INFO, AN2. 3. PZH. | -1- | 2017. 05. 16.

α VARIÁNS

1, $f(x,y) = \arcsin(x^2 y)$; $P\left(\frac{1}{2}, 2\right)$; $f(P) = \arcsin\frac{1}{2} = \underline{\underline{\frac{\pi}{6}}}$ (2)

a, [10]

$f'_x(x,y) = \dfrac{2xy}{\sqrt{1-x^4y^2}}$; (2) $\quad f'_x(P) = \dfrac{2}{\sqrt{1-\frac{1}{4}}} = \underline{\underline{\dfrac{4}{\sqrt{3}}}}$ (1)

$f'_y(x,y) = \dfrac{x^2}{\sqrt{1-x^4y^2}}$; (2) $\quad f'_y(P) = \dfrac{1/4}{\sqrt{1-\frac{1}{4}}} = \underline{\underline{\dfrac{1}{2\sqrt{3}}}}$ (1)

Érintő sík:

$$z - z_0 = f'_x \cdot (x-x_0) + f'_y (y-y_0) \implies \underline{\underline{z - \frac{\pi}{6} = \frac{4}{\sqrt{3}}\left(x-\frac{1}{2}\right) + \frac{1}{2\sqrt{3}}(y-2)}}$$ (2)

b, [6] A max. átlag: $\|\operatorname{grad} f(P)\| = \sqrt{f'^2_x(P) + f'^2_y(P)} = \sqrt{\frac{16}{3} + \frac{1}{12}} = \underline{\underline{\sqrt{\frac{65}{12}}}}$ (3)

$$\underline{e} = \frac{\operatorname{grad} f(P)}{\|\operatorname{grad} f(P)\|} = \left(\sqrt{\frac{12}{65}} \cdot \frac{4}{\sqrt{3}}, \sqrt{\frac{12}{65}} \cdot \frac{1}{2\sqrt{3}}\right) = \underline{\underline{\left(\frac{8}{\sqrt{65}}, \frac{1}{\sqrt{65}}\right)}}$$ (3)

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2, [16] $g(x,y) = \dfrac{f(x^2+3y)}{y}$

$g'_x(x,y) = \dfrac{2x}{y} f'(x^2+3y)$ (2); $\quad g'_y(x,y) = \dfrac{3f'(x^2+3y) \cdot y - f(x^2+3y)}{y^2}$ (2)

$g''_{xy}(x,y) \overset{(2)}{=} g''_{yx}(x,y) = \dfrac{-2x}{y^2} f'(x^2+3y) + \dfrac{6x}{y} f''(x^2+3y)$ (3)

$g''_{xx}(x,y) = \dfrac{2}{y} f'(x^2+3y) + \dfrac{4x^2}{y} f''(x^2+3y)$ (3)

$g''_{yy}(x,y) = \dfrac{\partial}{\partial y}\left(\dfrac{3}{y} f'(x^2+3y) - \dfrac{1}{y^2} f(x^2+3y)\right) =$

$= \dfrac{-3}{y^2} f'(x^2+3y) + \dfrac{9}{y} f''(x^2+3y) + \dfrac{2}{y^3} f(x^2+3y) - \dfrac{3}{y^2} f'(x^2+3y) =$

$= \dfrac{2}{y^3} f(x^2+3y) - \dfrac{6}{y^2} f'(x^2+3y) + \dfrac{9}{y} f''(x^2+3y)$ (4)

3, $\boxed{17}$ $f(x,y) = (2x-y)^2 + 4x^3 - 6y$

$$\left.\begin{aligned} f'_x(x,y) &= 4(2x-y) + 12x^2 = 0 \quad (2) \\ f'_y(x,y) &= -2(2x-y) - 6 = 0 \quad (2) \end{aligned}\right\} \Rightarrow \begin{aligned} &12x^2 + 8x - 4y = 0 \\ &3x^2 + 2x - y = 0 \\ &-2x + y - 3 = 0 \Rightarrow y = 3 + 2x \end{aligned}$$

$$3x^2 + 2x - (3+2x) = 0 \Rightarrow x^2 = 1 \Rightarrow \left.\begin{aligned} x_1 &= +1 \,;\; y_1 = 5 \\ x_2 &= -1 \,;\; y_2 = 1 \end{aligned}\right\} \quad (5)$$

$$|\underline{\underline{H}}(x,y)| = \begin{vmatrix} 8+24x & -4 \\ -4 & 2 \end{vmatrix} = 16 + 48x - 16 = +48x \quad (4)$$

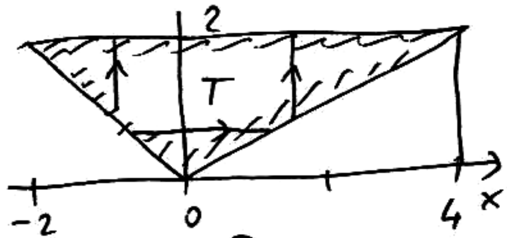
$(-1,1)$-ben; $\det \underline{\underline{H}}(-1,1) = -48 < 0 \Rightarrow$ Nyeregpont. (2)

$(+1,5)$-ben; $\left.\begin{aligned} &\det \underline{\underline{H}}(1,5) = 48 > 0 \\ &f''_{xx}(1,5) = 8 + 24 = 32 > 0 \end{aligned}\right\} \Rightarrow$ lokális minimum (2)

4, $\boxed{17}$

$$I = \int_{x=-2}^{0} \int_{y=-x}^{2} e^{y^2}\, dy\, dx + \int_{x=0}^{4} \int_{y=\frac{x}{2}}^{2} e^{y^2}\, dy\, dx =$$

