ij} \bar f_j}_{\mathcal{A}\bar e_i}$$

$$\begin{pmatrix} y_1 \\ \vdots \\ y_m \end{pmatrix} = \underbrace{\left[\mathcal{A}\bar e_1 \ \dots \ \mathcal{A}\bar e_n \right]}_{\text{felírva } \bar f_1, \dots, \bar f_m \text{ bázisban}} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$



$$\bar r = \bar s + \bar p$$
$$r = r_x \bar i + r_y \bar j + r_z \bar k$$
$$p = p_x \bar i + p_y \bar j + p_z \bar k$$
$$s = s_x \bar i + s_y \bar j + s_z \bar k$$
$$\bar i' = a_{11} \bar i + a_{21} \bar j + a_{31} \bar k$$
$$\bar j' = a_{12} \bar i + a_{22} \bar j + a_{32} \bar k$$
$$\bar k' = a_{13} \bar i + a_{23} \bar j + a_{33} \bar k$$

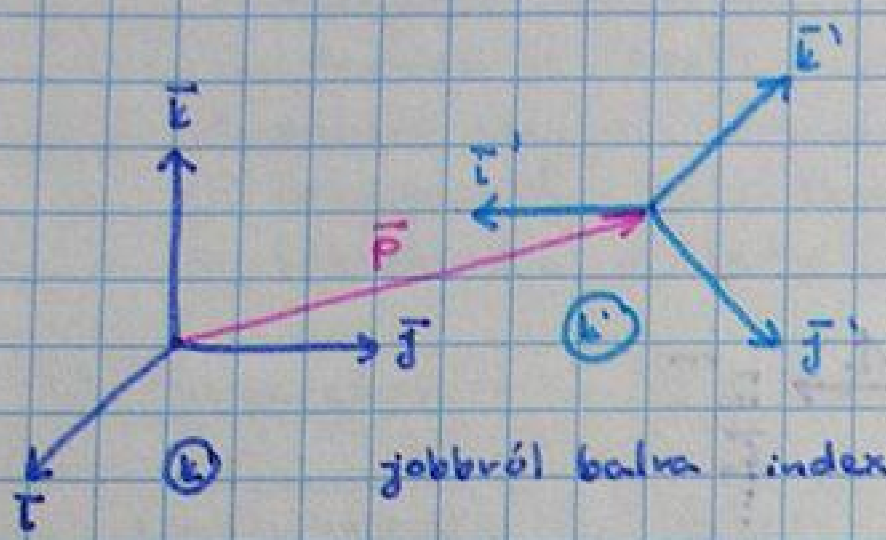
$$\begin{aligned} r_x \bar i + r_y \bar j + r_z \bar k = {} & s_x (a_{11} \bar i + a_{21} \bar j + a_{31} \bar k) + \\ & s_y (a_{12} \bar i + a_{22} \bar j + a_{32} \bar k) + \\ & s_z (a_{13} \bar i + a_{23} \bar j + a_{33} \bar k) + \\ & p_x \bar i + p_y \bar j + p_z \bar k \end{aligned}$$

$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \underbrace{\begin{array}{c} \begin{array}{ccc} \bar i & \bar j & \bar k \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \end{array}}_{\text{felírva } \bar i, \bar j, \bar k \text{ bázisban } \bar{\bar{A}}} \cdot \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} + \underbrace{\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix}}_{\bar p}$$

$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \bar{\bar{A}} \cdot \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} + \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} \Rightarrow \begin{array}{l} \bar{r} = \bar{\bar{A}} \cdot \bar{s} + \bar{p} \\ 1 = 1 \end{array}$$

homogén transzformáció lineáris

$$\begin{pmatrix} \bar{r} \\ 1 \end{pmatrix} = \underbrace{\left(\begin{array}{c|c} \bar{\bar{A}} & \bar{p} \\ \hline \bar{0}^T & 1 \end{array}\right)}_{\bar{\bar{T}}_{k,k'}} \cdot \begin{pmatrix} \bar{s} \\ 1 \end{pmatrix}$$



