α VARIÁNS

1, $\boxed{15}$ $f(x,y) = \begin{cases} \dfrac{y^2}{x+y} & , \text{ha } x+y \neq 0 \\ 0 & , \text{ha } x+y = 0 \end{cases}$ $P(2,1)$ $\underline{e} = -\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$

a, $\boxed{8}$ $f'_x(x,y) = \dfrac{-y^2}{(x+y)^2}$ ②; $f'_x(P) = f'_x(2,1) = -\frac{1}{9}$ ①

$f'_y(x,y) = \dfrac{2y(x+y) - y^2}{(x+y)^2} = \dfrac{2xy + y^2}{(x+y)^2}$ ②; $f'_y(2,1) = \frac{5}{9}$ ①

$\dfrac{df(P)}{d\underline{e}} = \operatorname{grad} f(P) \cdot \underline{e} = \left(-\frac{1}{9}\right)\left(-\frac{3}{5}\right) + \frac{5}{9} \cdot \frac{4}{5} = \underline{\underline{\frac{23}{45}}}$ ① ①

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b, $\boxed{7}$ $\dfrac{df(\underline{0})}{d\underline{e}} = \lim_{t \to 0+} \dfrac{f(t \cdot \underline{e}) - f(\underline{0})}{t}$ ③ $= \lim_{t \to 0+} \dfrac{1}{t}\left(\dfrac{\left(\frac{4}{5}t\right)^2}{-\frac{3}{5}t + \frac{4}{5}t} - 0\right)$ ② $= \underline{\underline{\frac{16}{5}}}$ ②

2, $\boxed{15}$ $g(x,y) = f\left(\frac{x}{y}\right)$; $f \in C^2(\mathbb{R})$

$g'_x(x,y) = f'\left(\frac{x}{y}\right) \cdot \frac{1}{y}$ ②; $g'_y(x,y) = f'\left(\frac{x}{y}\right) \cdot \frac{-x}{y^2}$ ②

$g''_{xx}(x,y) = f''\left(\frac{x}{y}\right) \cdot \frac{1}{y^2}$ ③; $g''_{xy}(x,y) = g''_{yx}(x,y) = f''\left(\frac{x}{y}\right) \cdot \frac{-x}{y^3} + f'\left(\frac{x}{y}\right) \cdot \frac{-1}{y^2}$ ② ③

$g''_{yy}(x,y) = f''\left(\frac{x}{y}\right) \cdot \frac{x^2}{y^4} + f'\left(\frac{x}{y}\right) \cdot \frac{2x}{y^3}$ ③

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3 $\boxed{20}$ $f(x,y) = x^3 + y^3 - 3xy$

f diff.-ható; a lok. szélsőérték szükséges feltétele, hogy $\operatorname{grad} f = \underline{0}$

$$\left.\begin{aligned} f'_x(x,y) &= 3x^2 - 3y \overset{!}{=} 0 \quad ② \\ f'_y(x,y) &= 3y^2 - 3x \overset{!}{=} 0 \quad ② \end{aligned}\right\} \Rightarrow y = x^2 \longrightarrow x^4 = x \Rightarrow \begin{aligned} &x_1 = 0,\ y_1 = 0 \quad ③ \\ &x_2 = 1,\ y_2 = 1 \quad ③ \end{aligned}$$

$$|\underline{\underline{H}}(x,y)| = \begin{vmatrix} f''_{xx} & f''_{xy} \\ f''_{yx} & f''_{yy} \end{vmatrix} = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = 36xy - 9 \quad ⑤$$

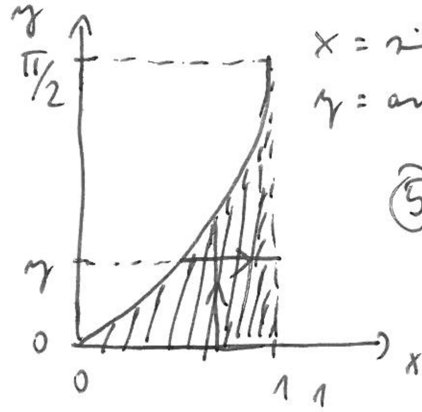
(0,0)-ban: $|\underline{\underline{H}}(0,0)| = -9 < 0 \Rightarrow$ itt nincs lokális széls. érték. ②

(1,1)-ben: $|\underline{\underline{H}}(1,1)| = 36 - 9 = 27 > 0 \Rightarrow$ itt van lok. széls. ért.

