MINTAMEGOLDÁS  t1-l Int. an. 2.  VD4. 2014. 06. 19.

---

1, $\boxed{12}$ $y\cdot y' = \dfrac{e^{-y^2}}{\sqrt{4+x^2}}$ , $y(0)=1$

$\int y\, e^{y^2}\, dy = \int \dfrac{dx}{\sqrt{4+x^2}}$ 2 ; $\frac{1}{2}\int 2y\, e^{y^2}\, dy = \frac{1}{2} e^{y^2} + C$ 3

$f' e^f$ alak

$\frac{1}{2}\int \dfrac{dx}{\sqrt{1+\left(\frac{x}{2}\right)^2}} = \frac{1}{\cancel{2}}\, \mathrm{ar\,sh}\left(\frac{x}{2}\right)\cdot \cancel{2} + c = \mathrm{ar\,sh}\left(\frac{x}{2}\right) + c$ 3

$\frac{1}{2} e^{y^2} = \mathrm{ar\,sh}\left(\frac{x}{2}\right) + C \Rightarrow y_{ált}(x) = \sqrt{\ln\left(2\,\mathrm{ar\,sh}\left(\frac{x}{2}\right) + \tilde{C}\right)}$ ; $\tilde{C}\in\mathbb{R}$ 2

$1 = \sqrt{\ln(0+\tilde{C})} \Rightarrow \tilde{C} = e \Rightarrow y_{kezd}(x) = \sqrt{\ln\left(2\,\mathrm{ar\,sh}\left(\frac{x}{2}\right) + e\right)}$ 2

---

2, a, $\boxed{4}$ Belső rezonancia: A karakterisztikus egyenletnek többszörös gyöke van. 2

Külső rezonancia: A zavaró függvény egy tagja a polinom szorzó nélkül megoldása a homogén egyenletnek. 2

b, $\boxed{6}$ $y'' + 6y' + Cy = 5e^{-2x} \Rightarrow \lambda^2 + 6\lambda + C = 0$

Belső rez.: $\lambda^2 + 6\lambda + 9 = (\lambda+3)^2 \Rightarrow C = 9$ 3

Külső rez.: $\lambda^2 + 6\lambda + C = (\lambda+2)(\lambda+4) = \lambda^2 + 6\lambda + 8$

$\lambda_1 = 2$ $\Rightarrow C = 8$ 3

c, $\boxed{10}$ $C = 5 \Rightarrow \lambda^2 + 6\lambda + 5 = (\lambda+5)(\lambda+1) = 0$ 2

$\Rightarrow y_{H,ált}(x) = C_1 e^{-5x} + C_2 e^{-x}$ 3

$\Rightarrow y_{I,p}(x) = A\, e^{-2x}$ 2 ; Beírva: $A(-2)^2 e^{-2x} + (-2)6A\, e^{-2x} + 5A e^{-2x} = 5e^{-2x}$ 1

$4A - 12A + 5A = 5$ ; $A = -\frac{5}{3}$ 2 ; $y_{I,á}(x) = C_1 e^{-5x} + C_2 e^{-x} - \frac{5}{3} e^{-2x}$

3, a, Legyen $a_n > 0$ $\forall n \in \mathbb{N}$ esetén.

[3] i, Ha $\forall n \in \mathbb{N}$ esetén $\frac{a_{n+1}}{a_n} \geq 1$, akkor $\sum_{n=0}^{\infty} a_n = \infty$ (div.)

ii, Ha $\exists C: \forall n \in \mathbb{N}$ esetén $\frac{a_{n+1}}{a_n} \leq C < 1$, akkor $\sum_{n=0}^{\infty} a_n < \infty$ (konv.)

(Mindkét helyen $\forall n \in \mathbb{N}$ helyett $\forall n > N_0$ is írható.)

b, $\sum_{n=1}^{\infty} \frac{5^{2n+1}}{(2n)!}$ ; $a_n = \frac{5^{2n+1}}{(2n)!} = \frac{5 \cdot 25^n}{(2n)!}$

[6]

$$\frac{a_{n+1}}{a_n} = \frac{5 \cdot 25^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{5 \cdot 25^n} = \frac{25}{(2n+1)(2n+2)} \xrightarrow{n\to\infty} 0 < 1$$

Tehát a sor konvergens!

— ∘ —

4, a, $x_0 = -2$

[4] $f(x) = e^{3x} = e^{3(x+2)-6} = \underline{e^{-6} \sum_{n=0}^{\infty} \frac{3^n}{n!} (x+2)^n}$ ③ ; $\underline{x \in \mathbb{R}}$ ①

b, $x_0 = 2$

[5] $g(x) = \frac{1}{1+x} = \frac{1}{3+x-2} = \frac{1}{3} \cdot \frac{1}{1-\left(-\frac{1}{3}(x-2)\right)} = \underline{\frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n (x-2)^n}$ ③

ha $\left|-\frac{1}{3}(x-2)\right| < 1$, azaz $\underline{-1 < x < 5}$ ②

— ∘ —

[10] 5, $f(x,y)$ az $(x_0, y_0)$-ban __totálisan deriválható__, ha $\exists \underline{A} = \begin{bmatrix} A_x \\ A_y \end{bmatrix} \in \mathbb{R}^2$,