jobbról balra indexel

$$\bar{\bar{T}}_1 \cdot \bar{\bar{T}}_2 = \left[\begin{array}{c|c} \bar{\bar{A}}_1 & \bar{p}_1 \\ \hline \bar{0}^T & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} \bar{\bar{A}}_2 & \bar{p}_2 \\ \hline \bar{0}^T & 1 \end{array}\right] = \left[\begin{array}{c|c} \bar{\bar{A}}_1 \cdot \bar{\bar{A}}_2 & \bar{\bar{A}}_1 \cdot \bar{p}_2 + \bar{p}_1 \\ \hline \bar{0}^T & 1 \end{array}\right]$$

$$\bar{\bar{T}} \cdot \bar{\bar{T}}^{-1} = \underbrace{\bar{\bar{I}}_4}_{\text{egységmátrix}} = \left[\begin{array}{c|c} \bar{\bar{A}} & \bar{p} \\ \hline \bar{0}^T & 1 \end{array}\right] \cdot \left[\begin{array}{c|c} \bar{\bar{A}}^{-1} & -\bar{\bar{A}}^{-1} \cdot \bar{p} \\ \hline \bar{0}^T & 1 \end{array}\right] = \left[\begin{array}{c|c} \bar{\bar{I}}_3 & 0 \\ \hline \bar{0}^T & 1 \end{array}\right]$$

$$\bar{\bar{T}}^{-1} = \left[\begin{array}{c|c} \bar{\bar{A}}^{-1} & -\bar{\bar{A}}^{-1} \cdot \bar{p} \\ \hline \bar{0}^T & 1 \end{array}\right]$$

Vektorszorzás: $\bar{a}$ fix, $\mathcal{A}\bar{x} = \bar{a} \times \bar{x}$

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{\mathcal{A}} & \mathbb{R}^3 \\ \bar{i} & & \bar{i} \\ \bar{j} & & \bar{j} \\ \bar{k} & & \bar{k} \\ \mathbb{R}^3 & \xrightarrow{[\bar{a}\times]} & \mathbb{R}^3 \end{array}$$

$$\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$[\bar{a}\times] = [\bar{a} \times \bar{i} \quad \bar{a} \times \bar{j} \quad \bar{a} \times \bar{k}]$ felírhatjuk $\bar{i}, \bar{j}, \bar{k}$ bázisban

$$\bar{a} \times \bar{i} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ 1 & 0 & 0 \end{vmatrix} = \bar{i} \cdot \underbrace{\begin{vmatrix} a_y & a_z \\ 0 & 0 \end{vmatrix}}_{0} - \bar{j} \cdot \underbrace{\begin{vmatrix} a_x & a_z \\ 1 & 0 \end{vmatrix}}_{-a_z} + \bar{k} \cdot \underbrace{\begin{vmatrix} a_x & a_y \\ 1 & 0 \end{vmatrix}}_{-a_y} = 0 \cdot \bar{i} + a_z \cdot \bar{j} - a_y \bar{k}$$

$$[\bar{a}\times] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

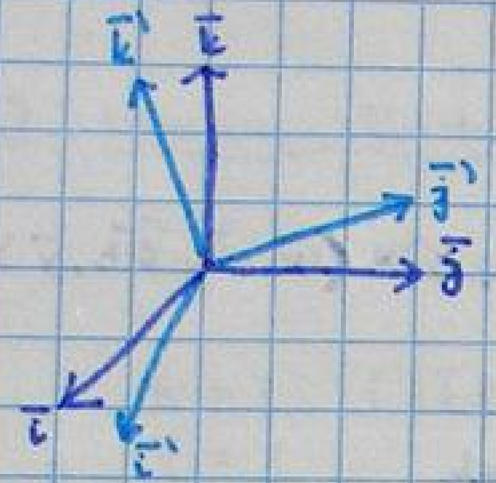
Diadikus szorzat: $\bar{a}, \bar{b}$ fix, $\mathcal{A}\bar{x} = \bar{a}\langle \bar{b}, \bar{x} \rangle$