$f''_{xx}(1,1) = 6 > 0 \Rightarrow$ lokális minimum van. ③

4, $\boxed{15}$

$$I = \int_{y=0}^{\pi/2} \int_{x=\sin y}^{1} \frac{\cos y \cdot e^x}{x}\, dx\, dy =$$



$x = \sin y$

$y = \arcsin x$

⑤ (rajz)

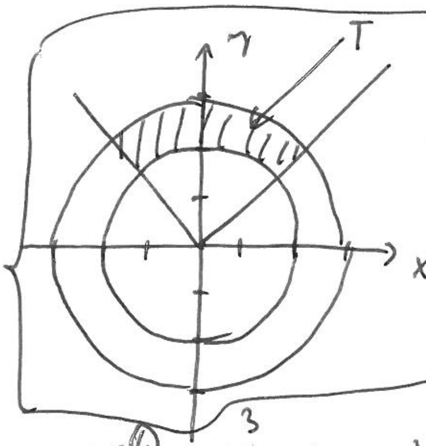
$$= \int_{x=0}^{1} \frac{e^x}{x} \int_{y=0}^{\arcsin x} \cos y\, dy\, dx \quad ⑤ \quad = \int_{x=0}^{1} \frac{e^x}{x} \left[\sin y\right]_{y=0}^{\arcsin x} dx = \int_{x=0}^{1} \frac{e^x}{\cancel{x}} \cdot \cancel{(x-0)}\, dx = \quad ③$$

$$= \left[e^x\right]_0^1 = \underline{\underline{e - 1}} \quad ②$$

5, $\boxed{15}$

$$T: \begin{cases} |x| \leq y \\ 4 \leq x^2 + y^2 \leq 9 \end{cases}$$

④



Síkbeli polár:

$T: \; 2 \leq r \leq 3$

$\frac{\pi}{4} \leq \varphi \leq \frac{3\pi}{4}$

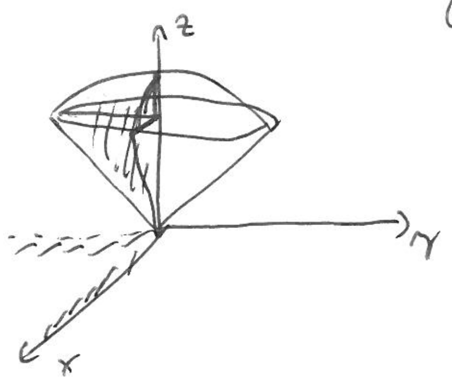
$$I = \iint_T x^2 y\, dT = \int_{r=2}^{3} \int_{\varphi=\frac{\pi}{4}}^{\frac{3\pi}{4}} r^2 \cos^2\varphi \; r \sin\varphi \; (r)\, d\varphi\, dr = \left(\int_{r=2}^{3} r^4\, dr\right) \cdot \left(\int_{\varphi=\frac{\pi}{4}}^{\frac{3\pi}{4}} \cos^2\varphi \sin\varphi\, d\varphi\right) = \quad ③$$

$$= \left[\frac{r^5}{5}\right]_{r=2}^{3} \cdot \left[-\frac{\cos^3\varphi}{3}\right]_{\varphi=\frac{\pi}{4}}^{3\pi/4} = \underline{\underline{\frac{3^5 - 2^5}{5} \cdot \frac{2}{3} \cdot \left(\frac{1}{\sqrt{2}}\right)^3}} = \frac{3^5 - 2^5}{5 \cdot 3 \cdot \sqrt{2}}$$

② ②

6, $\boxed{20}$

$$V\begin{cases} 0\le x,\quad y\le 0\\ x^2+y^2+z^2\le 1\\ \sqrt{x^2+y^2}\le z\end{cases}$$



