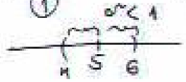
1) a, [14] [3]
$\forall \varepsilon$-hoz $\exists\, \delta(\varepsilon)$, hogy $|f(x) - \ell| < \varepsilon$, ha $0 < |x - x_0| < \delta(\varepsilon)$ } (2) $\Rightarrow \lim_{x \to x_0} f(x) = \ell$ (1)

b, [7] $\lim_{x \to 5} \frac{1}{x+2} = \frac{1}{7}$

$$\left|\frac{1}{x+2} - \frac{1}{7}\right| = \left|\frac{7-(x+2)}{7(x+2)}\right| = \left|\frac{7-x-2}{7x+14}\right| = \left|\frac{5-x}{7x+14}\right| = \left|\frac{x-5}{7x+14}\right| < \frac{|x-5|}{42} < \varepsilon$$

(1) (1) (1) (1) (2)

$\delta < 1$



$|x-5| < 42\varepsilon$ (1)

$\delta(\varepsilon) = \min(1, 42\varepsilon)$

2) $\left[-\frac{1}{2}, \frac{5}{2}\right]$ [15]

$$\frac{[x]\{x\}(x-\frac{3}{2})}{x^2|x-\frac{3}{2}|}$$

$x = \frac{3}{2}$ $\lim_{x \to \frac{3}{2}^+} (+1) \cdot \frac{1 \cdot \frac{1}{2}}{\frac{9}{4}} = \frac{1}{2} \cdot \frac{4}{9} = \frac{2}{9}$ (2)

$\lim_{x \to \frac{3}{2}^-} (-1) \frac{1 \cdot \frac{1}{2}}{\frac{9}{4}} = -\frac{2}{9}$ (2) } I. fajú véges ugrás (1)

$x = 0$ $\lim_{x \to 0^+} \frac{0 \cdot x(-\frac{3}{2})}{x^2 \cdot \frac{3}{2}} = 0$ (2)

$\lim_{x \to 0^-} \frac{(-1) \cdot 1 \cdot (-\frac{3}{2})}{x^2 \cdot \frac{3}{2}} = \infty$ (2) } II. fajú (1)

$x = 1$ $\lim_{x \to 1^+} \frac{1 \cdot 0 \cdot (-\frac{1}{2})}{1 \cdot \frac{1}{2}} = 0$ (2)

$\lim_{x \to 1^-} \frac{0 \cdot 1 \cdot (-\frac{1}{2})}{1 \cdot \frac{1}{2}} = 0$ (2) } I. fajú megszüntethető szakadás (1).

3) [13] a, $f(x) = (x+1)\sin\frac{1}{2x}$

$f'(x) = 1 \cdot \sin\frac{1}{2x} + (x+1) \cdot \cos\frac{1}{2x} \cdot \left(-\frac{1}{2}\right) \cdot \frac{1}{x^2}$ (5)

b, $g(x) = \frac{1}{\operatorname{arctg}\sqrt{x}} = (\operatorname{arctg}\sqrt{x})^{-1}$

$g'(x) = (-1)(\operatorname{arctg}\sqrt{x})^{-2} \cdot \frac{1}{1+x} \cdot \frac{1}{2} x^{-\frac{1}{2}}$ (5)

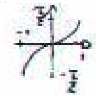
c, hol diffh.

f: $x \neq 0$ (1)

g: $(x \neq 0)$ $\boxed{x > 0}$ (2)

[20]

4) $f(x) = \pi + \arcsin(3x+5)$



a, [6] D_f $-1 \leq 3x+5 \leq 1$ (2)

$-6 \leq 3x \leq -4$

$-2 \leq x \leq -\frac{4}{3}$ (1) $[-2, -\frac{4}{3}]$

R_f: $-\frac{\pi}{2} \leq \arcsin(3x+5) \leq \frac{\pi}{2}$ (2)

$\frac{\pi}{2} \leq \pi + \arcsin(3x+5) \leq \frac{3\pi}{2}$ (1) $[\frac{\pi}{2}, \frac{3\pi}{2}]$

b, [6] $f'(x) = \frac{1}{\sqrt{1-(3x+5)^2}} \cdot 3$ (2)

érintőegyenes:

$x_0 = -\frac{9}{6}$ $f(x_0) = \pi + \arcsin(3x_0+5)$ $f'(-\frac{9}{6}) = \frac{1}{\sqrt{1-(\frac{1}{2})^2}} \cdot 3 = 2\sqrt{3}$ (1)

$f(-\frac{9}{6}) = \pi + \underbrace{\arcsin\left(\frac{1}{2}\right)}_{\frac{\pi}{6}} = \frac{7\pi}{6}$ (1)

$f(x) = f(x_0) + f'(x_0)(x - x_0)$ (1)

$f(x) = \frac{7\pi}{6} + 2\sqrt{3}\left(x + \frac{9}{6}\right)$ (1)

c, [8] $\exists$: arcsin szigorúan monoton fv. $+\pi$ } $\Rightarrow \exists\ f^{-1}$ } (2)

$f'(x) > 0 \overset{\text{vs}}{\Longrightarrow} \exists\ f^{-1}$

$y = \pi + \arcsin(3x+5)$

$x = \pi + \arcsin(3y+5)$ (1)

$x - \pi = \arcsin(3y+5)$

$\sin(x-\pi) = 3y+5$ (2)

$y = \frac{\sin(x-\pi) - 5}{3} = f^{-1}(x)$ (1)

$D_{f^{-1}} = R_f = [\frac{\pi}{2}, \frac{3\pi}{2}]$ (1) $R_{f^{-1}} = D_f = [-2, -\frac{4}{3}]$ (1)