$$\begin{array}{ccc} \mathbb{R}^3 & \xrightarrow{\mathcal{A}} & \mathbb{R}^3 \\ \bar{i} & & \bar{i} \\ \bar{j} & & \bar{j} \\ \bar{k} & & \bar{k} \\ \mathbb{R}^3 & \xrightarrow{[a \circ b]} & \mathbb{R}^3 \end{array}$$

$$[\bar{a} \circ \bar{b}] = [\bar{a}\overbrace{\langle \bar{b}, \bar{i} \rangle}^{b_x} \quad \bar{a}\overbrace{\langle \bar{b}, \bar{j} \rangle}^{b_y} \quad \bar{a}\overbrace{\langle \bar{b}, \bar{k} \rangle}^{b_z}] = \begin{bmatrix} a_x b_x & a_x b_y & a_x b_z \\ a_y b_x & a_y b_y & a_y b_z \\ a_z b_x & a_z b_y & a_z b_z \end{bmatrix}$$

2.) Orientáció jellemzői

a.) Direkt módszer:

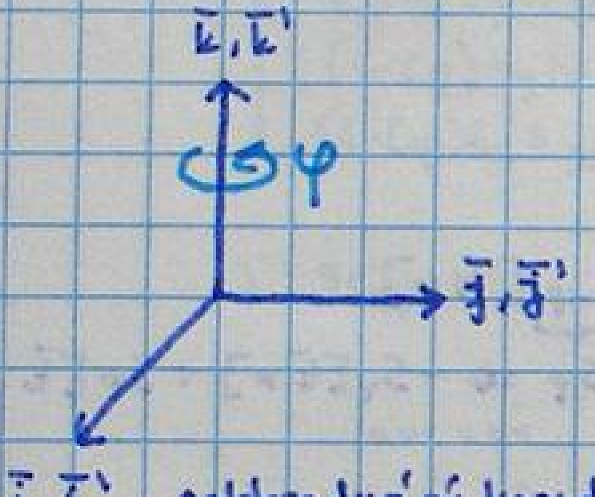


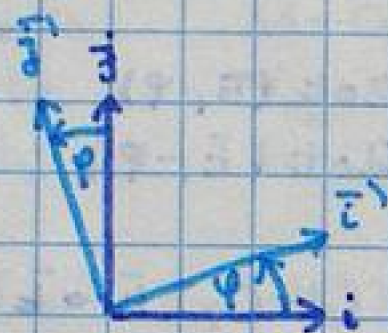
$$\overline{\overline{A}} = [\,\bar{i}' \quad \bar{j}' \quad \bar{k}'\,]$$

felírva az $\bar{i}, \bar{j}, \bar{k}$ bázisban

b.) Elemi forgatás:

- forgatás z körül φ szöggel





jobbsodrású koordináta rendszer

$$[\,\bar{i}_{elf} \quad \bar{j}_{elf} \quad \bar{k}_{elf}\,] = \begin{bmatrix} \cos\varphi & -\sin\varphi & 0 \\ \sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

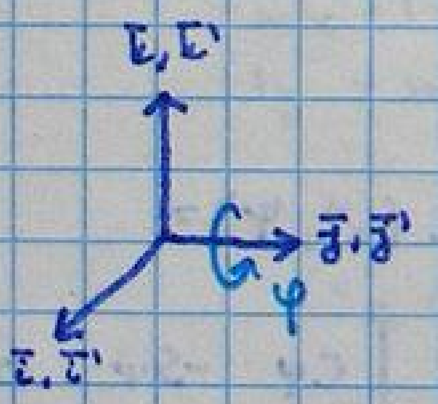
felírva $\bar{i}, \bar{j}, \bar{k}$ bázisban