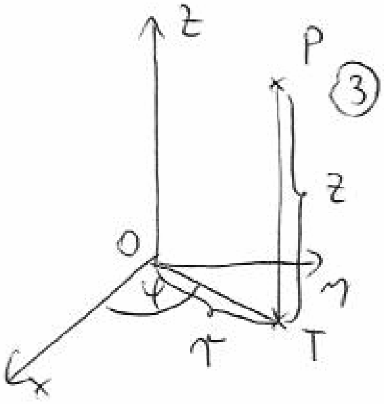
④ hogy $\Delta f = f(x,y) - f(x_0,y_0) = \underline{A} \cdot \Delta \underline{r} + \underline{\varepsilon}(\underline{r}) \cdot \Delta \underline{r}$,

ahol $\underline{r} = \begin{bmatrix} x \\ y \end{bmatrix}$, és $\lim_{\Delta \underline{r} \to (0,0)} \|\underline{\varepsilon}(\underline{r})\| = 0$ $\quad (\Delta \underline{r} = \underline{r} - \underline{r}_0)$

③ $f'_x(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0,y_0)}{h}$

③ Ha $f$ tot. diff.-ható $\underline{r}_0$-ban, akkor ott a parciális deriváltjai léteznek, és $\underline{A} = \operatorname{grad} f(\underline{r}_0) = \begin{bmatrix} f'_x(\underline{r}_0) \\ f'_y(\underline{r}_0) \end{bmatrix}$

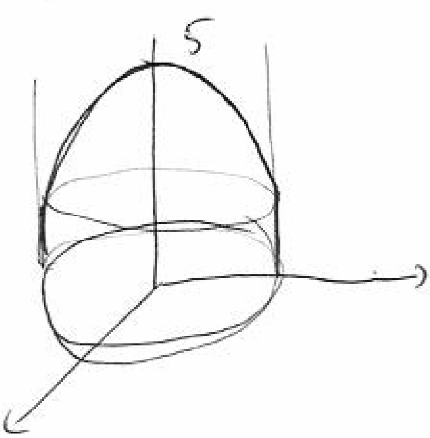
6, a, $\boxed{9}$




③ $\left.\begin{array}{l} x = r\cos\varphi \\ y = r\sin\varphi \\ z = z \end{array}\right\}$ ② $J(r,\varphi,z) = \begin{vmatrix} x'_r & x'_\varphi & x'_z \\ y'_r & y'_\varphi & y'_z \\ z'_r & z'_\varphi & z'_z \end{vmatrix} =$ ③

$$= \begin{vmatrix} \cos\varphi & -r\sin\varphi & 0 \\ \sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = r(\cos^2\varphi + \sin^2\varphi) = \underline{\underline{r}}$$ ①

b, $\boxed{9}$ $\left.\begin{array}{l} V: x^2+y^2 \le 4 \\ 0 \le z \le 5-x^2-y^2 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} 0 \le r \le 2 \\ 0 \le \varphi \le 2\pi \\ 0 \le z \le 5-r^2 \end{array}\right\}$ ② $I = \int_{r=0}^{2}\int_{\varphi=0}^{2\pi}\int_{z=0}^{5-r^2} r^2 \cdot r \, dz\, d\varphi\, dr =$ ②




$$= 2\pi \int_{r=0}^{2} r^3 (5-r^2)\, dr = 2\pi \left[ \frac{5r^4}{4} - \frac{r^6}{6} \right]_0^2 =$$ ① ②

$$= \underline{\underline{2\pi\left(20 - \frac{32}{3}\right)}}$$ ②

— o —

7, $\boxed{12}$ $f(x,y) = x^3 + y^2 - 3x$

$f'_x(x,y) = 3x^2 - 3 = 3(x+1)(x-1) = 0 \Rightarrow x = \pm 1$

$f'_y(x,y) = 2y = 0 \Rightarrow y = 0$

$\left.\begin{array}{l} (x_1, y_1) = (+1, 0) \\ (x_2, y_2) = (-1, 0) \end{array}\right\}$ ⑤

$$H(x,y) = \begin{vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{vmatrix} = \begin{vmatrix} 6x & 0 \\ 0 & 2 \end{vmatrix} = 12x$$ ②

$(-1,0)$-ban: $H(-1,0) = -12 < 0 \Rightarrow$ nincs lok. széls. érték. ②

$(+1,0)$-ban: $H(+1,0) = +12 > 0 \Rightarrow$ van lok. szél. ért.

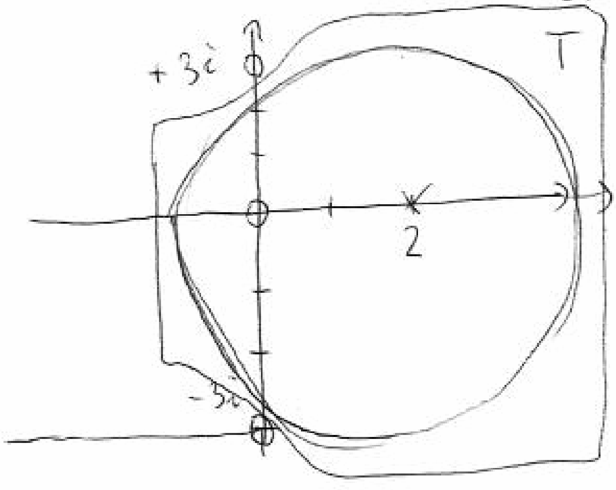
$f''_{xx} = +6 > 0 \Rightarrow$ lokális minimum van. ③

8, $\boxed{10}$

$$\oint_{|z-2|=3} \frac{\ln(z+3i)}{z(z-3i)}\,dz = \oint_{|z-2|=3} \frac{\left(\frac{\ln(z+3i)}{z-3i}\right)}{z}\,dz = ?$$

reg. T-n

Csak a z = 0 pólus esik a körön belül!




③

$$\hookrightarrow 2\pi i \, \frac{\ln(0+3i)}{0-3i} = \frac{-2\pi}{3}\ln(3i) =$$ ④

$$= -\frac{2\pi}{3}\left(\ln 3 + \frac{\pi}{2} i\right) = -\frac{2\pi}{3}\ln 3 - \frac{\pi^2}{3} i$$ ③