(5) a, $\lim\limits_{x\to 5} \operatorname{arctg} \dfrac{x-6}{(x-5)^2} = (\operatorname{arctg}(-\infty)) = -\dfrac{\pi}{2}$ (1)

$\boxed{15}$ $\boxed{4}$ $x-6 \to -1$ (1); $(x-5)^2 \to 0^+$ (1); (1)

b, $\lim\limits_{x\to 0} x \operatorname{arctg}\dfrac{1}{x} = 0$ $\quad 0\cdot$ korlátos $= 0$ (2)

$\boxed{4}$ $x \to 0$

KORLÁTOS, mert $-\frac{\pi}{2} < \operatorname{arctg} u < \frac{\pi}{2}$ (2)

c, $\lim\limits_{x\to 2\pi} \dfrac{\sin 2x}{4-\frac{x^2}{\pi^2}} = \lim\limits_{u\to 0} \dfrac{\sin(2u+4\pi)}{4-\frac{(u+2\pi)^2}{\pi^2}} = \lim\limits_{u\to 0} \dfrac{(\sin 2u)\cdot \pi^2 \cdot 2u}{(-u^2-4u\pi)\cdot 2u}$

$\boxed{7}$ $x-2\pi = u$ $\quad\to -\frac{1}{2}\pi$ $\quad \to 1$

$= \lim\limits_{u\to 0} \dfrac{2u\pi^2}{-u^2-4u\pi} = \lim\limits_{u\to 0} \dfrac{\cancel{u}\cdot 2\pi^2}{\cancel{u}(-u-4\pi)} = \lim\limits_{u\to 0} \dfrac{2\pi^2}{-u-4\pi} = -\dfrac{1}{2}\pi$

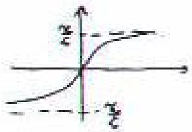
$\lim\limits_{x\to 2\pi} \dfrac{\sin 2x}{4-\frac{x^2}{\pi^2}} = -\dfrac{1}{2}\pi$

új változó bevezetése (2)

$\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$ felhasználása (2)

bővítés (2)

végeredmény (1)



(6) a, ha $\exists$ $\lim\limits_{h\to 0} \dfrac{f(x_0+h)-f(x_0)}{h} = \lim\limits_{x\to x_0} \dfrac{f(x)-f(x_0)}{x-x_0}$ (2)

$\boxed{12}$

jobboldali $\lim\limits_{x\to x_0^+} \dfrac{f(x)-f(x_0)}{x-x_0} = \lim\limits_{h\to 0^+} \dfrac{f(x_0+h)-f(x_0)}{h}$ (1)

baloldali $\lim\limits_{x\to x_0^-} \dfrac{f(x)-f(x_0)}{x-x_0} = \lim\limits_{h\to 0^-} \dfrac{f(x_0+h)-f(x_0)}{h}$ (1)

b, $f(x) = \sqrt{x \sin 2x}$

$\lim\limits_{x\to 0^+} \dfrac{\sqrt{x\sin 2x}-0}{x} = \lim\limits_{x\to 0^+} \sqrt{\dfrac{x \sin 2x \cdot 2}{x^2 \cdot 2}} = \sqrt{2}$ (4)

$\lim\limits_{x\to 0^-} \dfrac{\sqrt{x\sin 2x}-0}{-|x|} = \lim\limits_{x\to 0^-} -\sqrt{\dfrac{x \sin 2x \cdot 2}{x^2 \cdot 2}} = -\sqrt{2}$

$\sqrt{x^2} = |x|$
ha $x<0$, akkor
$|x| =$
$-|x| =$

... vagy páros fv. Ha a jobboldali derivált $\sqrt{2}$ akkor a baloldali $-\sqrt{2}$ (2)

$\lim\limits_{x\to 0^+} \neq \lim\limits_{x\to 0^-}$ $\Rightarrow \nexists \lim\limits_{x\to 0}$ $\Rightarrow$ nem diffh. $x_0 = 0$-ban

jobboldali $\neq$ baloldali

(7) $f(x) = x + \dfrac{4}{x}$ $\quad (x \neq 0)$

$\boxed{15}$ páratlan: $f(-x) = -f(x)$, $\forall x \in D_f$ (1)

$-x + \dfrac{4}{-x} \overset{?}{=} -\left(x + \dfrac{4}{x}\right)$

$-x - \dfrac{4}{x} \overset{?}{=} -x - \dfrac{4}{x}$ ✓ $\Rightarrow$ fv. páratlan (2)

$f'(x) = 1 - \dfrac{4}{x^2} = 0$ $\quad$ lehetséges szél. hely $1 = \dfrac{4}{x^2}$ $\quad x^2 = 4$ $\quad x_{1,2} = \pm 2$

$f''(x) = \dfrac{8}{x^3} = 0$ $\quad$ lehetséges inf. pont nincs

f': $1 - \dfrac{4}{x^2}$ (2) $\quad f' > 0 \Rightarrow$ szig. m. nő (2) $\quad 1 > \dfrac{4}{x^2}$ $\quad x^2 > 4$ $\quad (-\infty,-2)$,

$f' < 0 \Rightarrow$ szig. m. cs. $\quad x^2 < 4$ $\quad (-2,0), (0,2)$ (1)

$f''(x) = \dfrac{8}{x^3}$ (1)

$f'' > 0 \to$ alulról konvex (2) $\quad x > 0$ $\quad (0,\infty)$ (1)

$f'' < 0 \to$ alulról konkáv $\quad x < 0$ $\quad$ ~~$(0,\infty)$~~ $(-\infty, 0)$ (1)