Gömbi polárban:

$$\left.\begin{aligned} 0\le\ & r\le 1\\ 0\le\ & \vartheta\le \tfrac{\pi}{4}\\ \tfrac{3\pi}{2}\le\ & \varphi\le 2\pi\end{aligned}\right\}\ (6)$$

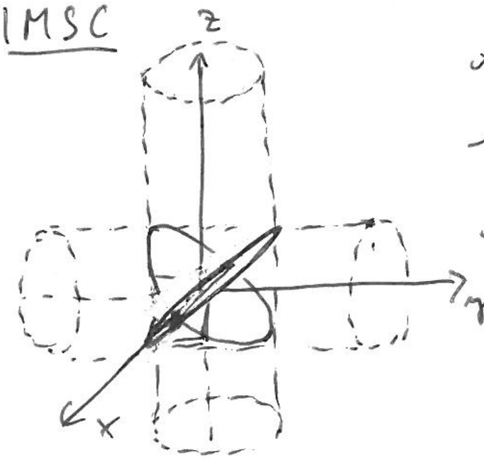
$$\iiint_V x^2yz\,dV=\int_{r=0}^{1}\int_{\vartheta=0}^{\pi/4}\int_{\varphi=\frac{3\pi}{2}}^{2\pi}\underbrace{(r\sin\vartheta\cos\varphi)^2}_{x}\underbrace{(r\sin\vartheta\sin\varphi)}_{y}\underbrace{(r\cos\vartheta)}_{z}\underbrace{r^2\sin\vartheta}_{|J|}\,d\varphi\,d\vartheta\,dr=\quad(4)$$

$$=\left(\int_{r=0}^{1}r^6\,dr\right)\cdot\left(\int_{\vartheta=0}^{\pi/4}\sin^4\vartheta\cos\vartheta\,d\vartheta\right)\cdot\left(\int_{\varphi=\frac{3\pi}{2}}^{2\pi}\cos^2\varphi\sin\varphi\,d\varphi\right)=\quad(3)$$

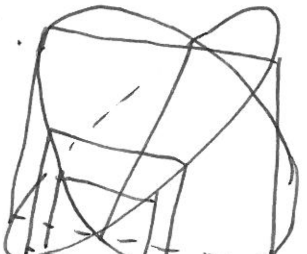
$$=\left[\frac{r^7}{7}\right]_0^1\cdot\left[\frac{\sin^5\vartheta}{5}\right]_0^{\pi/4}\cdot\left[-\frac{\cos^3\varphi}{3}\right]_{\varphi=\frac{3\pi}{2}}^{2\pi}=\frac{1}{7}\cdot\frac{1}{5\cdot4\cdot\sqrt2}\cdot\left(\frac{-1}{3}\right)\quad(7)$$

— o —

IMSC



A két henger palástja két ellipszisben metszi egymást, melyek síkjai merőlegesek.
A metszet alakzat x-re merőleges síkmetszetei → y négyzetek.



Descartes rendszerben számolunk:

$$V=\int_{x=-1}^{+1}\int_{y=-\sqrt{1-x^2}}^{+\sqrt{1-x^2}}\int_{z=-\sqrt{1-x^2}}^{+\sqrt{1-x^2}}1\,dz\,dy\,dx\quad(6)\quad=\int_{x=-1}^{+1}\int_{y=-\sqrt{1-x^2}}^{+\sqrt{1-x^2}}2\sqrt{1-x^2}\,dy\,dx=$$

$$=2\int_{x=-1}^{+1}2\cdot\sqrt{1-x^2}\cdot\sqrt{1-x^2}\,dx=4\int_{x=-1}^{1}(1-x^2)\,dx=4\left[x-\frac{x^3}{3}\right]_{-1}^{1}=8\left(1-\frac13\right)=\underline{\underline{\frac{16}{3}}}\quad(6)$$

Integrál felírása: 6p.; kiszámolás: 6p. (Ezeket lehet még bontani) β variáns (Csak végeredmények, részletes pontozás az α alapján)