$\cos\varphi = C\varphi, \quad \cos q_1 = C_1$

$\cos(\varphi + \psi) = C\varphi\psi, \quad \cos(q_1 + q_2) = C_{12}$

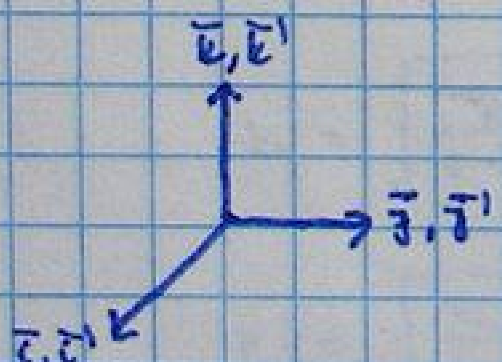
$$\overline{\overline{Rot}}(z, \varphi) = \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- forgatás y körül φ szöggel



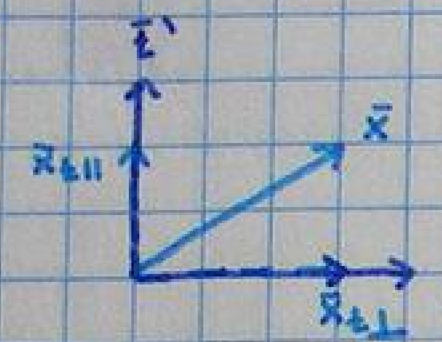
$$\overline{\overline{Rot}}(y, \varphi) = \begin{bmatrix} C\varphi & 0 & S\varphi \\ 0 & 1 & 0 \\ -S\varphi & 0 & C\varphi \end{bmatrix}$$

- forgatás x körül φ szöggel



$$\overline{\overline{Rot}}(x, \varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\varphi & -S\varphi \\ 0 & S\varphi & C\varphi \end{bmatrix}$$

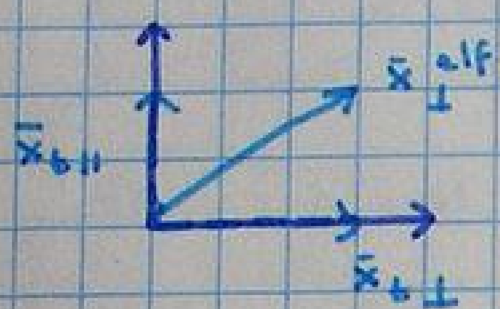
c.) Forgatás általános tengely körül $\quad \bar t' \; (|\bar t| = 1),\ \varphi$



$$\bar x \to \bar x_{t\parallel} = \underbrace{\bar t}_{\text{irány}} \underbrace{\langle \bar t, \bar x \rangle}_{\text{hossz}}$$

$$\bar x_t \cdot \bar x_{t\perp} = \bar x - \bar x_{t\parallel} = \bar x - \bar t \langle \bar t, \bar x \rangle$$

$$\bar x^{\,elf} = \bar x_{t\parallel} + \bar x_{t\perp}^{\,elf}$$



$$\bar x_{t\perp}^{\,elf} = C\varphi\, \bar x_{t\perp} + S\varphi\, \bar t \times \bar x_{t\perp} = C\varphi(\bar x - \bar t\langle \bar t, \bar x\rangle) + S\varphi\, \bar t' \times (\bar x - \bar t\langle \bar t, \bar x\rangle)$$

$$\bar x^{\,elf} = \bar t\langle \bar t, \bar x\rangle + C\varphi(\bar x - \bar t\underbrace{\langle \bar t, \bar x\rangle}_{\text{skalárja } \bar t\text{-nek és } \bar x\text{-nek}}) + S\varphi\, \bar t \times \bar x$$

$$\bar x^{\,elf} = C\varphi\, \bar x + (1 - C\varphi)\, \bar t\langle \bar t, \bar x\rangle + S\varphi\, \bar t \times \bar x$$

$$\mathbb{R}^3 \xrightarrow{Rot(\bar t, \varphi)} \mathbb{R}^3$$

$$\bar i, \bar j, \bar k \qquad\qquad \bar i, \bar j, \bar k$$

$$\mathbb{R}^3 \xrightarrow{\overline{\overline{Rot}}(\bar t, \varphi)} \mathbb{R}^3$$

$$\overline{\overline{Rot}}(\bar t, \varphi) = C\varphi\, \overline{\overline{I}} + (1 - C\varphi)\, \bar t \circ \bar t + S\varphi\, [\bar t \times] \quad \text{Rodriquez-képlet}$$