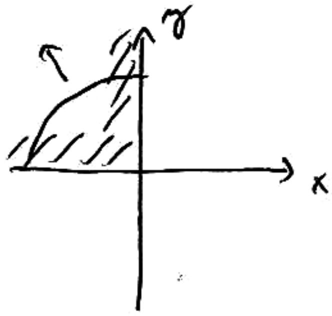
$$= \iint_T e^{-y^2}\, dT = \int_{y=0}^{2} \int_{x=-y}^{2y} e^{y^2}\, dx\, dy = \quad (6)$$



(6) (rajz)

$$= \int_{y=0}^{2} e^{y^2} \cdot 3y\, dy = \left[\frac{3}{2} e^{y^2}\right]_0^2 = \underline{\underline{\frac{3}{2}(e^4 - 1)}} \quad (5)$$

5, $\boxed{17}$



Improprius int.; a függvény az 0-ban ∞, a tartomány ∞.

$$T: \left.\begin{array}{l} 0 \le r < \infty \\ \frac{\pi}{2} \le \varphi \le \pi \end{array}\right\} (4)$$

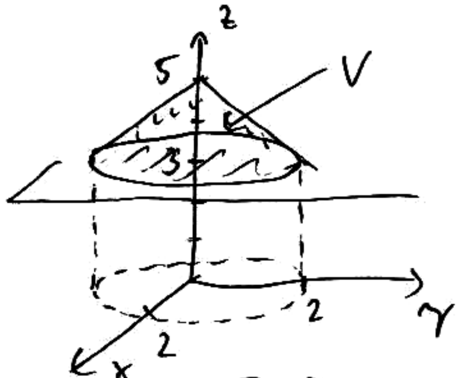
$$I = \int_{r=0}^{\infty} \int_{\varphi=\frac{\pi}{2}}^{\pi} \frac{1}{r + r^3}\, r \, d\varphi\, dr \overset{(4)}{=} \frac{\pi}{2} \lim_{\substack{r_1 \to 0+ \\ r_2 \to \infty}} \int_{r_1}^{r_2} \frac{1}{1+r^2}\, dr \overset{(3)}{=}$$

$$= \frac{\pi}{2} \cdot \lim_{\substack{r_1 \to 0+ \\ r_2 \to \infty}} \left[\operatorname{arctg} r \right]_{r_1}^{r_2} \overset{(3)}{=} \frac{\pi}{2} \left(\lim_{r_2 \to \infty} \operatorname{arctg} r_2 - \lim_{r_1 \to 0} \operatorname{arctg} r_1 \right) =$$

$$= \frac{\pi}{2} \cdot \left(\frac{\pi}{2} - 0 \right) = \underline{\underline{\frac{\pi^2}{4}}} \quad (3)$$

$\boxed{\text{Ha nincs limesz: } -3 \text{ p.}}$

6, $\boxed{17}$



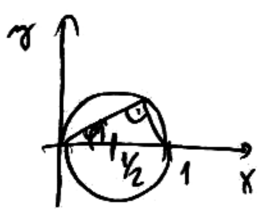
Henger-koordinátákkal:

$$V: \left.\begin{array}{l} 0 \le r \le 2 \\ 0 \le \varphi \le 2\pi \\ 3 \le z \le 5 - r \end{array}\right\} (5)$$

$$I = \int_{r=0}^{2} \int_{\varphi=0}^{2\pi} \int_{z=3}^{5-r} 2z \cdot r \, dz\, d\varphi\, dr \overset{(4)}{=} 2\pi \cdot \int_{r=0}^{2} r \underbrace{\left[z^2 \right]_3^{5-r}}_{r^2 - 10r + 16} dr \quad (2)\ (2) =$$

$$= 2\pi \int_{r=0}^{2} (r^3 - 10r^2 + 16r)\, dr = 2\pi \left[\frac{r^4}{4} - \frac{10r^3}{3} + 8r^2 \right]_0^2 = 2\pi \left(4 - \frac{80}{3} + 32 \right) \overset{(4)}{=} \underline{\underline{\frac{56\pi}{3}}}$$

$\boxed{\text{IMSC}}$ Henger-koordinátákkal



$$\left.\begin{array}{l} -\frac{\pi}{2} \le \varphi \le \frac{\pi}{2} \\ 0 \le r \le \cos\varphi \\ -\sqrt{1-r^2} \le z \le +\sqrt{1-r^2} \end{array}\right\} \quad V = \int_{\varphi=-\frac{\pi}{2}}^{\pi/2} \int_{r=0}^{\cos\varphi} \int_{z=-\sqrt{1-r^2}}^{\sqrt{1-r^2}} r \, dz\, dr\, d\varphi \overset{(6)}{=}$$

$$= \int_{\varphi=-\frac{\pi}{2}}^{\pi/2} \int_{r=0}^{\cos\varphi} 2r\sqrt{1-r^2}\, dr\, d\varphi = \int_{\varphi=-\pi/2}^{\pi/2} \left[-\frac{2}{3} (1-r^2)^{3/2} \right]_0^{\cos\varphi} d\varphi =$$

$$= \frac{-2}{3} \int_{\varphi=-\frac{\pi}{2}}^{\pi/2} \left(|\sin\varphi|^3 - 1 \right) d\varphi = \frac{4}{3} \int_{\varphi=0}^{\pi/2} (1 - \sin^3\varphi)\, d\varphi = \frac{4}{3} \int_0^{\pi/2} (1 - \sin\varphi + \cos^2\varphi \sin\varphi)\, d\varphi = \frac{4}{3} \left[\varphi + \cos\varphi - \frac{\cos^3\varphi}{3} \right]_0^{\pi/2}$$

$$= \frac{4}{3} \left(\frac{\pi}{2} - \frac{2}{3} \right) = \underline{\underline{\frac{2\pi}{3} - \frac{8}{9}}} \quad (6)$$