1, a, $f'_x(x,y)=\frac{2x(x-y)-x^2}{(x-y)^2}$ (2) ; $f'_x(1,-2)=\frac{5}{9}$ (1) ; $f'_y(x,y)=\frac{x^2}{(x-y)^2}$ (2) ; $f'_y(1,-2)=\frac{1}{9}$ (1)

$\boxed{8}$

$$\frac{df(P)}{d\underline{e}}=\begin{bmatrix}5/9\\1/9\end{bmatrix}\cdot\begin{bmatrix}3/5\\4/5\end{bmatrix} \text{ (1) } =\frac{15+4}{45}=\frac{19}{45} \text{ (1)}$$

b, $\boxed{7}$

$$\frac{df(\underline{0})}{d\underline{e}}=\lim_{t\to 0+}\frac{1}{t}\left(\frac{(3/5\,t)^2}{3/5\,t-4/5\,t}\right)=-\frac{9}{5}$$

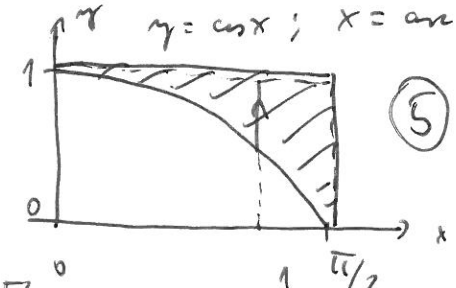
2, α_1-ből $x\leftrightarrow y$ cserével kapható meg.

3, $f_\beta=-f_\alpha$, tehát $(0,0)$-ban itt is nyeregpont van, de $(1,1)$-ben lokális maximum van.

4, $\boxed{15}$

$$I=\int_{x=0}^{\pi/2}\int_{y=\cos x}^{1}\frac{\sin x\cdot e^y}{y}\,dy\,dx=$$

$y=\cos x$; $x=\arccos y$

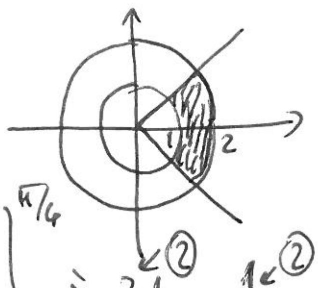


(5)

$$=\int_{y=0}^{1}\frac{e^y}{y}\int_{x=\arccos y}^{\pi/2}\sin x\,dx\,dy \text{ (5) } =\int_{y=0}^{1}\frac{e^y}{y}\underbrace{\left[-\cos x\right]_{x=\arccos y}^{\pi/2}}_{-0+y} \text{ (3) }\,dy=\int_0^1 e^y\,dy=e-1 \text{ (2)}$$

5, $\boxed{15}$

$$\underbrace{\int_{r=1}^{2}\int_{\varphi=-\frac{\pi}{4}}^{\pi/4}}_{(4)}\overbrace{r\cos\varphi\, r^2\sin^2\varphi\, r}^{(4)}\,d\varphi\,dr=$$



$$=\left(\int_1^2 r^4\,dr\right)\cdot\left(\int_{\varphi=-\frac{\pi}{4}}^{\pi/4}\cos\varphi\sin^2\varphi\,d\varphi\right) \text{ (3) } =\frac{2^5-1}{5}\left[\frac{\sin^3\varphi}{3}\right]_{-\frac{\pi}{4}}^{\pi/4}=\frac{31}{5}\text{ (2) }\cdot\frac{1}{3\sqrt{2}}\text{ (2)}$$

6, $\boxed{20}$

$$\int_{r=0}^{1}\int_{\vartheta=0}^{\pi/4}\int_{\varphi=\pi/2}^{\pi} r^6\sin^4\vartheta\cos\vartheta\cos\varphi\sin^2\varphi\,d\varphi\,d\vartheta\,dr= \text{ (10)}$$

$$=\left[\frac{r^7}{7}\right]_0^1\cdot\left[\frac{\sin^5\vartheta}{5}\right]_0^{\pi/4}\cdot\left[\frac{\sin^3\varphi}{3}\right]_{\pi/2}^{\pi}=\frac{1}{7}\cdot\frac{1}{5\cdot4\cdot\sqrt{2}}\cdot\left(\frac{-1}{3}\right) \text{ (10)}$$