Forgatás: $Rot(\bar t, \varphi)$

Forgatás inverze: $Rot(\bar t, -\varphi)$

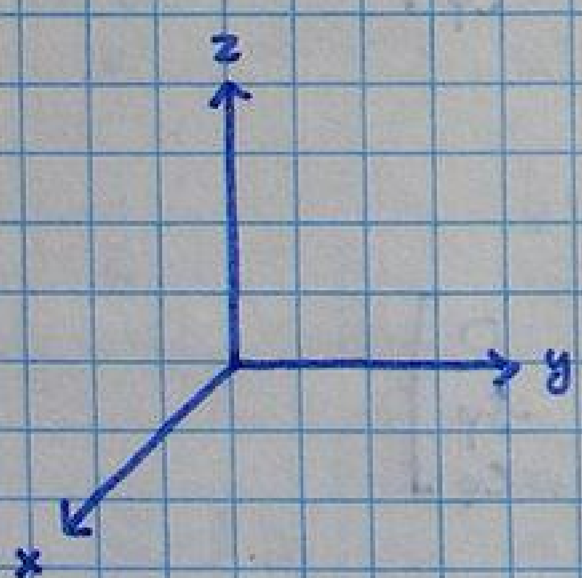
$$Rot(\bar t, -\varphi) = C\varphi\, \overline{\overline{I}} + (1 - C\varphi)\, \bar t \circ \bar t + \underbrace{S_{-\varphi}}_{-S\varphi}[\bar t \times]$$

$$-S\varphi \Rightarrow -S\varphi[\bar t \times] = S\varphi[\bar t \times]^T$$

$$\overline{\overline{Rot}}(\bar t, \varphi)^{-1} = \overline{\overline{Rot}}(\bar t, \varphi)^{-1}$$

ortonormált mátrix esetén: $\overline{\overline{A}}^{-1} = \overline{\overline{A}}^{T} \Rightarrow \begin{bmatrix} \overline{\overline{A}} & \bar p \\ \bar 0^T & 1 \end{bmatrix}^{-1} = \begin{bmatrix} \overline{\overline{A}}^{T} & -\overline{\overline{A}}^{T} \bar p \\ \bar 0^T & 1 \end{bmatrix}$

d.) Euler-szögek



$$\overline{\overline{Euler}}(\varphi, \vartheta, \psi) = \overline{\overline{Rot}}(z, \varphi)\, \overline{\overline{Rot}}(y', \vartheta)\, \overline{\overline{Rot}}(z'', \psi) =$$

$$= \begin{bmatrix} -C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\vartheta & 0 & S\vartheta \\ 0 & 1 & 0 \\ -S\vartheta & 0 & C\vartheta \end{bmatrix} \cdot \begin{bmatrix} C\psi & -S\psi & 0 \\ S\psi & C\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$= [$

e.) RPY szögek:

R: roll

P: pitch

Y: yaw

$$RPY(\varphi, \vartheta, \psi) = \overline{\overline{Rot}}(z, \varphi)\, \overline{\overline{Rot}}(y', \vartheta)\, \overline{\overline{Rot}}(x'', \psi) =$$

$$= \begin{bmatrix} C\varphi & -S\varphi & 0 \\ S\varphi & C\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} C\vartheta & 0 & S\vartheta \\ 0 & 1 & 0 \\ -S\vartheta & 0 & C\vartheta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi & -S\psi \\ 0 & S\psi & C\psi \end{bmatrix} =$$

$= [$

f.) Kvaternió: $\bar q = \left(\sin\frac{\varphi}{2} \cdot \bar t,\ \cos\frac{\varphi}{2} \cdot 1\right)$

g.) $e^{[\bar t, x]\varphi} \Leftrightarrow$ Rodriquez-képlet